BAULKHAM HILLS HIGH SCHOOL
2016
YEAR 11
HALF YEARLY EXAMINATIONS

## Mathematics Extension

## General Instructions

- Reading time - 5 minutes
- Working time - 90 minutes
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in Questions 6-8
- Marks may be deducted for careless or badly arranged work

Total marks - 50
Section I Pages 2 - 3
5 marks

- Attempt Questions 1 - 5
- Allow about 10 minutes for this section
Section II Pages 4-6
45 marks
- Attempt Questions 6-8
- Allow about 80 minutes for this section


## Section I

## 5 marks

Attempt Questions 1 - 5
Allow about 10 minutes for this section
Use the multiple-choice answer sheet for Questions 1 - 5
1 Which of the following is FALSE ?
(A) $\sin \left(90^{\circ}-\alpha\right)=\cos \alpha$
(B) $\sin \left(90^{\circ}+\alpha\right)=\cos \alpha$
(C) $\cos \left(90^{\circ}-\alpha\right)=\sin \alpha$
(D) $\cos \left(90^{\circ}+\alpha\right)=\sin \alpha$

2 Which expression is equal to ${ }^{n} \mathbf{C}_{2}$ ?
(A) $\frac{n}{2}$
(B) $\frac{n^{2}-n}{2}$
(C) $\frac{n^{2}+n}{2}$
(D) $n$

3 How many solutions does the equation $x^{\frac{1}{3}}=|x-2|-3$ have ?
(A) 0
(B) 1
(C) 2
(D) 3

4 A test is administered with 15 questions.
Students are allowed to answer any 10 questions.
How many ways can a student get 9 out of 10 questions correct?
(A) 30030
(B) 5005
(C) 3003
(D) 15

5 The cube below has sides four metres long. $M$ is the midpoint of $D C$.


The angle EMH is closest to
(A) $41.8^{\circ}$
(B) $48.2^{\circ}$
(C) $49.1^{\circ}$
(D) $54.7^{\circ}$

## Section II

## 45 marks

Attempt Questions 6 - 8
Allow about 80 minutes for this section
Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.
Question 6 ( $\mathbf{1 5}$ marks) Use a separate answer sheet
(a) Solve the inequation $\frac{x-2}{x+5}<2$

4 students are selected. How many of these selections contain at least 2 girls?
(c) If $a=\frac{5-\sqrt{5}}{5+\sqrt{5}}$, evaluate, showing all working:
(i) $a+\frac{1}{a}$
(ii) $a^{2}+\frac{1}{a^{2}}$
(d) 47271 is a five digit number whose digits sum to $4+7+2+7+1=21$.

How many five digit numbers are there whose digits sum to 43 ?
(e) On a number plane, shade the region where $(x, y)$ satisfies both of the inequalities

$$
y<\sqrt{16-x^{2}} \quad \text { and } \quad y \leq x
$$

Question 7 (15 marks) Use a separate answer sheet
(a) How many ways can 8 people be arranged in a line if:
(i) there are no restrictions?
(ii) Lance and Vinuja must be together?
(iii) Duvaraha must be at the beginning and Arpita must be at the end?
(b) Given that $3 \sin ^{2} x+2 \sin x=6 \cos x+9 \sin x \cos x$ and $-90^{\circ}<x<90^{\circ}$, find the possible values of $\tan x$.
(c) How many distinct arrangements of all of the letters of the word MISSISSIPPI, are there if:
(i) there are no restrictions? 1
(ii) all four I's do not come together?
(d) The diagram below shows two vertical towers $B D$ and $C E$ of heights $h$ and $2 h$ respectively, on a horizontal plane $A B C$.
Point $A$ is due south of point $B$, and the angles of elevation of the tops of the towers from $A$ are both $30^{\circ}$.
The angle of elevation of $E$ from $D$ is $15^{\circ}$

(i) Show that $B C=h \tan 75^{\circ}$
(ii) Find similar expressions for both $A B$ and $A C \quad 1$
(iii) Hence find the bearing of the taller tower from point $A$

## Marks

Question 8 (15 marks) Use a separate answer sheet
(a) 4 female and 4 male students are to be seated around a circular table. In how many ways can this be done if the males and females must alternate?
(b) Solve
(i) $\frac{x}{3 x-4} \leq \frac{1}{x-1}$
(ii) $x^{2}-|x|-3=2 x+|x|$
(c) In this question $f^{2}(x)$ denotes $f(f(x)), f^{3}(x)$ denotes $f(f(f(x)))$, and so on. If $f(x)=\frac{x+\sqrt{3}}{1-x \sqrt{3}}$;
(i) find $f^{2}(x)$ in terms of $x$
(ii) show that $f^{3}(x)=x$
(iii) find $f^{2016}(x)$
(d) A rectangular box has edges of lengths $a, b$ and $c$ units.

A diagonal of length $d$ is drawn through the box between opposite corners as shown.
The three different angles between this diagonal and the three edges $a, b$ and $c$ of the box are labelled $\alpha, \beta$ and $\gamma$ respectively.

(i) Express $d$ in terms of $a, b$ and $c$.
(ii) Hence, or otherwise, show that the angles $\alpha, \beta$ and $\gamma$ satisfy the identity

$$
\cos ^{2} \alpha+\cos ^{2} \beta+\cos ^{2} \gamma=1
$$

## End of paper

