NAME:_____ TEACHER: File

SYDNEY TECHNICAL HIGH SCHOOL

2 Unit Mathematics

Preliminary HSC

Year 11

Assessment Task 2 **July, 2011**

Time Allowed: **70 minutes**

General Instructions:

- Each question attempted is to be started on a NEW PAGE, clearly marked with the number of the question, your name and class on the top right hand side of the page
- Write using black or blue pen
- APPROVED CALCULATORS may be used
- All necessary working should be shown. Marks may be deducted if working is poorly set out or difficult to read.

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	TOTAL
/8	/8	/8	/8	/8	/8	/8	/8	/64

Question 1 (8 marks)

a) Evaluate $(\frac{1}{5})^{-1.2}$ correct to 3 significant figures. 2

b) Find a *and* b such that $\frac{1}{\sqrt{5}-2} = a + b\sqrt{5}$ 2

c) Write the equation of the circle with centre (-3,4) and radius 6 units. 1

d) Solve $|2x + 3| = 9$. 2

e) Write the domain of the function $f(x) = \sqrt{6 - x}$. 1

Question 2 (8 marks) Start a new page

a) Solve $x^2 = 5x$. 2

b) Factorise $3x^2 + x - 2$. 1

c) Find the exact value of θ such that $2cos\theta = 1$ if $0 \leq \theta \leq 90°$. 2

d) i) Sketch $y = 9 - x^2$ showing x and y intercepts. 2

ii) Find the range of $y = 9 - x^2$. 1

Question 3 (8 marks) Start a new page

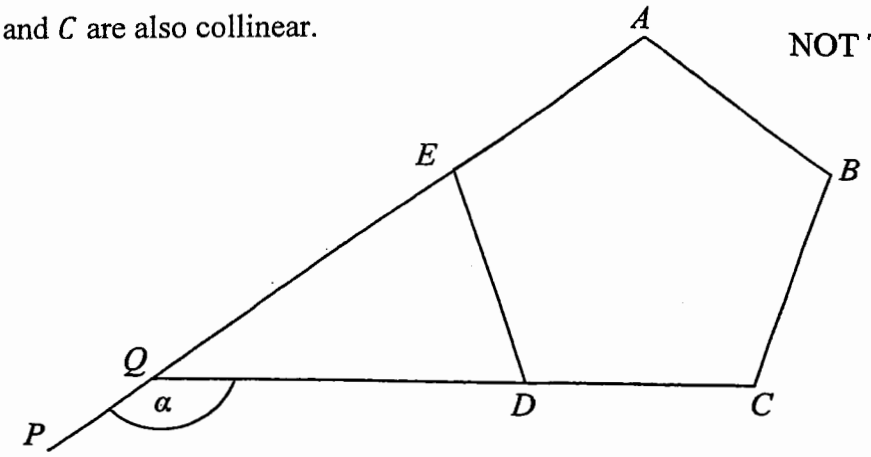
a) Let M be the midpoint of $(-1,4)$ and $(5,8)$. 3

Find the equation of the line through M with gradient $-\frac{1}{2}$ (write your answer in general form).

b) Prove that $\frac{sin\theta}{1-cos\theta} + \frac{sin\theta}{1+cos\theta} = 2cosec\ \theta$. 3

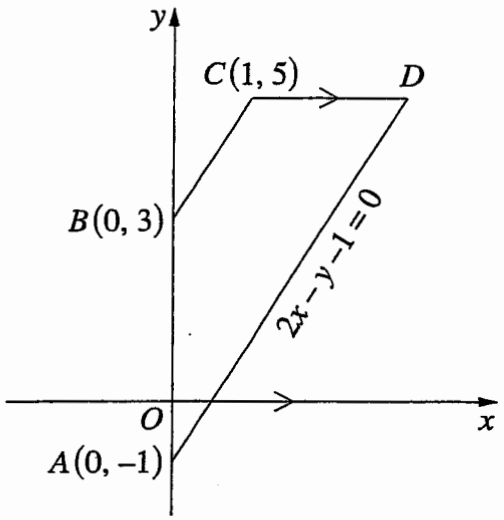
c) $ABCDE$ is a regular pentagon. The points P, Q, E and A are collinear. The points Q, D and C are also collinear.

NOT TO SCALE



Find the size of angle a giving reasons. 2

Question 4 (8 marks) Start a new page



NOT TO SCALE

In the diagram, $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 1 = 0$.

a) Show that $ABCD$ is a trapezium by showing that BC is parallel to AD. 2

b) The line CD is parallel to the x-axis. Find the coordinates of D. 1

c) Find the length of BC. 1

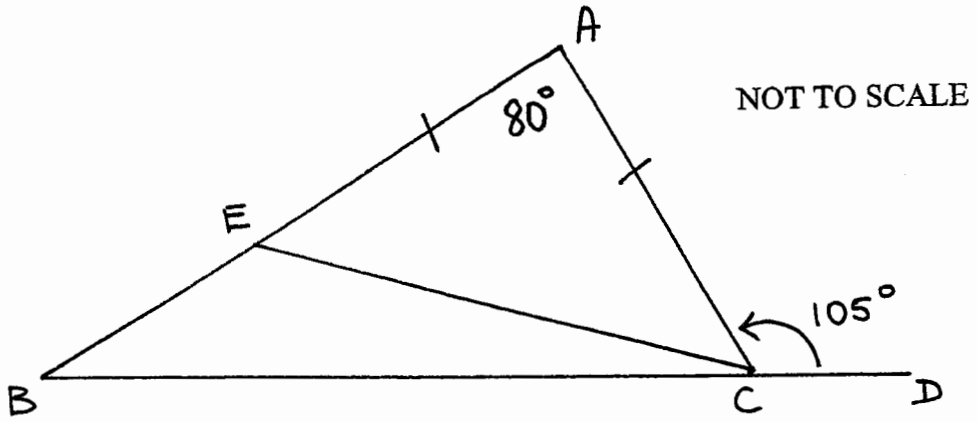
d) Show that the perpendicular distance from B to AD is $\frac{4}{\sqrt{5}}$. 2

e) Hence, or otherwise, find the area of the trapezium $ABCD$.
(simplify your answer fully) 2

Question 5 (8 marks) Start a new page

a) If $\sin(3x - 20°) = \cos(2x + 50°)$ find x. 1

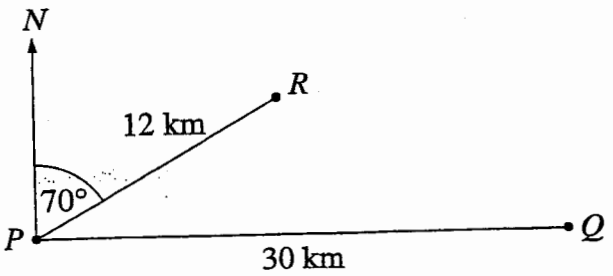
b) In the diagram below, $AE = AC, \hat{EAC} = 80°$ and $\hat{ACD} = 105°$.



3

Prove, with full reasoning, that $EB = EC$

c)



The diagram shows a point P which is 30 km due west of the point Q.

The point R is 12 km from P and has a bearing from P of 070°.

i) Find the distance of R from Q (to nearest km). 2

ii) Find the bearing of R from Q (to nearest degree). 2

Question 6 (8 marks) Start a new page

a) Find x and y such that $x + y - \sqrt{x - y} = 18 - 2\sqrt{3}$. 2

b) Solve the simultaneous equations

$4x - y = 9$

$3xy = -6$ 2

c) i) Sketch the function defined as follows

$$f(x) = \begin{cases} 0 \ if \ x \leq -3 \\ -1 \ if \ -3 < x < 0 \\ x^2 \ if \ x \geq 0 \end{cases}$$ 3

ii) Find $f(a^2)$ 1

Question 7 (8 marks) Start a new page

a) Solve $5x^2 - 12x + 4 < 0$. 2

b) If $sin\theta = \frac{-1}{\sqrt{13}}$ and $\tan\theta > 0$ find the <u>exact</u> value of $cos\theta$. 2

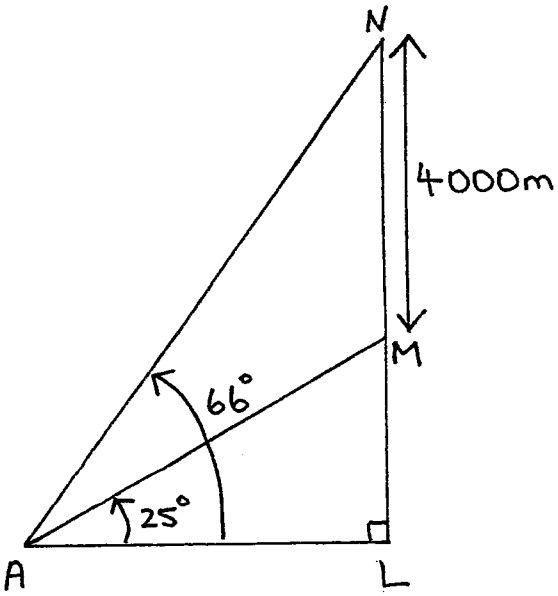
c) Solve in the domain $0 \leq \theta \leq 360°$

i) $2cos^2\theta = 1$ 2

ii) $(sin\theta - 1)(2sin\theta - 1) = 0$ 2

Question 8 (8 marks) Start a new page

a) A rocket launched vertically from L is observed from A. Soon after launch when at position M its angle of elevation is 25°. After it climbs 4000 metres from this position to N its angle of elevation is 66°.



i) Find $\hat{ANL}$ *and* $\hat{NAM}$. 2

ii) Find AM to the nearest metre. 2

iii) Find how far the observer is, horizontally from the launch pad. 2
(answer to nearest metre)

b) Solve $3sin\theta = 2cos\theta$ if $-180° \leq \theta \leq 180°$.

Write your solutions correct to the nearest minute. 2

2unit MATHS YEAR 11 2011 Task 2 (August)

Question 1

a) 6.90 (3 sig. fig)

b) $\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{1}$

$\therefore a = 2 \quad b = 1$

c) $(x+3)^2+(y-4)^2=36$

d) $2x+3=9 \qquad 2x+3=-9$

$2x=6 \qquad 2x=-12$

$x=3$ and $x=-6$

e) $6-x \geqslant 0$

$-x \geqslant -6$

D: $x \leqslant 6$

Question 2

a) $x^2-5x=0$

$x(x-5)=0 \quad \therefore x=0, \; x=5$

b) $3x^2+x-2$

$3x \times -2$, $x \times +1$

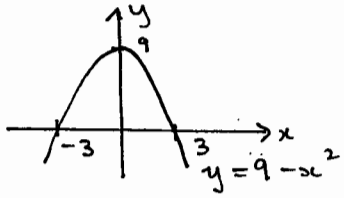
$(3x-2)(x+1)$

c) $2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\therefore \theta = 60°$

d) i)



ii) R: $y \leqslant 9$

Question 3

a) $M(2,6)$

$y-6 = -\frac{1}{2}(x-2)$

$2y-12 = -x+2$

$x+2y-14=0$

b) LHS $= \frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{\sin\theta(1+\cos\theta)+\sin\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{\sin\theta+\sin\theta\cos\theta+\sin\theta-\sin\theta\cos\theta}{1-\cos^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta}$

$= \frac{2}{\sin\theta}$

$= 2\operatorname{cosec}\theta$

$=$ RHS

c) $Q\hat{E}D = Q\hat{D}E = 72°$ (ext. angle of regular pentagon)

$\therefore \alpha = 144°$ (angle sum of triangle)

Question 4

a) $2x-y-1=0$

$\therefore y = 2x-1 \qquad m_{AD} = 2$

$m_{BC} = \frac{5-3}{1-0} = 2$

$\therefore m_{AD} = m_{BC} \; \therefore$ AD $\parallel$ BC

b) $D(3,5)$

c) $BC = \sqrt{(1-0)^2+(5-3)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$ units

d) $P = \left| \frac{2\times 0 - 1\times 3 - 1}{\sqrt{2^2+1^2}} \right|$

$= \left| \frac{-4}{\sqrt{5}} \right|$

$\therefore P = \frac{4}{\sqrt{5}}$

e) area ABCD $= \frac{1}{2} \cdot \frac{4}{\sqrt{5}} (BC + AD)$

$BC = \sqrt{5}$ from above

$AD = \sqrt{(3-0)^2+(5-(-1))^2}$

$= \sqrt{9+36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$\therefore$ Area $= \frac{1}{2} \cdot \frac{4}{\sqrt{5}} (\sqrt{5} + 3\sqrt{5})$

$= \frac{1}{2} \cdot \frac{4}{\sqrt{5}} (4\sqrt{5})$

$= 8$ unit^2

Question 5

a) $3x-20+(2x+50)=90$

$5x+30=90$

$5x=60$

$x=12°$

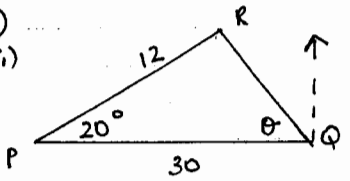
b) $E\hat{B}C = 25°$ (ext. angle of triangle)

$A\hat{C}E = 50°$ (opp equal sides isos. triangle)

$\therefore E\hat{C}B = 25°$ (angles on straight line)

$\therefore EB = EC$ (opp equal angles in isosceles triangle)

c) i)



$RQ^2 = 12^2+30^2-2\times 12\times 30\times \cos 20°$

RQ = 19km (to nearest km)

ii) $\frac{\sin\theta}{12} = \frac{\sin 20°}{19}$

$\sin\theta = \frac{12\sin 20°}{19}$

$\theta = 12.47°$

$\theta = 12°$ to nearest deg.

$\therefore$ bearing R from Q

N 78° W OR 282° T

(if using cosine Rule and 19km $\rightarrow$ 282° T)

Question 6

a) $\left.\begin{aligned} x+y &= 18 \\ x-y &= 12 \end{aligned}\right\}$

$2x = 30$

$\therefore x = 15 \qquad y = 3$

b) $4x-y=9$

$\therefore y = 4x-9$

$3x(4x-9) = -6$

$12x^2-27x+6=0$

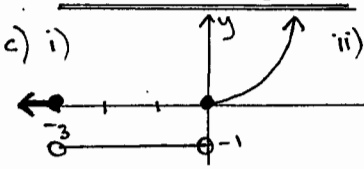
$4x^2-9x+2=0$

$4x \times -1$, $x \times -2$

$(4x-1)(x-2)=0$

$\therefore \left\{\begin{aligned} x &= \tfrac{1}{4} \\ y &= -8 \end{aligned}\right.$ and $\left\{\begin{aligned} x &= 2 \\ y &= -1 \end{aligned}\right.$

c) i)



ii) $f(a^2) = a^4$

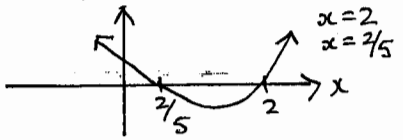
Question 7

a) $5x^2 - 12x + 4 < 0$

$5x \times -2$

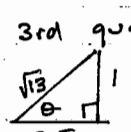
$x \times -2$

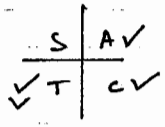
$(5x - 2)(x - 2) < 0$



$\frac{2}{5} < x < 2$

b) θ in 3rd quad





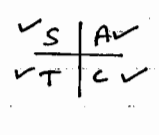
$\therefore \cos\theta = \frac{-2\sqrt{3}}{\sqrt{13}}$

c)

i) $2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

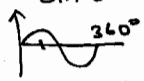
$\cos\theta = \pm\frac{1}{\sqrt{2}}$



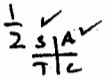
acute $\theta = 45°$

$\therefore \theta = 45°, 135°, 225°, 315°$

ii) $\sin\theta = 1$ & $\sin\theta = \frac{1}{2}$ ✓



acute $\theta = 30°$



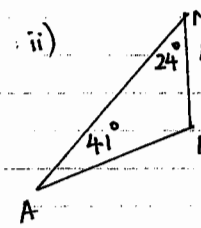
$\therefore \theta = 90°$ and $\theta = 30°, 150°$

Question 8

a) i) $\hat{ANL} = 24°$

$\hat{NAM} = 41°$

ii)

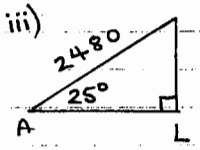


$\frac{AM}{\sin 24°} = \frac{4000}{\sin 41°}$

$AM = \frac{4000 \sin 24°}{\sin 41°}$

$AM = 2480$ to nearest m

iii)



$\cos 25° = \frac{AL}{2480}$

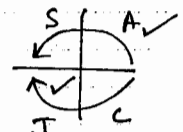
$AL = 2247.6$ m

$\therefore AL = 2248$ m (nearest metre)

b) $3\sin\theta = 2\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{2}{3}$

$\therefore \tan\theta = \frac{2}{3}$



$\therefore$ acute angle $33°41'$

$\therefore \theta = 33°41', -146°19'$