BAULKHAM HILLS HIGH SCHOOL

2018

YEAR 11 HALF YEARLY ASSESSMENTS

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours 30 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- In Questions 11 – 18, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 130

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 120 marks (pages 6 – 18)

- Attempt Questions 11 – 18
- Allow about 2 hours 15 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1 Which one of the following expressions represents the factored form of $8x^3 + 27$?

(A) $(2x + 3)(4x^2 + 6x + 9)$

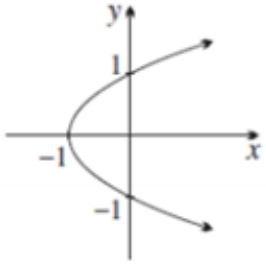
(B) $(2x + 3)(4x^2 - 6x + 9)$

(C) $(2x - 3)(4x^2 - 6x - 9)$

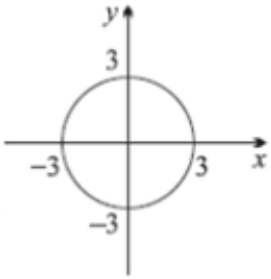
(D) $(2x - 3)(4x^2 + 6x - 9)$

2 Which of the following is a function?

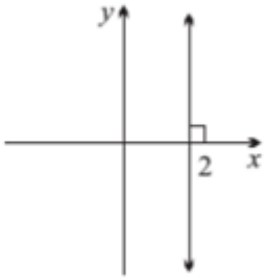
(A)



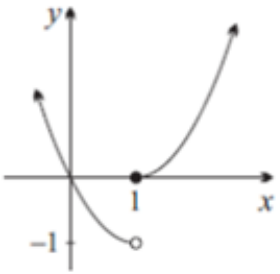
(B)



(C)



(D)



3 $\dfrac{1+\sqrt{3}}{2-\sqrt{3}} =$

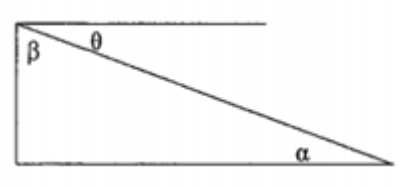
(A) $5 + 3\sqrt{3}$

(B) $\dfrac{5+3\sqrt{3}}{7}$

(C) $\sqrt{3} - 1$

(D) $\dfrac{\sqrt{3}-1}{7}$

4



In the diagram above;

(A) The angle of elevation is α and the angle of depression is β

(B) The angle of elevation is β and the angle of depression is θ

(C) The angle of elevation is θ and the angle of depression is β

(D) The angle of elevation is α and the angle of depression is θ

5 Given $\cos\theta = -\frac{3}{5}$ and $\sin\theta < 0$, the ratio for $\tan\theta$ =

(A) $-\frac{4}{5}$

(B) $-\frac{4}{3}$

(C) $\frac{4}{3}$

(D) $\frac{4}{5}$

6 In simplified form, the algebraic expression $\frac{x+5}{(x-3)(x+1)} - \frac{x-1}{x^2-x-2}$ can be written as

(A) $\frac{7x-13}{(x+1)(x-2)(x-3)}$

(B) $\frac{-x-13}{(x+1)(x-2)(x-3)}$

(C) $\frac{7x-7}{(x+1)(x-2)(x-3)}$

(D) $\frac{-x-7}{(x+1)(x-2)(x-3)}$

7 $\frac{\sin(360° - A)}{\sin(90° - A)} =$

(A) -1

(B) 1

(C) $-\tan A$

(D) $\tan A$

8 Two functions are defined as $f(x) = 3x^2 - 4$ and $g(y) = y^2 - 2y$. How many solutions are there to the equation $f(a) = g(2a)$?

(A) 0

(B) 1

(C) 2

(D) 3

9 The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k ?

(A) 5

(B) 6

(C) 25

(D) 36

10 Let r and s be integers, then $\dfrac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$ is an integer if

(A) $r + s \leq 0$

(B) $s \leq 0$

(C) $r \leq 0$

(D) $r \geq s$

END OF SECTION I

Section II

120 marks
Attempt Questions 11 – 18
Allow about 2 hours 15 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

In Questions 11 to 18, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 ***(15 marks)*** Use a *separate* answer sheet

(a) Write $\sqrt[4]{x^5}$ in index form. *1*

(b) Evaluate $\sqrt{\dfrac{4.81 \times 10^5}{7.36 \times 10^9}}$ correct to two significant figures. *2*

(c) If $f(x) = 7 - 2x^2$, find the value of $f(-1)$. *1*

(d) Simplify $\sqrt{75} - 2\sqrt{27}$. *2*

(e) Expand and simplify $(3x - 4)(x - 2)(x + 2)$. *2*

(f) Solve the inequation $\dfrac{2}{x-1} \leq 1$. *3*

(g) Factorise

(i) $2x^2 + 3x - 2$ *1*

(ii) $x^3 + 5x^2 + x + 5$ *1*

(iii) $4a^2(x^3 + 18ab^2) - (32a^5 + 9b^2x^3)$ *2*

Marks

Question 12 ***(15 marks)*** Use a *separate* answer sheet

(a) Express as a single fraction in simplest terms

(i) $\dfrac{125a^3 - 8}{a^2 - 7a + 10} \times \dfrac{a - 5}{25a^2 - 4}$ *2*

(ii) $\dfrac{2}{x^2 - 1} - \dfrac{1}{x^2 - x} + \dfrac{x - 1}{x^2 + x}$ *3*

(b) Solve

(i) $\dfrac{3x - 2}{4} - \dfrac{2x - 1}{8} = 5$ *2*

(ii) $|2x + 6| = 3x - 1$ *3*

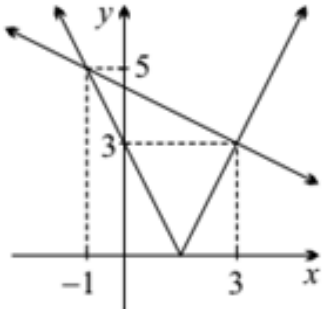
(iii) Solve $2x^2 - 6x \leq 0$ *2*

(iv) $\dfrac{x + 1}{x^2 - 4} \geq 0$ *3*

Marks

<u>Question 13</u> ***(15 marks)*** Use a *separate* answer sheet

(a) The graph below shows the absolute value function $y = |2x - 3|$ and the straight line $y = \dfrac{9 - x}{2}$. *1*



Use the graph to solve the inequation $|2x - 3| \geq \dfrac{9 - x}{2}$.

(b) Rationalise the denominator of $\dfrac{10}{\sqrt{5} - \sqrt{3}}$. *2*

(c) Express $0.01\dot{2}\dot{3}$ as a fraction in its simplest form, without the aid of a calculator. *2*

(d) Make *v* the subject of the formula $F = \dfrac{mv^2}{gr}$ *2*

(e) Use the method of "completing the square" to solve the equation *2*

$$x^2 - 8x + 9 = 0$$

Question 13 continues on page 9

Marks

Question 13 (continued)

(f) For the equation $(x + 2y)(2x - y) + (x - y)(3x + 4y) = 22$

(i) Verify that $x = 2$ and $y = 1$ is a possible solution *1*

(ii) Find any other value(s) of x which makes the equation true when $y = 1$ *2*

(g) Consider the function $f(x) = x^2 - y^2 + x + 3y - 2$.

(i) Show that $f(x) = (x - y + 2)(x + y - 1)$ *1*

(ii) Hence, or otherwise, sketch the region $x^2 - y^2 + x + 3y > 2$. *2*

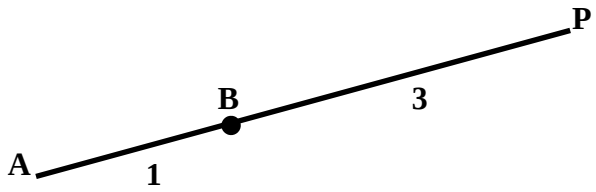
End of Question 13

Marks

Question 14 ***(15 marks)*** Use a *separate* answer sheet

(a) The diagram below can be described by the statement; *1*

"B divides AP in the ratio 1:3"



Write down two different statements, other than the one given above, that would also describe the diagram

(b) Consider the function $y = f(x)$

(i) State the condition for the function to be even. *1*

(ii) Give an example of an even function. *1*

(c) The function $f(x)$ is defined as *2*

$$f(x) = \begin{cases} x^2 + 1 & : x > 3 \\ 3x & : -2 \leq x \leq 3 \\ 2 & : x < -2 \end{cases}$$

Find $f(2) + f(5) - f(-2)$.

(d) Consider the function $y = \sqrt{x^2 - 16}$

(i) Find the domain of the function. *1*

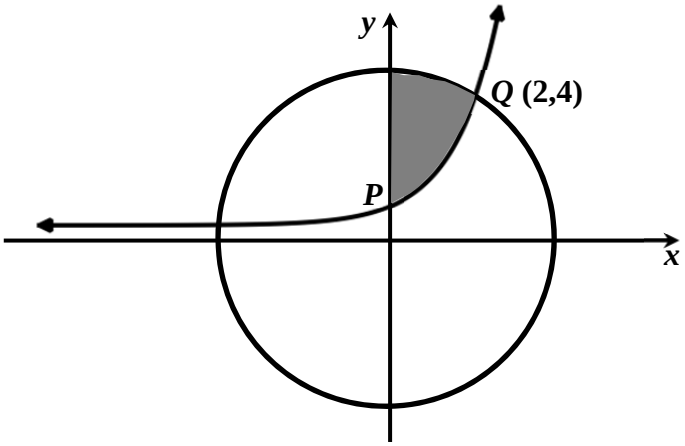
(ii) Find the range of the function. *1*

Question 14 continues on page 11

Marks

Question 14 (continued)

(e) A circle, centred at the origin and an exponential of the form $y = a^x$ are shown below.



P and *Q* lie on the curves as shown, *Q* has coordinates $(2,4)$

(i) Write down the coordinates of *P*. *1*

(ii) Find the equation of the circle. *2*

(iii) Find the equation of the exponential function. *1*

(iv) Give the three inequations which combine to define the shaded region. *2*

(f) Is the function $f(x) = |x - 2| - |x + 2|$ odd, even or neither? *2*
Justify your answer.

End of Question 14

Marks

Question 15 ***(15 marks)*** Use a *separate* answer sheet

(a) It is known that $\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$.

Using this value, find the following;
(*Note: your working MUST show how you used* $\sin 15°$)

(i) $\cos 75°$. *1*

(ii) $\text{cosec}\ 15°$. *1*

(iii) $\sin 195°$. *1*

(b) Solve the simultaneous equations *2*

$$2x + y = 4$$
$$5x + 2y = 9$$

(c) Prove the relationship $\sin^4 x - \cos^4 x \equiv 1 - 2\cos^2 x$. *2*

(d) A student is asked to solve $\tan x = \sin x$, giving all solutions in the range $0° \leq x \leq 360°$.
Below is the student's working;

$$\tan x = \sin x$$
$$\frac{\sin x}{\cos x} = \sin x \qquad (\text{write } \tan x \text{ as } \frac{\sin x}{\cos x})$$
$$\sin x = \sin x \cos x \qquad (\text{multiply by } \cos x)$$
$$1 = \cos x \qquad (\text{cancel } \sin x)$$
$$x = 0°, 360°$$

Whilst the student did find two correct answers, there is another answer that they did not find.

(i) What did the student do wrong in their working? *1*

(ii) Find the missing answer. *1*

(e) Solve, to the nearest degree where necessary, for $0° \leq \theta \leq 360°$

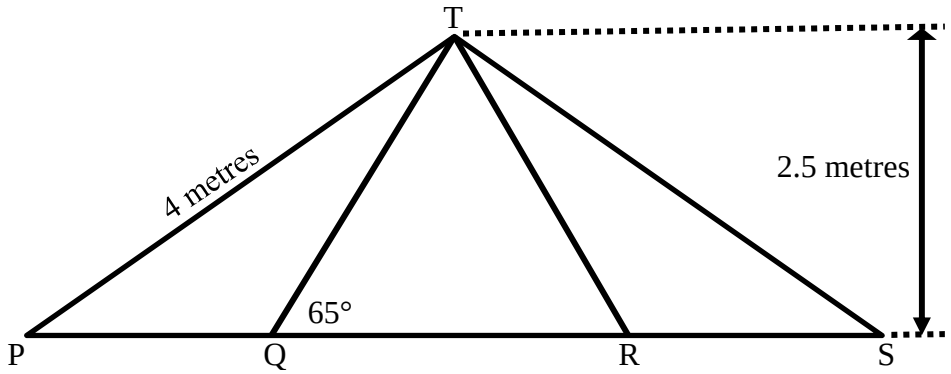
(i) $\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta$. *3*

(ii) $\sin(20° - 2\theta) = \dfrac{1}{7}$. *3*

Marks

Question 16 ***(15 marks)*** Use a *separate* answer sheet

(a) The diagram below represents a framework for supporting a roof. The outer beams TP and TS are each 4 metres in length, the inner beams TQ and TR are inclined at $65°$ to the horizontal and the height of the framework is 2.5 metres.



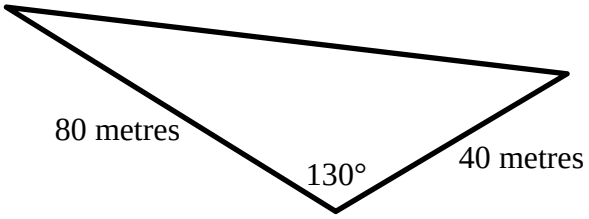
Calculate the;

(i) angle TPS, correct to the nearest degree *1*

(ii) width PS, correct to the nearest centimetre. *1*

(iii) length of TQ, correct to the nearest centimetre. *1*

(b) A field is triangular, with two sides of 80 metres and 40 metres, enclosing an angle of $130°$.



(i) Calculate the area of the field, correct to the nearest metre squared. *1*

(ii) Without calculating its length, how do you know that the third side must be the largest side of the triangle? *1*

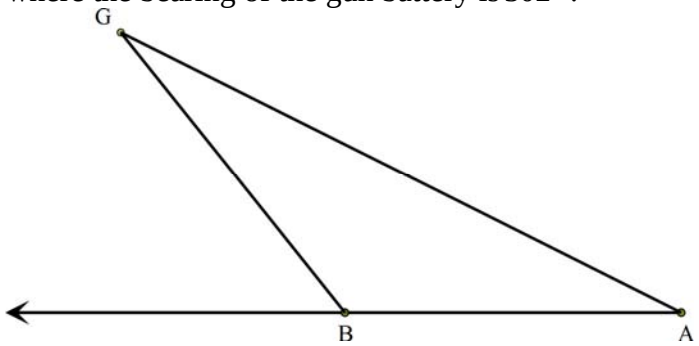
(ii) Use the cosine rule to calculate the length of the third side, correct to the nearest metre. *1*

Question 16 continues on page 14

Marks

Question 16 (continued)

(c) A and B are two points 1500 metres apart on a road running due west. A soldier at A observes that the bearing of an enemy's gun battery at G is 296° and heads towards B, where the bearing of the gun battery is 302°.



(i) Copy this diagram onto your answer sheet, showing the given information. *1*

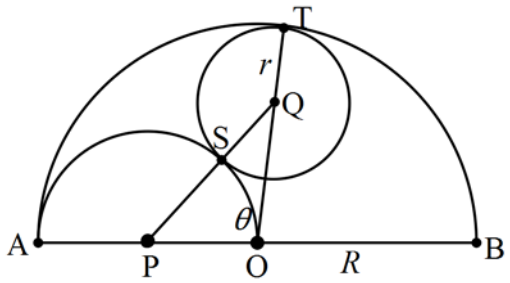
(ii) Explain why $\angle AGB = 6^\circ$ *1*

(iii) Show that $BG = \dfrac{1500\sin 26^\circ}{\sin 6^\circ}$ *1*

(iv) The range of the guns in the enemy's battery is 5 km. How far past B can the soldier travel before being in range of the enemy's guns? Give your answer correct to the nearest metre. *3*

(d) Inside a large semicircle with centre O and diameter AOB of length $2R$, a smaller semicircle is drawn with diameter AO and centre P. A small circle is drawn with centre Q and radius r which is tangent to the large semicircle at T and the small semicircle at S. Let $\angle POQ = \theta$. *3*

(Note: P, S and Q and O, Q and T are collinear)



Show that $\dfrac{r}{R} = \dfrac{1-\cos\theta}{3-\cos\theta}$

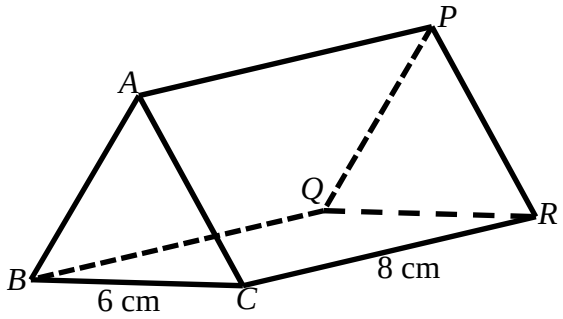
End of Question 16

Marks

Question 17 ***(15 marks)*** Use a *separate* answer sheet

(a) Given the points $A(-2,4)$ and $B(3,11)$, find the coordinates of the point P, which divides the interval AB in the ratio 2:3. *2*

(b) The base of the triangular prism shown below is a rectangle 8 cm long and 6 cm wide. The vertical faces ABC and PQR are equilateral triangles.



Calculate, to the nearest degree;

(i) the angle between the plane PBC and the base. *2*

(ii) the angle between the diagonal PC and the base. *2*

(c) Find the vertical and horizontal asymptotes of the graph of the function; *2*

$$y = \frac{x^2}{x^2 + 5x + 6}$$

(d) It is known that the difference of two squares factorisation is given by; *1*

$$a^2 - b^2 = (a - b)(a + b)$$

This factorisation can be generalised for higher powers to;

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + b^{n-1})$$

Using this generalisation, or otherwise, determine whether $3^{2018} - 2^{2018}$ is a prime number.

Question 17 continues on page 16

Marks

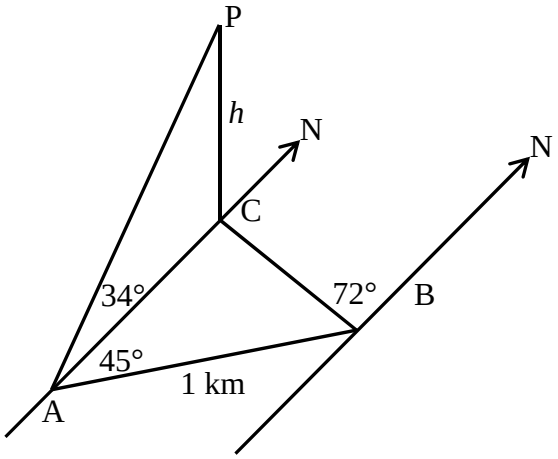
Question 17 (continued)

(e) The point $K(x,y)$ divides the interval joining $M(1,2)$ and $N(-1,-4)$ in the ratio k:1.

(i) Write expressions for x and y in terms of k. *1*

(ii) Find the value of k, such that the line $3x - 4y - 5 = 0$ divides MN in the ratio k:1. *2*

(f) *3*



An aeroplane is sighted simultaneously from two stations A and B, B being one kilometre north-east of A. To the observer at A the aeroplane appears due north at an elevation of $34°$; to the observer at B it appears in a direction $\text{N}72°\text{W}$.

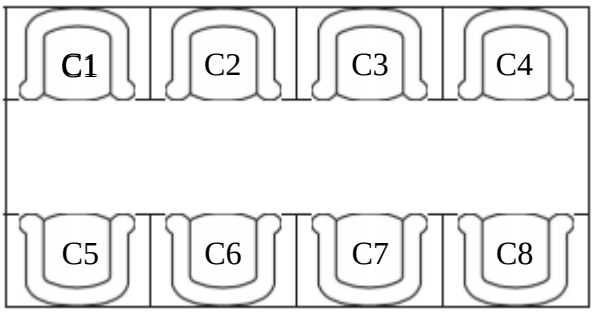
Find the height of the aeroplane, correct to the nearest metre..

End of Question 17

Marks

Question 18 ***(15 marks)*** Use a *separate* answer sheet

(a) A railway carriage compartment consists of two bench seats facing each other, with each bench being able to accommodate four people.



(i) How many ways can a party of eight people be seated in this compartment? *1*

(ii) How many ways can the eight passengers take their seats if Aiden and Alex must sit next to each other? *2*

(b) There are thirteen boys and nine girls in a class. From this class, five students are randomly selected to form the class committee.

(i) How many possibilities are there for the committee? *1*

(ii) How many of the committees would contain at least one boy and one girl? *2*

(iii) The committee is to be seated around a table. In how many ways could the committee be seated? *1*

(c) Consider the letters of **ZOOLOGICAL**.

(i) How many distinct arrangements of all of the letters are possible? *1*

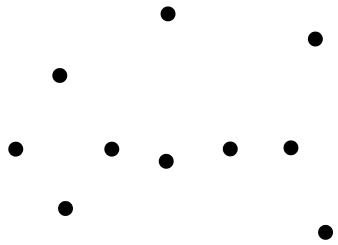
(ii) How many arrangements are possible if no letter **O** is placed next to another letter **O**? *2*

Question 18 continues on page 18

Marks

Question 18 (continued)

(d) *2*



There are ten points in a plane, four of which are collinear. How many triangles can be found using these points?

(e) If you have one white, two blue and three red flags, find how many signals you can make, each containing four flags arranged one above the other. *3*

End of paper