File

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 TERM 3

2013

Mathematics

General Instructions

- Working time – 75 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in Questions 6 to 13
- Start each question on a new page

Total marks – 70 marks

Section 1 – 5 marks

Attempt questions 1 -5
Allow about 7 minutes for this section

Section 2 – 65 marks

Attempt questions 6 to 13
Allow about 63 minutes for this section

Name: _____

Teacher: _____

Section I (5 marks)

Attempt questions 1 to 5.

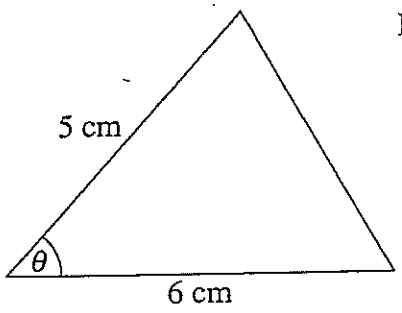
Allow approximately 7 minutes for this section.

Use the multiple choice answer sheet provided in your answer books.

Select the alternative A, B, C or D that best answers the question.

Fill in the response oval completely.

Question 1

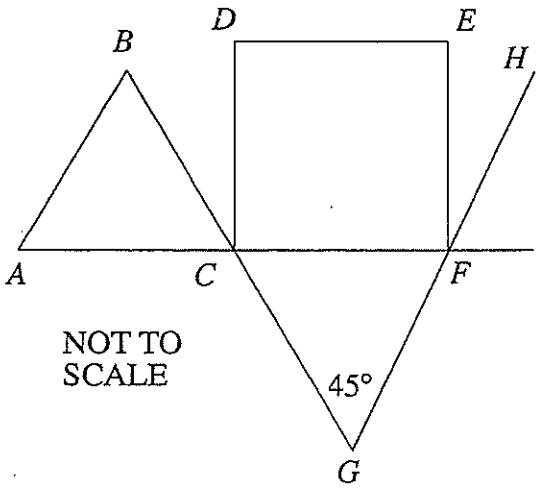


NOT TO SCALE

Find the value of $\sin\theta$ if the area of this triangle is 10 cm^2.

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 3

Question 2



NOT TO SCALE

ABC is an equilateral triangle.

$CDEF$ is a square.

BCG, ACF, and GFH are straight lines.

$\angle CGF = 45°$.

The size of $\angle EFH$ is

(A) 15°

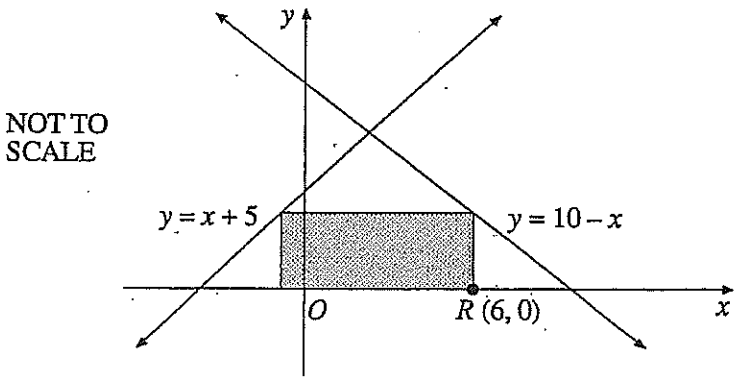
(B) $22\frac{1}{2}°$

(C) 30°

(D) 45°

Question 3

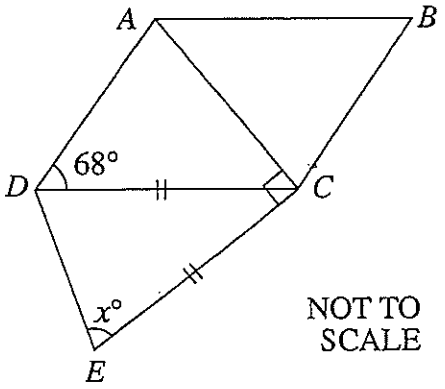
R is the point with coordinates (6, 0).



The area of the shaded rectangle in square units is

(A) 20 (B) 24 (C) 28 (D) 60

Question 4



$ABCD$ is a rhombus.

$DC = EC$.

$\angle ADC = 68°$ and $\angle ACE = 90°$.

The value of x is

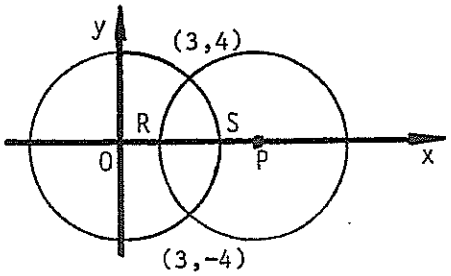
(A) 22

(B) 68

(C) 73

(D) 79

Question 5



O and P are centres of equal circles which intersect at (3,4) and (3,-4). The circles cut the x axis as shown at R and S.

RS = ?

A. 1 B. 2 C. 4 D. 5

Section Ⅱ - Attempt questions 6 – 13.
All answers and working to be done in your answer book.

Question 6 (8marks)

a) Simplify giving the answer to 2 significant figures **2**

$$\frac{4.7 + \sqrt{3.2}}{1 + 4.15^2}$$

b) Solve $-3 \leq 1 - 2x < 5$ **2**

c) If $(2 + \sqrt{3})^2 = a + \sqrt{b}$ Find the value of a and b **2**

d) Solve $x^2 \geq (x + 1)(x + 5)$ **2**

Question 7 (8 marks) (Start a new page)

a) Factorise fully

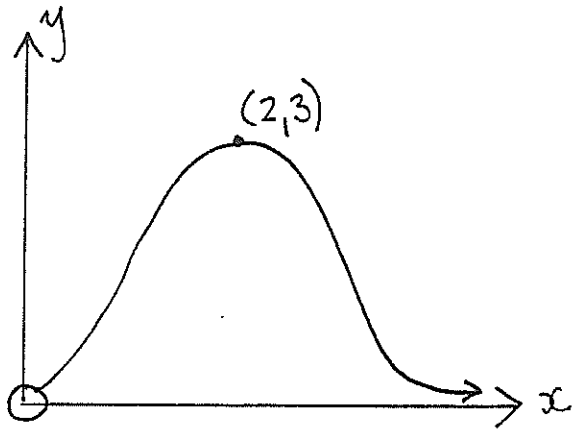
i. $8\,mn^2 + 6m^2n + 10mn$ **1**

ii. $5x^2 - 33x + 18$ **1**

iii. $125 - 27x^3$ **2**

b) If $\sin\theta = \frac{4}{7}$ and $\tan\theta < 0$ find the exact value of $\cos\theta$ **1**

c)



3

Consider the graph of $y = f\,(x)$

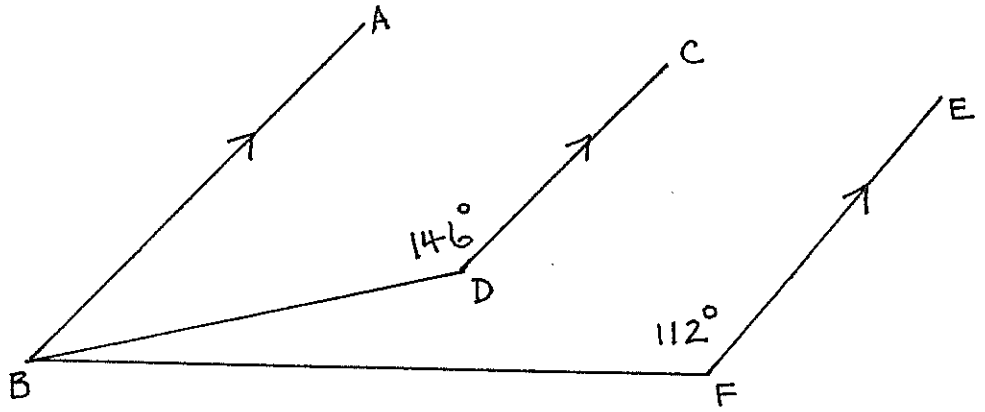
i. State its domain

ii. State its range

iii. Write down the equation (s) of any asymptotes

<u>Question 8</u> (9 marks) (Start a new page)

a)



3

In the diagram above AB$\|$ $CD \parallel EF$, $\angle BDC = 146°$ *and* $\angle EF\hat{B} = 112°$. *Show, giving reasons, that* BD *bisects* $\angle ABF$.

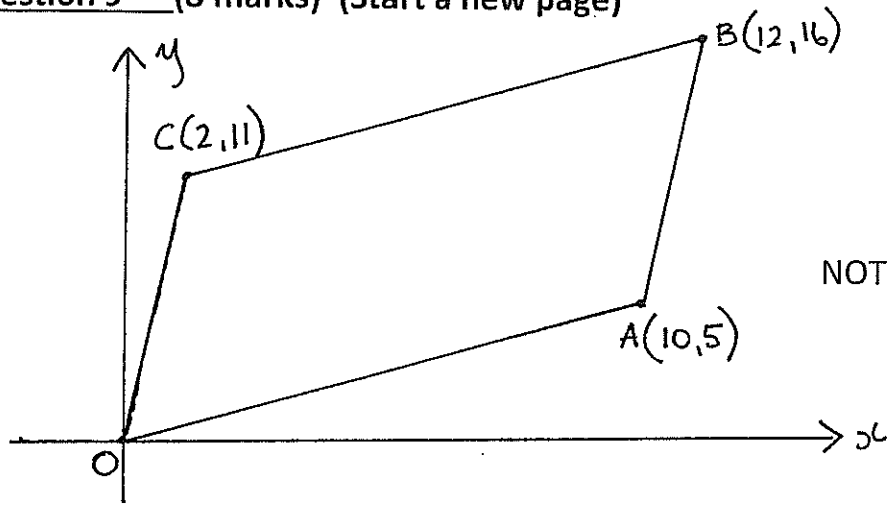
b) Write $\frac{1}{x+1} - \frac{1}{x-1}$ as a single fraction **2**

c) Solve for x

i. $\frac{5}{8}(x+4) = 4x - \frac{1}{2}$ **2**

ii. $x + \frac{2}{x} = 3$ **2**

Question 9 (8 marks) (Start a new page)



NOT TO SCALE

In the diagram above A, B and C are the points (10,5), (12,16) and (2,11) respectively

Copy the diagram into your answer sheets

a) Find the distance AC **1**

b) Find the midpoint of AC **1**

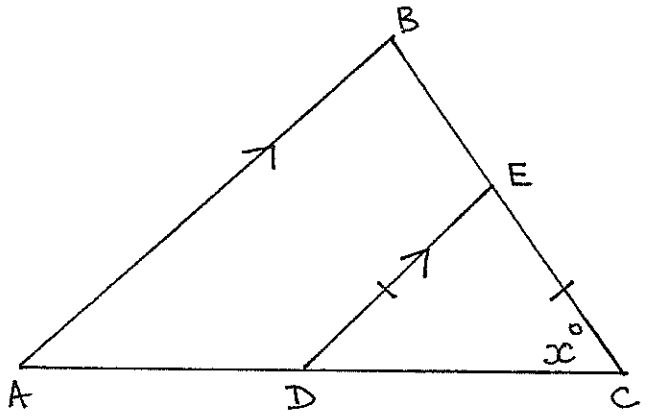
c) Show the OB ⊥ AC **2**

d) Find the midpoint of OB and explain why OABC is a rhombus **2**

e) Hence find the area of OABC **2**

Question 10 (8 marks) (Start a new page)

a)



In the diagram above AB ∥ DE and DE = EC. Let ∠ ECD = $x°$

i. Copy the diagram above onto your answer sheets

ii. Prove AB = BC (reasons required) **3**

b) i. Sketch $y = |x - 1|$ and $y = |x|$ on the same axes (Label each graph carefully) **2**

ii. Hence solve $|x - 1| = |x|$ **1**

c) Solve $|x + 3| = 2x - 1$ **2**

Question 11 (8 marks) (Start a new page)

a) Find the acute angle θ, correct to the nearest minute, given that $\tan\theta = 1\frac{1}{3}$ **2**

b) Simplify $\frac{\cos(90-\theta)}{\cos(180-\theta)}$ **2**

c) Solve $2sin\theta - 1 = 0\ for\ 0 \leq \theta \leq 360°$ **2**

d) Simplify $sec^2\theta - tan^2\theta$ **2**

Question 12 (8 marks) (Start a new page)

a) Prove the identity **3**

$$\frac{1}{1+sin\theta} + \frac{1}{1-sin\theta} = 2sec^2\theta$$

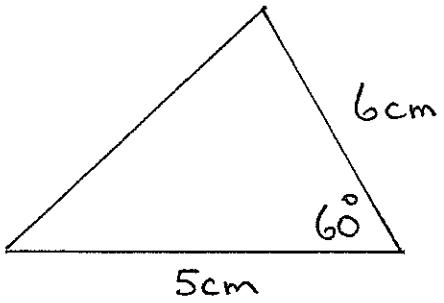
b) Solve the following trigonometric equations in the domain $0° \leq \theta \leq 360°$

i. $3cos^2\theta + 5cos = 0$ **3**

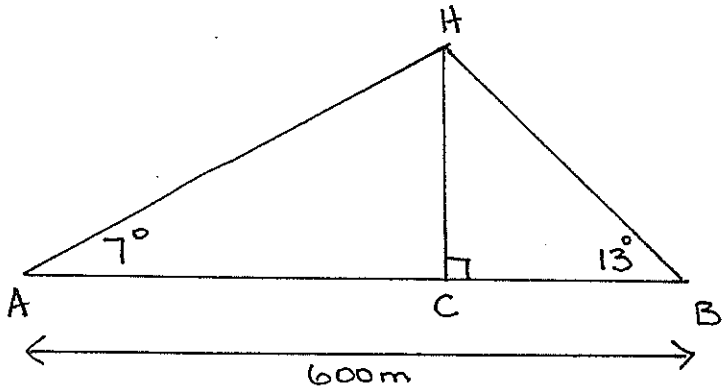
ii. $\sqrt{3}\sin\theta - \cos\theta = 0$ **2**

Question 13 (8marks) (Start a new page)

a) Find the exact area of the triangle below. 1



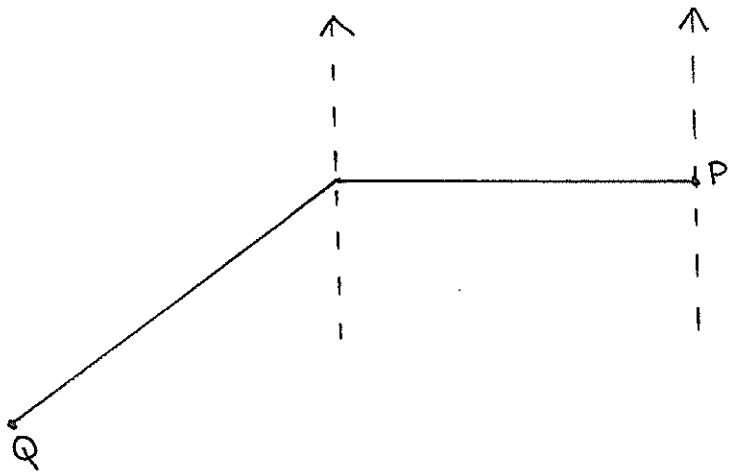
b) In the diagram below, a helicopter H is hovering above a straight, horizontal road AB of length 600 metres. The angles of elevation of H from A and B are 7° *and* 13° respectively. The point C lies on the road directly below H.



i. Use the sine rule to show that HB= $\frac{600 \sin 7°}{\sin 160°}$ 1

ii. Hence, find the height CH of the helicopter above the road, correct to the nearest metre. 1

c) A ship sails 24 km west of port P and then 17 km on a bearing of S 42°17′ W to port Q.



i. Copy the diagram above onto your answer sheets and show the information on it.

ii. Calculate the distance of Q from P to one decimal place. 2

iii. Calculate the TRUE bearing of the ship from P (to the nearest degree) 3

SOLUTIONS YR 11 TERM 3 (WEEK 2) MATHEMATICS 2 UNIT

Question 1 B

Question 2 A

Question 3 C

Question 4 C

Question 5 C

Question 6

a) 0.36 (2 sig. fig)

b) $-3 \leq 1-2x < 5$

$-3 \leq -2x \quad 1-2x < 5$

$2x \leq 4 \quad -4 < 2x$

$x \leq 2 \quad -2 < x$

$-2 < x \leq 2$

c) $(2+\sqrt{3})^2 = a+\sqrt{b}$

$4 + 4\sqrt{3} + 3 = a+\sqrt{b}$

$7 + \sqrt{48} = a+\sqrt{b}$

$a=7 \quad b=48$

d) $x^2 \geq (x+1)(x+5)$

$x^2 \geq x^2 + 6x + 5$

$-6x \geq 5$

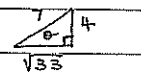
$x \leq -5/6$

Question 7

a) i) $2mn(4n+3m+5)$

ii) $(5x-3)(x-6)$

iii) $(5-3x)(25+15x+9x^2)$

b) 

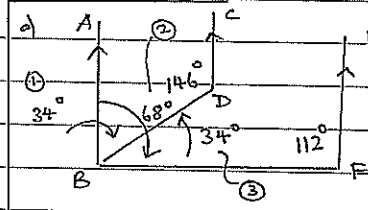
$\therefore \cos\theta = \frac{\sqrt{33}}{7}$

c) i) $D: x > 0$

ii) $R: 0 < y \leq 3$

iii) $y=0$ (asymptote)

Question 8

a) 

$A\hat{B}D = 34°$ (co-interior angles $AB \parallel CD$)

$A\hat{B}F = 68°$ ($AC \parallel EF$, co-interior angles)

$\therefore D\hat{B}F = 34°$

$\therefore$ BD bisects $A\hat{B}F$

b) $\frac{1}{x+1} - \frac{1}{x-1} = \frac{x-1-(x+1)}{(x+1)(x-1)}$

$= \frac{-2}{(x^2-1)}$

c) i) $\frac{5}{8}(x+4) = 4x - \frac{1}{2}$

$5(x+4) = 32x - 4$

$5x + 20 = 32x - 4$

$24 = 27x$

$x = \frac{8}{9}$

ii) $x + \frac{2}{x} = 3$

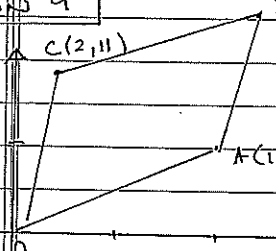
$x^2 + 2 = 3x$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$\therefore x = 2, \quad x = 1$

Question 9



a) $AC = \sqrt{(10-2)^2 + (5-11)^2}$

$= \sqrt{64+36}$

$= \sqrt{100}$

$AC = 10$ units

b) Midpt AC $(6, 8)$

c) $m_{OB} = \frac{16}{12} = \frac{4}{3}$

$m_{AC} = \frac{6}{-8} = \frac{3}{-4}$

$\frac{4}{3} \times \frac{3}{-4} = -1 \quad \therefore OB \perp AC$

d) midpt OB $(6, 8)$

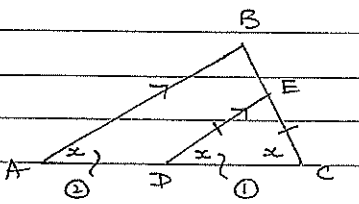
both diagonals bisected and meet at $90°$ $\therefore$ OABC $\rightarrow$ rhombus

e) $OB = \sqrt{12^2 + 16^2}$

$OB = 20$ units

$\therefore$ Area OABC $= \frac{20 \times 10}{2} = 100$ unit^2

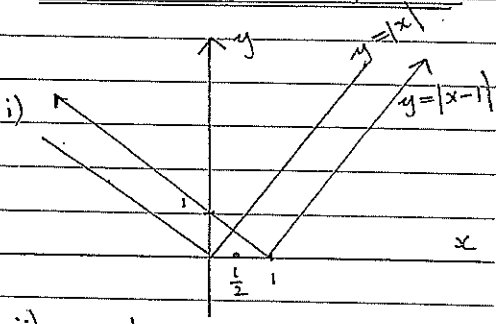
Question 10

a) 

$E\hat{D}C = E\hat{C}D$ (opposite equal sides in isosceles $\triangle DEC$)

$B\hat{A}D = E\hat{D}C = x$ (corresponding angles $AB \parallel DE$)

$\therefore AB = BC$ (opposite equal angles in isosceles $\triangle ABC$)

b) 

ii) $x = \frac{1}{2}$

c) $|x+3| = 2x-1$

$x+3 = 2x-1 \qquad x+3 = -(2x-1)$

$4 = x \qquad x+3 = -2x+1$

$3x = -2$

$x = -\frac{2}{3}$

check solutions:

$|4+3| = 8-1 \qquad \left|-\frac{2}{3}+3\right| = 2x-\frac{2}{3}-1$

$7 = 7 \qquad 2\frac{1}{3} \neq -2\frac{1}{3}$

$\therefore$ true $x=4$ only solution

Question 11

a) $\tan\theta = 1\frac{1}{3}$ $\quad \theta = 53^\circ 8'$

(nearest min required)

b) $\dfrac{\cos(90-\theta)}{\cos(180-\theta)} = \dfrac{\sin\theta}{-\cos\theta}$

$= -\tan\theta$

c) $2\sin\theta - 1 = 0$

$\therefore \sin\theta = \frac{1}{2}$ $\quad$ S | A ✓ ✓ / T | C

acute $\theta = 30^\circ$

$\therefore \theta = 150^\circ, 30^\circ$

d) $\sec^2\theta - \tan^2\theta$

$1 + \tan^2\theta - \tan^2\theta$

1

Question 12

a) $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

LHS $= \dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$= \dfrac{(1-\sin\theta)+(1+\sin\theta)}{1-\sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

$=$ RHS

b) i) $3\cos^2\theta + 5\cos\theta = 0$

$\cos\theta(3\cos\theta + 5) = 0$

$\cos\theta = 0$ $\quad \cos\theta = -5/3$ no solutions

$\theta = 90^\circ, 270^\circ$

ii) $\sqrt{3}\sin\theta - \cos\theta = 0$

$\sqrt{3}\sin\theta = \cos\theta$ $\quad$ A ✓ / T | acute $\theta = 30^\circ$

$\dfrac{\sqrt{3}\sin\theta}{\cos\theta} = 1$ $\quad \therefore \theta = 30^\circ, 210^\circ$

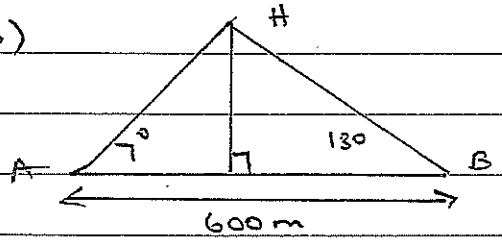
$\therefore \tan\theta = 1/\sqrt{3}$

Question 13

a) $A = \frac{1}{2} \cdot 6 \cdot 5 \cdot \frac{\sqrt{3}}{2}$

$A = \dfrac{15\sqrt{3}}{2}$ cm^2

b)



i) $\dfrac{HB}{\sin 7^\circ} = \dfrac{600}{\sin 160^\circ}$

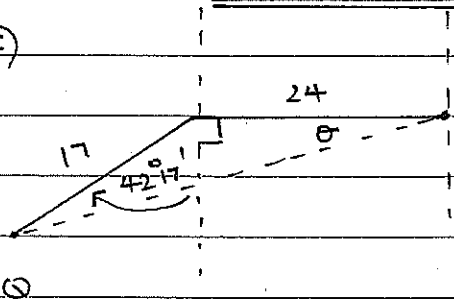
$HB = \dfrac{600\sin 7^\circ}{\sin 160^\circ}$

ii) $\sin 13^\circ = \dfrac{CH}{HB}$ ← from above

$\therefore CH = HB \cdot \sin 13^\circ$

$= 48$m

c)



i) $QP^2 = 17^2 + 24^2 - 2\times17\times24\times\cos 132^\circ 17'$

$\therefore QP =$ 37.6 km (1 dec pl)

ii) $\dfrac{\sin\theta}{17} = \dfrac{\sin 132^\circ 17'}{QP}$

$\therefore \theta = 20^\circ$ (to nearest deg)

true bearing ship from P $= 250^\circ$T