|Year 11 - Half-yearly| 2015

Question 6

a) Prove $\dfrac{2\sin^3 x + 2\cos^3 x}{\sin^2 x + \sin x\cos x} = 2\operatorname{cosec} x - 2\cos x$

Proof: $LHS = \dfrac{2\sin^3 x + 2\cos^3 x}{\sin^2 x + \sin x\cos x} = \dfrac{2(\sin^3 x + \cos^3 x)}{\sin x(\sin x + \cos x)}$

(1) $= \dfrac{2\cancel{(\sin x + \cos x)}(\sin^2 x - \sin x\cos x + \cos^2 x)}{\sin x\cancel{(\sin x + \cos x)}}$

$= \dfrac{2(1 - \sin x\cdot\cos x)}{\sin x} = \dfrac{2 - 2\sin x\cos x}{\sin x}$

(1) $= \dfrac{2}{\sin x} - \dfrac{2\cancel{\sin x}\cdot\cos x}{\cancel{\sin x}} = \dfrac{2}{\sin x} - 2\cos x$

$= 2\operatorname{cosec} x - 2\cos x$ = RHS ∴ proven

not shown

b) $f(x) = x^2 + \cos x$

(1) $f(-x) = (-x)^2 + \cos(-x)$ but $\cos(-x) = \cos x$ even function

$= x^2 + \cos x$ (1)

$= f(x)$ ∴ even function

(if) do ~~$f(-x) = x^2 + \cos x$~~
but then state $f(x) \neq f(-x)$
and $-f(x) \neq f(-x)$ ∴ neither ∴ $1/2$

c) $\dfrac{x^2+2}{x} \geq 2x - 1$ $\times x^2$ $x \neq 0$

$x(x^2+2) \geq x^2(2x-1)$

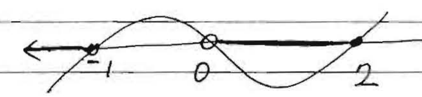
$0 \geq x^2(2x-1) - x(x^2+2)$ (1)

$0 \geq x[x(2x-1) - (x^2+2)]$

$0 \geq x[2x^2 - x - x^2 - 2]$

$0 \geq x[x^2 - x - 2]$

$0 \geq x[x-2][x+1]$ (1)



∴ $x \leq -1$, $0 < x \leq 2$ (1)

d) i) 6 men → pick 3 ∴ ${}^6C_3$
8 women → pick 2 ∴ ${}^8C_2$

∴ answer ${}^6C_3 \times {}^8C_2 = 560$ (1)

ii) At least one women chosen (1) for recognising all combos getting at least 1 correct

∴ 1w or 2w or 3w or 4w or 5w

$= {}^8C_1 \times {}^6C_4 + {}^8C_2 \times {}^6C_3 + {}^8C_3 \times {}^6C_2 + {}^8C_4 \times {}^6C_1 + {}^8C_5 = 1996$

OR All possible − No women on committee ∴ only men

$= {}^{14}C_5 - {}^6C_5$

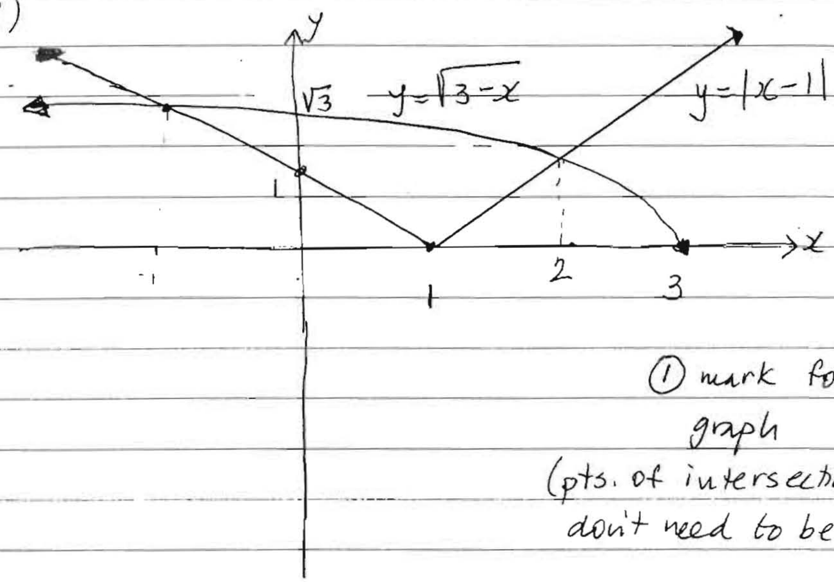
$= 2002 - 6 = \underline{1996}$

(if) ${}^{14}P_5 - {}^6P_5 = 239520$ 1 mark

Question 7

a) $y = \sqrt{3-x}$ $\qquad$ $y = |x-1|$

(i)



① mark for each graph
(pts. of intersection don't need to be shown)

(ii) $\sqrt{3-x} = |x-1|$ first

$3 - x = (x-1)^2$

$0 = x^2 - 2x + 1 + x - 3$

$0 = x^2 - x - 2 = (x-2)(x+1)$

$\therefore x = 2 \quad x = -1$ ①

$\therefore \sqrt{3-x} \le |x-1|$

① soln: $x \le -1$ or $2 \le x \le 3$

b) $3\cot\theta = \tan\theta + 2 \qquad 0° \le \theta \le 360°$

$\frac{3}{\tan\theta} = \tan\theta + 2 \;/\; \times \tan\theta$

$3 = \tan^2\theta + 2\tan\theta$

$0 = \tan^2\theta + 2\tan\theta - 3$

$0 = (\tan\theta + 3)(\tan\theta - 1)$

$\therefore \tan\theta = -3$ or $\tan\theta = 1$

for simplifying identities & creating quad. equation ①

$\theta = 108°26', 288°26'$ ① $\qquad \theta = 45°, 225°$

c) $x^5 + x^2y^2(y-x) - y^5$

$= x^5 + x^2y^3 - x^3y^2 - y^5$

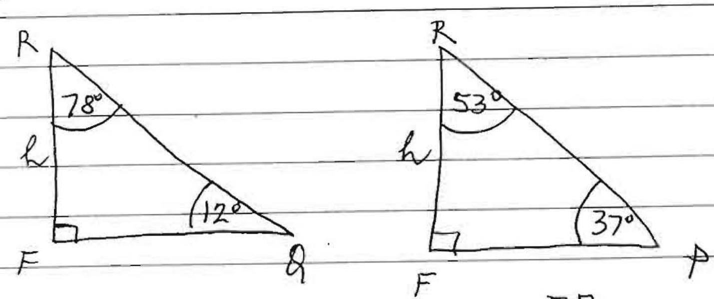
$= x^2(x^3+y^3) - y^2(x^3+y^3)$ ①

$= (x^2-y^2)(x^3+y^3) = (x-y)(x+y)(x+y)(x^2-xy+y^2)$

① $\qquad$ ①

or $= (x+y)^2(x-y)(x^2-xy+y^2)$

Question 8

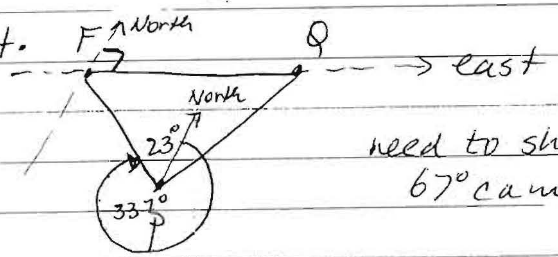


① (i) $\tan 53^\circ = \frac{FP}{h}$

need to show some evidence

need to show $FQ = h \cdot \tan 78^\circ$ ① $\therefore FP = h \cdot \tan 53^\circ$

(ii) cont.



need to show where 67° came from ①

$\therefore \angle QFP = 180^\circ - 23^\circ - 90^\circ = 67^\circ$

$\therefore$ by cosine rule in $\triangle FQP$

$PQ^2 = FP^2 + FQ^2 - 2 \times FP \times FQ \times \cos 67^\circ$

$\therefore 200^2 = h^2 \cdot \tan^2 53^\circ + h^2 \cdot \tan^2 78^\circ - 2 \times h \tan 53^\circ \cdot h \cdot \tan 78^\circ \times \cos 67^\circ$

$\therefore 200^2 = h^2 \cdot \tan^2 53^\circ + h^2 \tan^2 78^\circ - 2h^2 \tan 53^\circ \tan 78^\circ \cdot \cos 67^\circ$

$\therefore$ proven

a) iii)

$200^2 = h^2 \left[\tan^2 53^\circ + \tan^2 78^\circ - 2 \tan 53^\circ \tan 78^\circ \cos 67^\circ\right]$

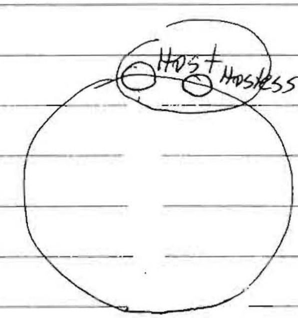
$\therefore h^2 = \frac{200^2}{\tan^2 53^\circ + \tan^2 78^\circ - 2 \tan 53^\circ \tan 78^\circ \cos 67^\circ}$

$\therefore h = \frac{200}{\sqrt{\tan^2 53^\circ + \tan^2 78^\circ - 2 \tan 53^\circ \tan 78^\circ \cos 67^\circ}}$

$\therefore h = 45.864$ ① (3 d.p) [ignore rounding]

$\doteqdot 46$m

b)



Answer = All possibilities − Host Hostess together

$= (8-1)! - 2 \times (7-1)!$

$= 5040 - 1440 = 3600$

2 - marks → correct solution with working

1 - mark → well presented methods with incorrect conclusion

(c) 4 digit numbers – must start with

6
7
8 _ _ _ 3 more digits

$\therefore 3 \times {}^4P_3 = 72$ (1)

or

5 digit numbers $= 5! = {}^5P_5 = 120$

$\therefore$ Total $= 72 + 120 = 192$

2 marks - correct answer with working
1 mark – 4 digits or 5 digit numbers
Or coherant working – mistake obvious

Question 9

ISOSCELES

(a) (i) $\dfrac{9!}{3!\,2!} = 30240$

S repeats 3 times 3!
E repeats 2 times 2!

2 - marks – correct solution with working
1 - mark $= \dfrac{9!}{\text{wrong repetition}}$

ii) SSS _ _ _ _ _ _ any remaining letter
7 entities $\therefore \dfrac{7!}{2!\,(EE)} = 2520$

2 - marks - correct solution with working
1 - mark – seeing 7! in the working
or dividing by 2!

iii) S _ _ _ _ _ _ _ S
fixed 7 places $\therefore$ 7! fixed $\dfrac{7!}{2!\,(EE)} = 2520$

2 - marks – correct solution with working
1 mark – working correctly towards solution

(b) $y = \dfrac{x}{9-x^2}$

(i) vertical asymptotes $9-x^2 \neq 0$

$\therefore\ x = \pm 3$ (1)

horizontal asymptote

$x \to +\infty$ (pick a large number, sub. in)

$\therefore\ y \to 0^-$

if $x \to -\infty$ (pick a small number eg. $x=-100$, sub in)

$\therefore\ y \to 0^+$

$\therefore$ horizontal asymptote is $y = 0$ (1)

ii) $f(x) = y = \dfrac{x}{9-x^2}$

$f(-x) = \dfrac{-x}{9-(-x)^2} = \dfrac{-x}{9-x^2} = -f(x)$ (1)

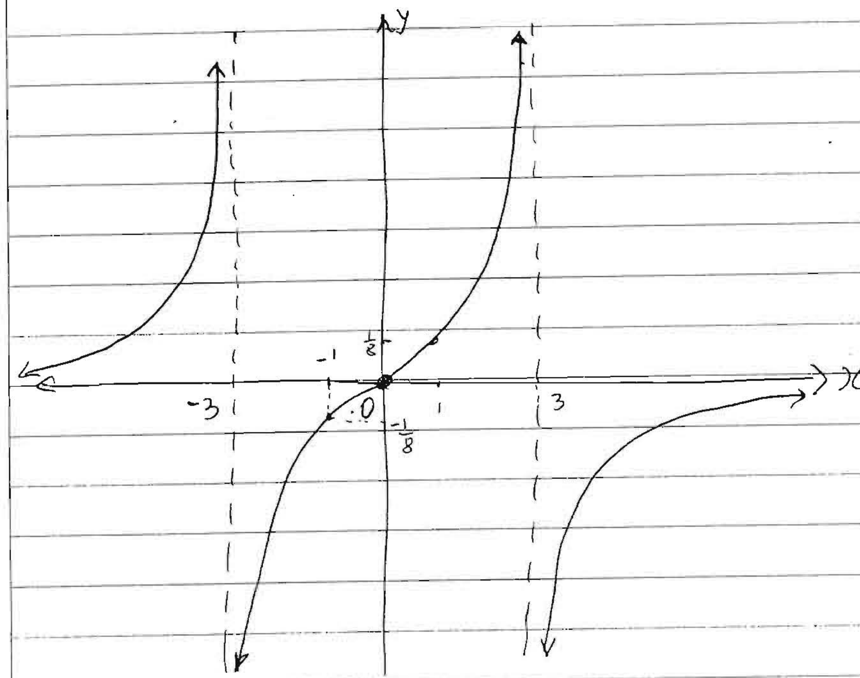
must show substitution

$\therefore$ odd function

iii)

x	-1	0	1
y	$\frac{-1}{8}$	0	$\frac{1}{8}$

- By using table of values and the property of the odd function drawing the shape



2marks - correct shape & asymptotes & x-int.
1mark - showing asymptotes correctly or odd function features

$\boxed{\text{Multiple choice answers}}$

1. A
2. B
3. D
4. B
5. D