YEAR 11 HALF YEARLY EXAMINATION 2017
MATHEMATICS EXTENSION 1
MARKING GUIDELINES

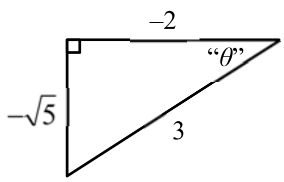
Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	C
3	C
4	A
5	B

Question	Answer
6	B
7	B
8	D
9	D
10	B

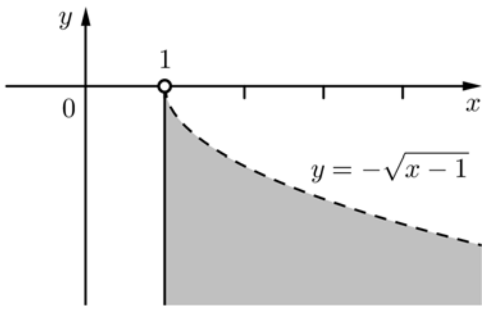
Questions 1 – 10

Sample solution	
1.	Number over $40000 = 3 \times 4!$ $= 72$ $\therefore$ 72 numbers are greater than 40000.
2.	2 ways to arrange the two youngest family members 6! ways to arrange the remaining six family members as well as the group of two around a circular table.
3.	$4 - x^2 > 0$ $4 > x^2$ $\therefore -2 < x < 2$
4.	$\cot\theta > 0$ if $\tan\theta > 0$ $2\sec\theta + 3 = 0$ $\sec\theta = \frac{-3}{2}$ $\cos\theta = \frac{2}{-3}$ $\therefore \cos\theta < 0$ and $\tan\theta > 0$ (i.e. θ is in quadrant 3) $\sin\theta = -\frac{\sqrt{5}}{3}$ 
5.	$\sin\theta\cos\theta = \sin\theta$ $\sin\theta(\cos\theta - 1) = 0$ $\therefore \sin\theta = 0$ or $\cos\theta = 1$ For $0° \le \theta \le 360°$, $\theta = 0°, 180°, 360°$
6.	$AH = AF = FH$ (equal diagonals of the square faces) ie. ΔAFH is equilateral. By Pythagoras' theorem, $AH = p\sqrt{2}$. Using the sine rule for area, $A_{\Delta AFH} = \frac{1}{2} \times p\sqrt{2} \times p\sqrt{2} \times \sin 60°$ $= \frac{p^2\sqrt{3}}{2}$

7.	$\dfrac{30}{3\sin^2\alpha+2\sin^2(90°-\alpha)}=\dfrac{30}{3\sin^2\alpha+2\cos^2\alpha}$ $=\dfrac{30}{\sin^2\alpha+2}$ Least value occurs when denomiator is largest, i.e. $\sin^2\alpha=1$. Least value of $\dfrac{30}{\sin^2\alpha+2}=\dfrac{30}{1+2}$ $=10$
8.	Student 1 multiplied both sides by $(x-1)$, which could be negative, the inequality sign may needed to have been flipped. Student 2 multiplied both sides by $\sqrt{x}(x-1)^2$, which is positive, the inequality is ok. Student 3 multiplied both sides by $(\sqrt{x})^2(x-1)^2$, which is positive, the extra $\sqrt{x}$ does not introduce any extra solutions in the natural domain, the inequality is ok.
9.	Let $x=-1$: $f(0)=(-1)^4-2\times(-1)+1$ $=1+2+1$ $=4$
10.	The number of ways of choosing 1 item from a set of *n* items does not depend on order, so ${}^nP_1={}^nC_1$. By definition, ${}^nC_r=\dfrac{n!}{r!(n-r)!}$ $=\dfrac{1}{r!}{}^nP_r$ ${}^nC_r\times r!={}^nP_r$ The number of ways of choosing *r* items from a set of *n* items where order is important is always more than the number of ways of choosing *r* items from a set of *n* items where order is not important, i.e. ${}^nP_r\geq{}^nC_r$, with equality when $r=1$. By elimination, ${}^nP_n\neq{}^nC_n$ is not always true. (Note: For the given conditions, this statement is only true for $n=1$.)

Section II

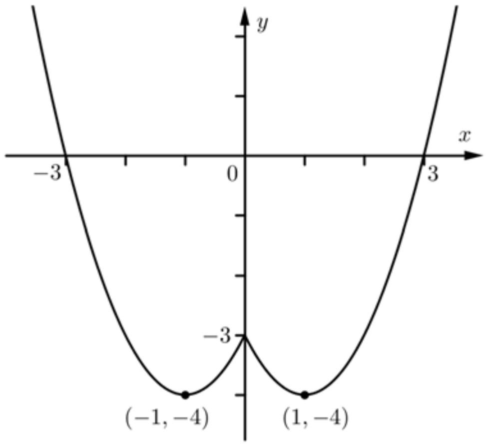
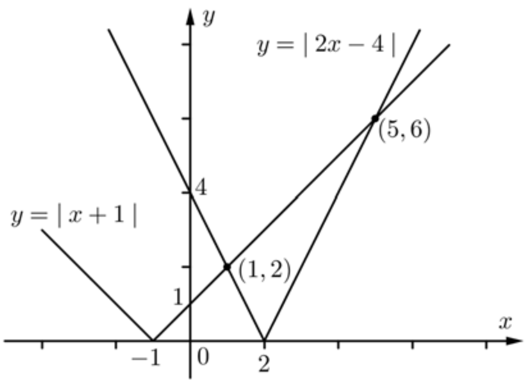
Question 11

Sample solution		Suggested marking criteria
(a)	Natural domain for $\dfrac{x}{x-4} : x \neq 4$ $\dfrac{x}{x-4} \leq 5$ $x(x-4) \leq 5(x-4)^2$ $0 \leq (x-4)[5(x-4)-x]$ $0 \leq 4(x-4)(x-5)$ $\therefore x < 4$ or $x \geq 5$	• 3 – correct solution • 2 – correctly identifies the two critical points $x = 4$ and $x = 5$ • 1 – attempts to solve the inequation using a suitable method – recognises a restriction in the domain
(b)	(i) $\dfrac{11!}{2!2!2!} = 4\ 989\ 600$	• 1 – correct answer, or equivalent numerical expression
	(ii) There are $\dfrac{4!}{2!} = 12$ ways of arranging the vowels. Treating the vowels as a single entity, the number of ways to arrange the remaining 7 letters and the single group of vowels is: $\dfrac{8!}{2!2!} = 10\ 080$ Total number of arrangements $= 10\ 080 \times 12$ $= 120\ 960$	• 1 – correct answer, or equivalent numerical expression
	(iii) Treating **MA** as a single entity, the number of ways to arrange **(MA)THE(MA)TICS** in a line is: $\dfrac{9!}{2!2!} = 90\ 720$	• 1 – correct answer, or equivalent numerical expression
(c)	(i) Let $u = x^2$: $u^2 - 4u + 3 = 0$ $(u-3)(u-1) = 0$ $u = 1$ or $u = 3$ $x^2 = 1$ $\quad x^2 = 3$ $x = \pm 1$ $\quad x = \pm\sqrt{3}$	• 1 – correct solution
	(ii) Using the results from (i), and letting $x = \tan\theta$: $\tan\theta = \pm 1$ or $\tan\theta = \pm\sqrt{3}$ $\theta = 45°, 135°, 225°, 315°$ $\quad \theta = 60°, 120°, 240°, 300°$ $\therefore \theta = 45°, 60°, 120°, 135°, 225°, 240°, 300°, 315°$	• 2 – correct solution • 1 – correct attempt at solving an appropriate trigonometric equation
(d)	 	• 2 – correct region and boundaries • 1 – significant progress towards the correct region and boundaries

Question 12

Sample solution			Suggested marking criteria
(a)	(i)	In ΔDBC, $\tan 60^\circ = \dfrac{h}{BC}$ $BC = \dfrac{h}{\tan 60^\circ}$ $= \dfrac{h}{\sqrt{3}}$	• 1 – correct solution
	(ii)	In ΔDAC, $\tan 30^\circ = \dfrac{h}{AC}$ $AC = \dfrac{h}{\tan 30^\circ}$ $= h\sqrt{3}$	• 1 – correct solution
	(iii)	In ΔABC, $AB^2 = BC^2 + AC^2 - 2 \times BC \times AC \times \cos 60^\circ$ $49 = \left(\dfrac{h}{\sqrt{3}}\right)^2 + \left(h\sqrt{3}\right)^2 - 2 \times \dfrac{h}{\sqrt{3}} \times h\sqrt{3} \times \dfrac{1}{2}$ $49 = \dfrac{h^2}{3} + 3h^2 - h^2$ $49 = \dfrac{7h^2}{3}$ $21 = h^2$ $h = \sqrt{21}$ $(\text{since } h > 0)$	• 2 – correct solution • 1 – uses cosine rule, showing appropriate substitution – finds AC in terms of h
	(iv)	By sine area rule, $A_{\Delta ABC} = \dfrac{1}{2} \times \dfrac{h}{\sqrt{3}} \times h\sqrt{3} \times \sin 60^\circ$ $= \dfrac{h^2}{2} \times \dfrac{\sqrt{3}}{2}$ $= \dfrac{21\sqrt{3}}{4}$ $A_{\Delta ABC} = \dfrac{1}{2} \times 7 \times CE$ $\dfrac{21\sqrt{3}}{4} = \dfrac{7}{2} \times CE$ $\dfrac{3\sqrt{3}}{2} = CE$	• 3 – correct solution • 2 – finds the area of ΔABC – significant progress towards solution using $\cos\angle CBA = \frac{1}{2\sqrt{7}}$ or $\sin\angle CAB = \frac{\sqrt{21}}{14}$, or equivalent merit • 1 – attempts to use the sine area rule to find the area of ΔABC
(b)	(i)	${}^{14}C_5 = 2002$	• 1 – correct answer, or equivalent numerical expression
	(ii)	${}^{6}C_2 \times {}^{8}C_3 = 840$	• 1 – correct answer, or equivalent numerical expression
	(iii)	There are ${}^{6}C_5 = 6$ committees that consist of men only. Therefore, there are $2002 - 6 = 1996$ committees with at least 1 woman.	• 1 – correct answer, or equivalent numerical expression
	(iv)	There are ${}^{6}C_5 = 6$ committees that consist of men only. There are ${}^{8}C_5 = 56$ committees that consist of women only. Therefore, there are $56 + 6 = 62$ committees that are entirely made up of members of the same gender.	• 1 – correct answer, or equivalent numerical expression

Question 13

Sample solution			Suggested marking criteria
(a)	(i)	$f(-x)=(-x)^2-\lvert -2x\rvert-3$ $=x^2-\lvert 2x\rvert-3$ (since $\lvert -A\rvert=\lvert A\rvert$ for all A) $=f(x)$ Therefore, $f(x)$ is an even function.	• 1 – correct solution
	(ii)		• 2 – correct solution • 1 – correctly sketches one branch of $y=f(x)$
	(iii)	The vertex of the parabola $y=x^2-2x-3$ is $(1,-4)$. By symmetry of the even function, the range of $f(x)=x^2-\lvert 2x\rvert-3$ is $y\geq -4$.	• 1 – correct solution
(b)	(i)		• 2 – correct graphs • 1 – one correct graph
	(ii)	$1\leq x\leq 5$	• 2 – correct solution • 1 – correctly identifies two critical points
(c)	(i)	By difference of two cubes, $64k^6-1=\left(4k^2-1\right)\left(16k^4+4k^2+1\right)$ $=(2k+1)(2k-1)\left(16k^4+4k^2+1\right)$ By difference of two squares, $64k^6-1=\left(8k^3+1\right)\left(8k^3-1\right)$ $=(2k+1)\left(4k^2-2k+1\right)(2k-1)\left(4k^2+2k+1\right)$	• 2 – correct solution • 1 – correct expression for one of the factorisations
	(ii)	$16k^4+4k^2+1=\left(4k^2-2k+1\right)\left(4k^2+2k+1\right)$	• 1 – correct solution

Question 14

Sample solution			Suggested marking criteria
(a)	(i)	$^{13}C_4 = 715$	• 1 – correct answer, or equivalent numerical expression
	(ii)	Let S represent a "stop" and P represent a "pass". The problem is equivalent to arranging 9 P's and 4 S's so that no S is adjacent to one another. With 9 P's, there are 10 spaces that the 4 S's could go: $\_P\_P\_P\_P\_P\_P\_P\_P\_P\_$ Number of ways $= {}^{10}C_4$ $= 210$	• 1 – correct answer, or equivalent numerical expression
(b)	(i)	$\text{LHS} = \dfrac{\left(\sin^2\alpha - \cos^2\alpha\right)(1-\sin\alpha\cos\alpha)}{\cos\alpha(\sec\alpha - \operatorname{cosec}\alpha)\left(\sin^3\alpha + \cos^3\alpha\right)}$ $= \dfrac{\cancel{(\sin\alpha+\cos\alpha)}(\sin\alpha-\cos\alpha)\cancel{(1-\sin\alpha\cos\alpha)}}{\cos\alpha(\sec\alpha-\operatorname{cosec}\alpha)\cancel{(\sin\alpha+\cos\alpha)}\cancel{\left(\sin^2\alpha-\sin\alpha\cos\alpha+\cos^2\alpha\right)}}$ $= \dfrac{\sin\alpha-\cos\alpha}{\cos\alpha\left(\dfrac{1}{\cos\alpha}-\dfrac{1}{\sin\alpha}\right)}$ $= \dfrac{\cancel{\sin\alpha-\cos\alpha}}{\cancel{\cos\alpha}\left(\dfrac{\cancel{\sin\alpha-\cos\alpha}}{\sin\alpha\cancel{\cos\alpha}}\right)}$ $= \sin\alpha$ $= \text{RHS}$	• 3 – correct solution • 2 – significant progress towards a valid solution • 1 – correctly factorises the difference of two squares – correctly factorises the sum of two cubes – correctly simplifies $\cos\alpha(\sec\alpha - \operatorname{cosec}\alpha)$
(c)	(i)	In any ΔABC, $A+B+C=180°$ $\therefore \tan(A+B+C)=0$ $\dfrac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}=0$ $\tan A+\tan B+\tan C-\tan A\tan B\tan C=0$ $\tan A+\tan B+\tan C=\tan A\tan B\tan C$	• 2 – correct solutions • 1 – recognising $\tan(A+B+C)=0$
	(ii)	$\dfrac{\tan X}{5}=\dfrac{\tan Y}{6}=\dfrac{\tan Z}{7}=k$ Therefore, $\tan X=5k$, $\tan Y=6k$ and $\tan Z=7k$. $\tan X+\tan Y+\tan Z=\tan X\tan Y\tan Z$ $5k+6k+7k=5k\times 6k\times 7k$ $18k=210k^3$ $0=6k\left(35k^2-3\right)$ $k\neq 0$ as this implies $\tan X=\tan Y=\tan Z=0$, which yields a degenerate triangle. $k \not< 0$ as X, Y and Z cannot be all obtuse, therefore $k=\sqrt{\dfrac{3}{35}}$	• 3 – correct solution • 2 – justifies why k is positive • 1 – attempts to solve for k using an appropriate method
	(iii)	$\tan X = 5\sqrt{\dfrac{3}{35}}$ $X = 55°40'$	• 1 – correct solution