BAULKHAM HILLS HS

PERSEVERE

2017 Preliminary
Half Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour and 30 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks – 54

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 8

44 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 15 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

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1 How many numbers greater than 40000 can be formed with the digits 2, 3, 4, 5 and 6 if no digit is used more than once?

(A) 48 (B) 72 (C) 96 (D) 120

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2 In how many ways can a family of eight sit around a circular table if the two youngest family members want to sit together?

(A) $7!$ (B) $2\times5!$ (C) $2\times6!$ (D) $2\times7!$

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3 What is the domain of the function $y=\dfrac{1}{\sqrt{4-x^2}}$?

(A) $x>2$ (B) $x<2$

(C) $-2<x<2$ (D) $x<-2$ or $x>2$

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4 If $2\sec\theta+3=0$, and $\cot\theta>0$, what is the exact value of $\sin\theta$?

(A) $-\dfrac{\sqrt{5}}{3}$ (B) $-\dfrac{\sqrt{5}}{2}$ (C) $\dfrac{\sqrt{5}}{3}$ (D) $\dfrac{\sqrt{5}}{2}$

5 How many value(s) of θ, for $0° \le \theta \le 360°$, satisfy the equation $\sin\theta\cos\theta = \sin\theta$?

(A) 2 (B) 3 (C) 4 (D) 5

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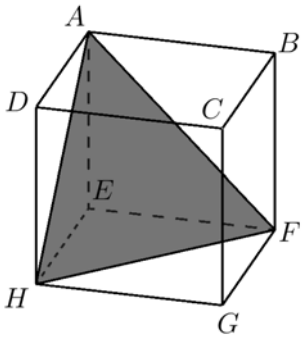
6 *ABCDEFGH* is a cube of side length *p*, what is the area of ΔAFH in terms of *p*?



(A) $\frac{p^2\sqrt{3}}{4}$ (B) $\frac{p^2\sqrt{3}}{2}$

(C) $\frac{p^2}{4}$ (D) $\frac{p^2}{2}$

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7 For $0° \le \alpha \le 90°$, the least value of $\frac{30}{3\sin^2\alpha + 2\sin^2(90° - \alpha)}$ is:

(A) 15 (B) 10 (C) 6 (D) 5

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8 When solving $\frac{x-1}{\sqrt{x}} > \frac{2}{x-1}$ within the natural domain, three students obtain the following inequalities:

Student 1: $(x-1)^2 > 2\sqrt{x}$

Student 2: $(x-1)^3 > 2(x-1)\sqrt{x}$

Student 3: $(x-1)^3\sqrt{x} > 2x(x-1)$

Which student(s) will obtain the correct solution to the original inequality?

(A) Student 1 only (B) Student 2 only

(C) Student 3 only (D) Both students 2 and 3

9 If $f(x+1) = x^4 - 2x + 1$, what is the value of $f(0)$?

(A) 0 (B) 1 (C) 2 (D) 4

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10 Given that n and r are both positive integers such that $n \geq r$, which of the following is **NOT** always true?

(A) ${}^nP_1 = {}^nC_1$ (B) ${}^nP_n = {}^nC_n$

(C) ${}^nP_r = {}^nC_r \times r!$ (D) ${}^nP_r \geq {}^nC_r$

Section II

44 marks
Attempt Questions 11 – 14
Allow about 1 hour and 15 minutes for this section

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (11 marks) Use the Question 11 section of the writing booklet.

(a) Solve $\dfrac{x}{x-4} \leq 5$. **3**

(b) The letters A, E, I, O and U are vowels. In how many ways can the letters of the word **MATHEMATICS** be arranged in a line if:

(i) there are no restrictions? **1**

(ii) the vowels are all together? **1**

(iii) both the letter **M**'s must be immediately followed by the letter **A**? **1**

(c)

(i) Solve $x^4 - 4x^2 + 3 = 0$. **1**

(ii) Hence, or otherwise, solve $\tan^4\theta - 4\tan^2\theta + 3 = 0$ for $0° \leq \theta \leq 360°$. **2**

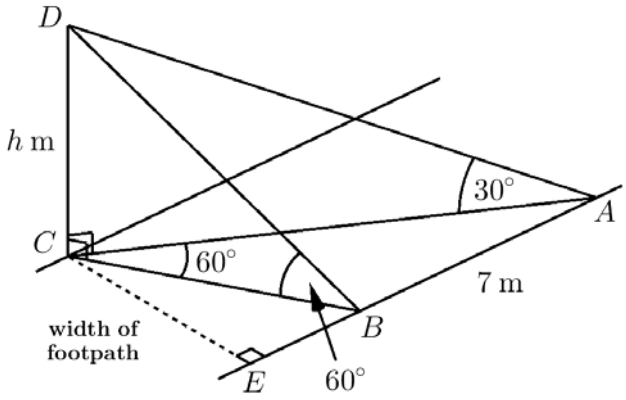
(d) Sketch the region on the Cartesian plane satisfying the inequality $y < -\sqrt{x-1}$. **2**

End of Question 11

Question 12 (11 marks) Use the Question 12 section of the writing booklet.

(a) A footpath on horizontal ground has two parallel edges. CD is a vertical flagpole of height h metres which stands with its base, C, on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB = 7$ m and $\angle ACB = 60°$.

From A and B, the angles of elevation of the top of the flagpole, D, are $30°$ and $60°$ respectively.



(i) Show that $BC = \dfrac{h}{\sqrt{3}}$. **1**

(ii) Find a similar expression for AC. **1**

(iii) Hence, find the exact height of the flagpole. **2**

(iv) Find the exact width of the footpath. **3**

(b) From a group of 6 men and 8 women, a committee of 5 people is to be chosen, how many ways can this committee be formed if:

(i) there are no restrictions? **1**

(ii) the committee consists of 2 men and 3 women? **1**

(iii) the committee must have at least 1 woman? **1**

(iv) the entire committee is of the same gender? **1**

End of Question 12

Question 13 (11 marks) Use the Question 13 section of the writing booklet.

(a) Consider the function $f(x) = x^2 - |2x| - 3$.

(i) Show that $f(x)$ is an even function. **1**

(ii) Hence, or otherwise, sketch the graph of $y = f(x)$, showing all intercepts. **2**

(iii) What is the range of $y = f(x)$? **1**

(b)

(i) On the same set of axes, sketch the graphs of $y = |2x - 4|$ and $y = |x + 1|$, showing all intercepts. **2**

(ii) Hence, or otherwise, solve the inequality $|2x - 4| \leq |x + 1|$. **2**

(c)

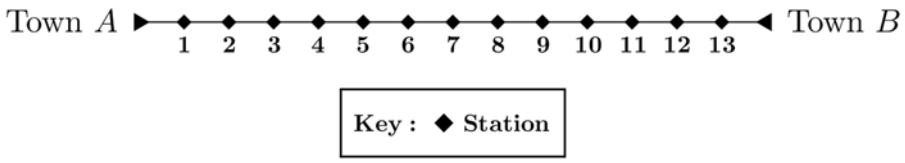
(i) Fully factorise $64k^6 - 1$ as a difference of two squares and as a difference of two cubes. **2**

(ii) Hence, or otherwise, factorise $16k^4 + 4k^2 + 1$. **1**

End of Question 13

Question 14 (11 marks) Use the Question 14 section of the writing booklet.

(a) A train is leaving Town *A*, heading towards Town *B* without turning around. There are 13 train stations between the two towns:



(i) In how many ways can the train stop at 4 of the 13 stations? **1**

(ii) In how many ways can the train stop at 4 of the 13 stations if the train does not stop at consecutive stations? **1**

(b) Show that $\dfrac{\left(\sin^2\alpha-\cos^2\alpha\right)\left(1-\sin\alpha\cos\alpha\right)}{\cos\alpha\left(\sec\alpha-\operatorname{cosec}\alpha\right)\left(\sin^3\alpha+\cos^3\alpha\right)}=\sin\alpha$. **3**

(c) It can be shown that:

$$\tan\left(A+B+C\right)=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$$ (DO NOT PROVE THIS.)

(i) For any ΔABC, explain why $\tan A+\tan B+\tan C=\tan A\tan B\tan C$. **2**

(ii) It is given, for ΔXYZ, that $\dfrac{\tan X}{5}=\dfrac{\tan Y}{6}=\dfrac{\tan Z}{7}=k$ for some constant k, show that $k=\sqrt{\dfrac{3}{35}}$. **3**

(iii) Hence calculate the size of the smallest angle in ΔXYZ correct to the nearest minute. **1**

End of Paper