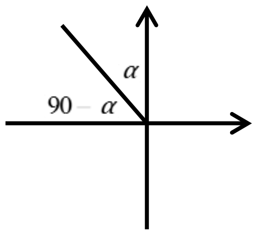
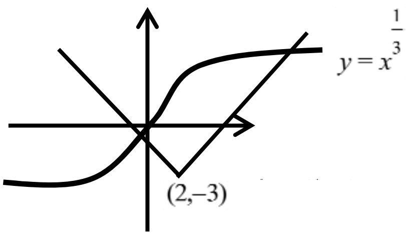
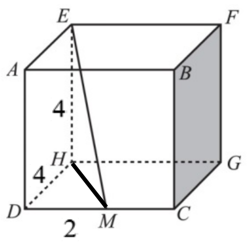
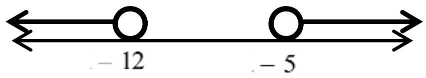
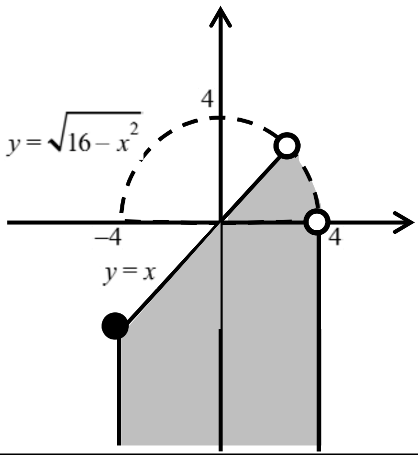
BAULKHAM HILLS HIGH SCHOOL
YEAR 11 MATHEMATICS EXTENSION HALF YEARLY EXAMINATION 2016 SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. D - $\cos(90^\circ+\alpha) = -\cos(90^\circ-\alpha)$ $= -\sin\alpha$ $\neq \sin\alpha$  	**1**	
2. B - ${}^n\mathbf{C}_2 = \frac{n!}{2!(n-2)!}$ $= \frac{n(n-1)}{2}$ $= \frac{n^2-n}{2}$	**1**	
3. C - 2 solutions  	**1**	
4. A - Ways $= {}^{15}\mathbf{C}_{10} \times 10$ $= 3003 \times 10$ $= 30030$ *OR* let correct $= C$ wrong $= W$ blank $= B$ How many words can be made from CCCCCCCCCWBBBBB # Ways $= \frac{15!}{9!5!}$ $= 30030$	**1**	
5. A - $HM^2 = 2^2 + 4^2$ $HM = \sqrt{20}$ $\tan\angle EMH = \frac{4}{\sqrt{20}}$ $\angle EMH = 41.8^\circ$  	**1**	

Solution	Marks	Comments
SECTION II		
QUESTION 6		
6(a) (i) $\frac{x-2}{x+5}<2$ $x+5\neq 0$ $x\neq -5$ $x-2=2(x+5)$ $x-2=2x+10$ $x=-12$  $x<-12$ or $x>-5$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
6 (b) # selections $={}^6\mathbf{C}_2\times{}^{14}\mathbf{C}_2+{}^6\mathbf{C}_3\times{}^{14}\mathbf{C}_1+{}^6\mathbf{C}_4\times{}^{14}\mathbf{C}_0$ $=1365+280+15$ $=1660$	2	**2 marks** • Correct solution **1 mark** • Correctly calculates one case
6 (c) (i) $a+\frac{1}{a}=\frac{5-\sqrt{5}}{5+\sqrt{5}}+\frac{5+\sqrt{5}}{5-\sqrt{5}}$ $=\frac{(5-\sqrt{5})^2+(5+\sqrt{5})^2}{(5+\sqrt{5})(5-\sqrt{5})}$ $=\frac{2\left(5^2+(\sqrt{5})^2\right)}{5^2-(\sqrt{5})^2}$ $=\frac{60}{20}$ $=3$	2	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution
6 (c) (ii) $a^2+\frac{1}{a^2}=\left(a+\frac{1}{a}\right)^2-2\times a\times\frac{1}{a}$ $=3^2-2$ $=7$	2	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution
6(d) Only two possible combination of digits = 43 ***Case 1:*** $9+9+9+9+7=43$ Ways $=\frac{5!}{4!}$ $=5$ ***Case 2:*** $9+9+9+8+8=43$ Ways $=\frac{5!}{3!2!}$ $=10$ $\therefore$ total five digit numbers $=5+10$ $=15$	3	**3 marks** • Correctly solution **2 marks** • Identifies two correct cases • Correctly evaluates the number of possibilities in one case **1 mark** • Attempts to evaluate one of the correct cases
6(e) 	3	**3 marks** • Correct region making note of which boundaries and points of intersection are included **2 marks** • Both boundaries correct with only one region correct • Region correct, however boundary incorrectly identified **1 mark** • One boundary correctly identified

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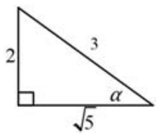
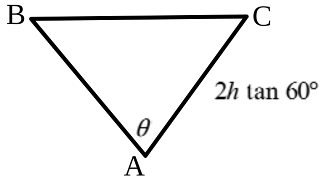
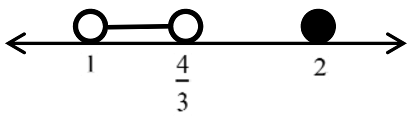
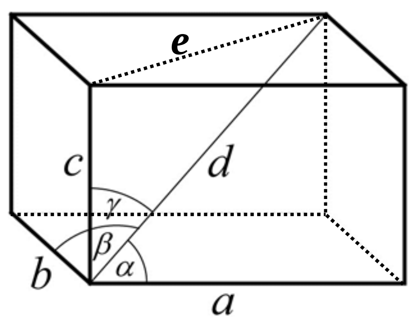
QUESTION 7			
7(a) (i)	$\text{Ways} = 8!$ $= 40320$	**1**	**1 mark** • Correct answer
7(a) (ii)	$\text{Ways} = 2! \times 7!$ $= 10080$	**2**	**2 marks** • Correct solution **1 mark** • Treats L & V as one object • Calculates # of arrangements of L & V
7(a) (iii)	$\text{Ways} = 1 \times 1 \times 6!$ $= 720$	**2**	**2 marks** • Correct solution **1 mark** • Correctly handles restriction
7 (b)	$3\sin^2 x + 2\sin x = 6\cos x + 9\sin x\cos x$ $3\sin^2 x + 2\sin x - 6\cos x - 9\sin x\cos x = 0$ $\sin x(3\sin x + 2) - 3\cos x(2 + 3\sin x) = 0$ $(3\sin x + 2)(\sin x - 3\cos x) = 0$  $\sin x = -\dfrac{2}{3}$ OR $\tan x = 3$ Quadrant 4 $\therefore \tan x = -\dfrac{2}{\sqrt{5}}$ $\therefore \tan x = -\dfrac{2}{\sqrt{5}}$ or $\tan x = 3$	**3**	**3 marks** • Correct solution **2 marks** • Finds one correct answer for tan*x* **1 mark** • Correctly factorises the terms or equivalent merit
7(c) (i)	Ways no restrictions $= \dfrac{11!}{4!4!2!}$ $= 34650$	**1**	**1 mark** • Correct answer
7(c) (ii)	Ways I's together $= \dfrac{8!}{4!2!}$ $= 840$ Ways I's not together $= 34650 - 840$ $= 33510$	**2**	**2 marks** • Correct solution **1 mark** • Calculates # ways I's together • Uses the complementary event idea
7 (d) (i)	$\dfrac{DF}{h} = \tan 75°$ $DF = h\tan 75°$ But BC=FD $\therefore BC = h\tan 75°$	**1**	**1 marks** • Correct solution
7 (d) (ii)	$AB = h\tan 60°$ $AC = 2h\tan 60°$	**1**	**1 marks** • Correct answer
7 (d) (iii)	$\cos\theta = \dfrac{h^2\tan^2 60° + 4h^2\tan^2 60° - h^2\tan^2 75°}{4h^2\tan^2 60°}$ $= \dfrac{5\tan^2 60° - \tan^2 75°}{4\tan^2 60°}$ $= \dfrac{15 - \tan^2 75°}{12}$ $= 0.89316.....$ $\theta = 84°53'$ $\therefore$ bearing of the tower is $N\ 85°\ E$ 	**2**	**2 marks** • Correct solution **1 mark** • Correct substitution into the cosine rule

Solution	Marks	Comments
QUESTION 8		
8 (a) # Ways $= 3! \times 4!$ $= 144$	**2**	**2 marks** • Correct solution **1 mark** • Correctly deals with circle arrangement as opposed to line arrangement • Correctly deals with the alternation
8(b) (i) $\frac{x}{3x-4} \leq \frac{1}{x-1}$ $3x-4 \neq 0$ $\quad x-1 \neq 0$ $x \neq \frac{4}{3}$ $\quad x \neq 1$ $x(x-1) = 3x-4$ $x^2 - x = 3x - 4$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $x = 2$  $1 < x < \frac{4}{3}$ or $x = 2$	**3**	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the three correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with both denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
8(b) (ii) $x^2 - \|x\| - 3 = 2x + \|x\|$ $2\|x\| = x^2 - 2x - 3$ $2x = x^2 - 2x - 3$ $x^2 - 4x - 3 = 0$ $x = \frac{4 \pm \sqrt{28}}{2}$ $x = 2 \pm \sqrt{7}$ however $x = 2 - \sqrt{7}$ is not a solution $\therefore$ $x = 2 + \sqrt{7}$ $-2x = x^2 - 2x - 3$ $x^2 - 3 = 0$ $x = \pm\sqrt{3}$ however $x = \sqrt{3}$ not a solution $\therefore$ $x = -\sqrt{3}$ $\therefore x = -\sqrt{3}$ or $x = 2 + \sqrt{7}$	**3**	**3 marks** • Correctly identifies the two correct answers **2 marks** • Finds one answer after rejecting its conjugate • Finds four answers including the correct two **1 mark** • Identifies two correct cases
8 (c) (i) $f^2(x) = \frac{\left(\frac{x+\sqrt{3}}{1-x\sqrt{3}}\right) + \sqrt{3}}{1 - \left(\frac{x+\sqrt{3}}{1-x\sqrt{3}}\right)\sqrt{3}}$ $= \frac{x+\sqrt{3}+\sqrt{3}-3x}{1-x\sqrt{3}-x\sqrt{3}-3}$ $= \frac{-2x+2\sqrt{3}}{-2-2x\sqrt{3}}$ $= \frac{x-\sqrt{3}}{1+x\sqrt{3}}$	**1**	**1 marks** • Correct solution
8 (c) (ii) $f^3(x) = \frac{\left(\frac{x-\sqrt{3}}{1+x\sqrt{3}}\right) + \sqrt{3}}{1 - \sqrt{3}\left(\frac{x-\sqrt{3}}{1+x\sqrt{3}}\right)}$ $= \frac{x-\sqrt{3}+\sqrt{3}+3x}{1+x\sqrt{3}-x\sqrt{3}+3}$ $= \frac{4x}{4}$ $= x$	**1**	**1 marks** • Correct solution
8 (c) (iii) As $f^3(x) = x$ then $f^4(x) = f(x)$, $f^5(x) = f^2(x)$, $f^6(x) = f^3(x) = x$, etc $\therefore f^{2016}(x) = f^3(x) = x$	**1**	**1 marks** • Correct solution

Solution	Marks	Comments
QUESTION 8…*continued*		
8 (d) (i)  $d^2 = e^2 + c^2$ $e^2 = a^2 + b^2$ $\therefore d^2 = a^2 + b^2 + c^2$ $d = \sqrt{a^2 + b^2 + c^2}$	2	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution
8 (d) (ii) $\cos\alpha = \frac{a}{d}$ Similarly $\cos\beta = \frac{b}{d}$ and $\cos\gamma = \frac{c}{d}$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{a^2}{d^2} + \frac{b^2}{d^2} + \frac{c^2}{d^2}$ $= \frac{a^2 + b^2 + c^2}{d^2}$ $= \frac{d^2}{d^2}$ $= 1$	2	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution