SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

Year 11

2 Unit

July 2010

Common Test

Time allowed: 70 mins

Name: ______________________

Teacher: ______________________

Instructions:

- Begin each question on a new page
- Marks shown are approximate and may be varied
- Show necessary working
- Full marks may not be awarded if working is poorly set out or difficult to read

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
7	/7	/8	/7	/8	/8	/7	/8	/60

QUESTION 1 **MARKS**

(a) Solve for x: $\dfrac{3}{x-2} = \dfrac{4}{2x+5}$. **1**

(b) Express $0.6\dot{8}\dot{1}$ as a fraction in simplest terms. **2**

(c) Use your calculator to find the value of $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$ correct to 3 significant figures. **2**

(d) Simplify $\dfrac{2x^3 + 128}{x^2 - 9} \div \dfrac{x+4}{x+3}$. **2**

QUESTION 2 *(Start a new page)*

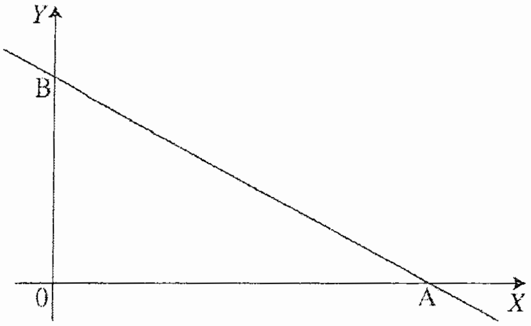
(a) Solve $|7 - 3x| = 4$. **2**

(b) The lines $ax + 2y = 6$ and $4y = bx - 9$ are parallel. Find the value of $\dfrac{a}{b}$. **3**

(c) Solve $6x^2 = x + 2$. **2**

QUESTION 3 *(Start a new page)*

(a)



B is the point (0, 6) and A is a point on the x-axis.

(i) If $AB = 10$ units, find the coordinates of A. **1**

(ii) Find the coordinates of M, the mid-point of AB. **1**

(iii) Show that AB has equation $3x + 4y - 24 = 0$. **2**

(iv) Find the shortest distance from 0 to the line AB. **2**

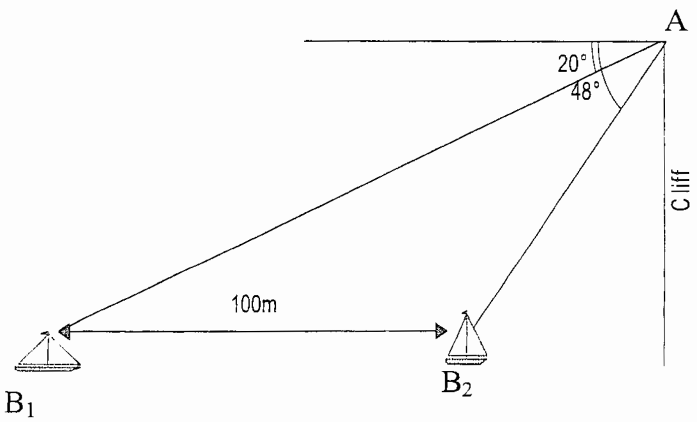
(b) Solve the simultaneous equations $2x - 3y - 12 = 0$ and $5x + 2y - 11 = 0$ **2**

QUESTION 4 *(Start a new page)*

(a) From the top of a cliff an observer measures the angles of depression of two boats on a lake, one behind the other in a straight line to be 20° and 48°. The boats are exactly 100 m apart.

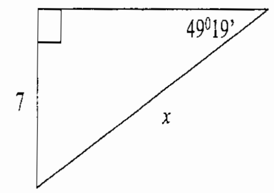
i. Find A to B_2 **2**

ii. Find the height of the top of the cliff above sea level (to the nearest metre). **2**



(b) Given $f(x) = 5 - x^2$, find $f(-2)$ **1**

(c) Evaluate x to one decimal place. **2**



QUESTION 5 *(Start a new page)*

(a) State the domain and range of the function $y = -\sqrt{16 - x^2}$. **2**

(b) Given $\sin\theta = \dfrac{\sqrt{3}}{10}$ and $\tan\theta < 0$, find the exact values of $\cos\theta$ and $\cot\theta$. **2**

(c) Solve $\sin^2\theta = \dfrac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$. **2**

(d) $4^{x+2} \times 2^{2x-3} = 8^x$ **2**

QUESTION 6 *(Start a new page)*

(a) If $\sin \alpha = 0.6$ and $0^{\circ} < \alpha < 90^{\circ}$ write down the exact value of:-

i. $\tan(180^{\circ} - \alpha)$ **1**

(b) Find the exact value of $\sin 225^{\circ}$ **1**

(c) A function is defined by the rule:

$$f(x) = \begin{cases} x+2, x \geq 0 \\ \dfrac{1}{x}, x < 0 \end{cases}$$

Find:

(i) $f(3)$ (ii) $f(-2)$ **2**

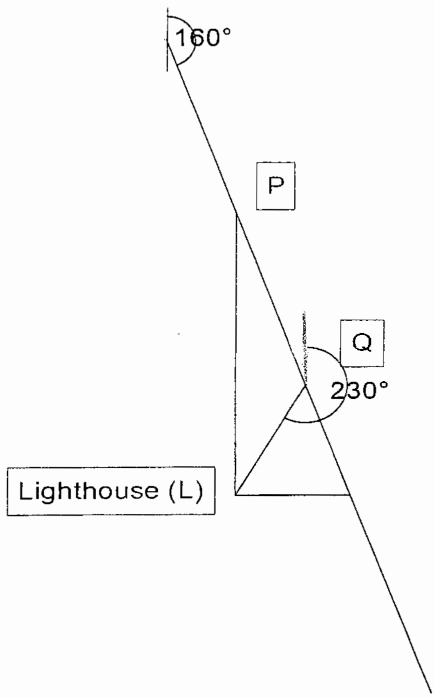
(d) Solve for $-180^{0} \leq \theta \leq 180^{0}$

i. $\sin \theta = -\dfrac{\sqrt{3}}{2}$ **2**

ii. $\tan \theta + 1 = 0$ **2**

QUESTION 7 *(Start a new page)*

A boat sailing on a bearing of 160° at 12 km/h arrives at a point P due north of a lighthouse L at precisely noon. Forty minutes later the boat is at a point Q where the lighthouse is on a bearing of 230°.



(i) Copy the diagram onto your book and find the distance PQ. **1**

(ii) Calculate the size of angle PQL. **1**

(iii) Hence calculate the distance PL to the nearest 100 m. **2**

(iv) Find the time to the nearest minute, at which the lighthouse will be due west of the boat. **3**

QUESTION 8 *(Start a new page)*

(a) Prove the identity $(\sec\theta - \cos\theta)^2 = \tan^2\theta - \sin^2\theta$. **2**

(b) If $x + y = 1$ show that $(x^2 - y^2)^2 + xy = x^3 + y^3$. **2**

(c) (i) If $u = 3 + \sqrt{7}$ show that $u + \dfrac{2}{u} = 6$. **2**

(ii) Hence evaluate $u^2 + \dfrac{4}{u^2}$. **2**

CORRECTED

SOLUTIONS

QUESTION 1	QUESTION 2
(a) $\frac{3}{x-2} = \frac{4}{2x+5}$ $3(2x+5) = 4(x-2)$ $6x + 15 = 4x - 8$ $2x = -23$ $x = \frac{-23}{2}$	**(a)** $\lvert 7 - 3x \rvert = 4$ $7 - 3x = 4$ or $-7 + 3x = 4$ $3x = 3$ $3x = 11$ $x = 1$ $x = \frac{11}{3}$
(b) Let $0.6\overline{81} = x$ Then $10x = 6.818181818181\ldots$ $1000x = 681.8181818181\ldots\ldots$ $990x = 675$ $x = \frac{675}{990}$ $x = \frac{15}{22}$ (c) 0.040 993 $\therefore$ 0.0410 3 significant figures	(b) $ax + 2y = 6$ $2y = -ax + 6$ $y = -\frac{ax}{2} + 3$ $\therefore$ Gradient $= -\frac{a}{2}$ $4y = bx - 9$ $y = \frac{bx}{4} - \frac{9}{4}$ $\therefore$ Gradient $= \frac{b}{4}$ Since lines are parallel, gradients are equal. $\therefore\ -\frac{a}{2} = \frac{b}{4}$ $\therefore\ \frac{a}{b} = \frac{-2}{4} = \frac{-1}{2}$
(d) $\frac{2x^3+128}{x^2-9} \div \frac{x+4}{x+3}$ $= \frac{2(x^3+64)}{x^2-9} \times \frac{x+3}{x+4}$ $= \frac{2(x+4)(x^2-4x+16)(x+3)}{(x-3)(x+3)(x+4)}$ $= \frac{2(x^2-4x+16)}{x-3}$	(c) $6x^2 = x + 2$ $6x^2 - x - 2 = 0$ $(3x-2)(2x+1) = 0$ $3x - 2 = 0$ or $2x + 1 = 0$ $x = \frac{2}{3}$ or $x = \frac{-1}{2}$

QUESTION 3

(i)

$AB^2 = OA^2 + OB^2$ *(Pythagoras' Theorem)*

$100^2 = OA^2 + 36$

$OA^2 = 64$

$OA = 8$

$\therefore$ Coordinates of A are (8, 0)

(ii)

$x_M = \frac{8+0}{2}$ $\quad$ $y_M = \frac{0+6}{2}$

$\therefore M(4, 3)$

(iii) Equation of AB: $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\frac{y-0}{x-8} = \frac{6-0}{0-8}$

$\therefore y = -\frac{6}{8}(x-8)$

$8y = -6x + 48$

$3x + 4y - 24 = 0$

(iv) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

$= \left| \frac{-24}{\sqrt{9+16}} \right|$

$= \frac{24}{5}u$

$2x - 3y - 12 = 0 \ldots\ldots\ldots(1)\ldots\times 5$

$5x + 2y - 11 = 0 \ldots\ldots\ldots(2)\ldots\times 2$

$10x - 15y - 60 = 0 \ldots..(3)$

$-(10x + 4y - 22 = 0)$

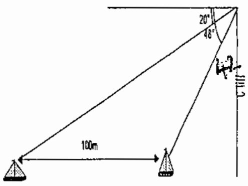
$-19y - 38 = 0$

$y = -2 \quad sub\ldots\ldots(1)$

$\therefore x = 3$

QUESTION 4

(a)



i

$x \div \sin 20° = 100 \div \sin 28°$

$x = 100 x \sin 20° \div \sin 28°$

$x = 72.85\ldots$

ii

$\cos 42 = cliff \div 72.85$

$cliff = \cos 42 \; x \; 72.85\ldots$

cliff = 54.138 m

(b)

$f(x) = 5 - x^2$

$f(-2) = 1$

(c)

$\sin \; 49° \, 19' = \frac{7}{x}$

$x = \frac{7}{\sin 49° 19'}$

$x = 9.23 \ldots$

$x = 9.2$

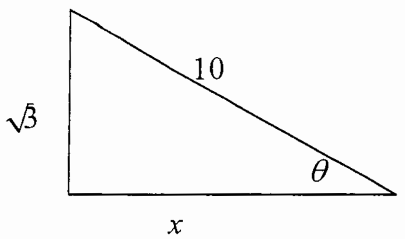
QUESTION 5

(a)

Domain: $-4 \leq x \leq 4$

Range: $-4 \leq y \leq 0$

(b)



$$(\sqrt{3})^2 + x^2 = 10^2$$

$$x = \sqrt{97}$$

and $\tan\,\theta < 0$, $\therefore$ $2nd\ quadrant$

$$\cos\theta = \frac{-\sqrt{97}}{10}$$

$$\cot\,\theta = \frac{-\sqrt{97}}{\sqrt{3}}$$

(c)

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ\ 135^\circ\ 225^\circ\ 315^\circ$$

(d)

$$4^{x+2}\ x\ 2^{2x-3} = 8^x$$

$$2^{2x+4}\ x\ 2^{2x-3} = 2^{3x}$$

Equating coefficents

$$2x+4 + 2x - 3 = 3x$$

$$4x + 1 = 3x$$

$$x = -1$$

QUESTION 6

(a) $\alpha = -0.75$

(b)

$$\sin 225^\circ = \sin(180^\circ + 45^\circ)$$

$$= -\frac{1}{\sqrt{2}}$$

(c)

$$f(3) = 5$$

$$f(-2) = -\frac{1}{2}$$

(d) (i)

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = -60^\circ\ -120^\circ$$

$$\tan\theta + 1 = 0$$

$$\theta = 135^\circ\ -45^\circ$$

QUESTION 7 If the bearing of the lighthouse, L, from Q is 230°, then the bearing of the point Q from the lighthouse is (230° – 180°) = 50°. The boat travels 12 km/h; so in 40 minutes it covers 8 km.	**QUESTION 8** (a) LHS $= (\sec\theta - \cos\theta)^2$ $= \sec^2\theta - 2\sec\theta\cos\theta + \cos^2\theta$ $= \sec^2\theta - 2 + \cos^2\theta$ $= \sec^2\theta - 1 + \cos^2\theta - 1$ $= \sec^2\theta - 1 - (1 - \cos^2\theta)$ $= \tan^2\theta - \sin^2\theta$ = RHS
(ii) From $\triangle PQL$, angle $PQL = 180 - (50 + 20) = 110°$.	**(b)** LHS $= (x^2 - y^2)^2 + xy$ $= [(x-y)(x+y)]^2 + xy$ $= (x-y)^2 + xy$ $\qquad (x+y) = 1$ $= x^2 - 2xy + y^2 + xy$ $= x^2 - xy + y^2$ RHS $= x^3 + y^3$ $= (x+y)(x^2 - xy + y^2)$ $= x^2 - xy + y^2$ $\qquad (x+y) = 1$ = LHS
(iii) Applying the sine rule in $\triangle PQL$: $\frac{PL}{\sin 110°} = \frac{8}{\sin 50°}$ $PL = \frac{8\sin 110°}{\sin 50°}$ = 9.8 km to the nearest 100 m **(iv)** Let R be the point due east of the boat. In $\triangle PLR$: $\cos 20° = \frac{PL}{PR}$ $\therefore PR = \frac{PL}{\cos 20°}$	**(c)** (i) $u = 3 + \sqrt{7}$ $\therefore u + \frac{2}{u} = 3 + \sqrt{7} + \frac{2}{3+\sqrt{7}}$ $= \frac{(3+\sqrt{7})^2 + 2}{3+\sqrt{7}}$ $= \frac{9 + 6\sqrt{7} + 7 + 2}{3+\sqrt{7}}$ $= \frac{18 + 6\sqrt{7}}{3+\sqrt{7}}$ $= \frac{6(3+\sqrt{7})}{3+\sqrt{7}}$ $= 6$ (ii) $u + \frac{2}{u} = 6$ $\therefore (u + \frac{2}{u})^2 = 36$

$= \frac{9.8}{\cos 20°}$ ≈ 10.4 km $\therefore$ Time $= \frac{\text{distance}}{\text{speed}}$ $= \frac{10.4}{12}$ $= 0.8\dot{6}$ h $= 52$ min $\therefore$ Time taken is 52 min and actual time will be 12.52 p.m.	$u^2 + \frac{4}{u^2} + 4 = 36$ $u^2 + \frac{4}{u^2} = 32$