## **BAULKHAM HILLS HIGH SCHOOL**

## YEAR 11 MATHEMATICS EXTENSION HALF YEARLY ASSESSMENTS 2018 SOLUTIONS

Solution	Marks	Comments
SECTION I		
<b>1. B</b> - $8x^3 + 27 = (2x)^3 + 3^3$		
$= (2x + 3)((2x)^{2} - (2x)(3))$ = (2x + 3)(4x^{2} - 6x + 9)	1	
$= (2x+3)(4x^2-6x+9)$		
2. D – the only graph that satisfies the "vertical line test"	1	
<b>3.</b> $\mathbf{A} - \frac{1+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+3\sqrt{3}+3}{4-3}$		
<b>3.</b> A - $\frac{1}{2-\sqrt{3}} \times \frac{1}{2+\sqrt{3}} = \frac{1}{4-3}$	1	
$=5+3\sqrt{3}$		
4. <b>D</b> – from the diagram;		
Angle of elevation is $\alpha$	1	
Angle of depression is $\theta$		
5. C - $\cos\theta < 0$ and $\sin\theta < 0 \Rightarrow$ quadrant 3, $\therefore \tan\theta > 0$	1	
$\tan \theta = \frac{4}{2}$	1	
$\tan \theta = \frac{4}{3}$		
x+5 $x-1$ $x+5$ $x-1$		
6. $\mathbf{A} - \frac{x+5}{(x-3)(x+1)} - \frac{x-1}{x^2 - x - 2} = \frac{x+5}{(x-3)(x+1)} - \frac{x-1}{(x-2)(x+1)}$		
(x - 5)(x - 1) - (x - 2)(x - 2) - (x - 1)(x - 3)		
$=\frac{1}{(x-3)(x-2)(x+1)}$	1	
$\frac{x^2 + 3x - 10 - x^2 + 4x - 3}{3}$	1	
$=\frac{(x+5)(x+1)}{(x-2)(x+1)} = \frac{(x+5)(x-2) - (x-1)(x-3)}{(x-3)(x-2)(x+1)} = \frac{x^2 + 3x - 10 - x^2 + 4x - 3}{(x-3)(x-2)(x+1)} = \frac{7x - 13}{7x - 13}$		
$=\frac{1/x-13}{1}$		
(x-3)(x-2)(x+1)		
7. C - $\frac{\sin(360^\circ - A)}{\sin(90^\circ - A)} = \frac{-\sin A}{\cos A}$	-	
	1	
$= -\tan A$ 8. <b>B</b> - $f(a) = g(2a)$		
$3a^{2} - 4 = (2a)^{2} - 2(2a)$		
$3a^{2} - 4 = (2a)^{2} - 2(2a)^{2}$ $3a^{2} - 4 = 4a^{2} - 4a$		
$3a^2 - 4a - 4a^2 - 4a^2 = 4a$	1	
$(a-2)^2 = 0$ $a=2 \implies$ only one solution		
9. C - $x^2 + y^2 - 12x - 10y + k = 0$		
x + y = 12x - 10y + k = 0 (x - 6) <sup>2</sup> + (y - 5) <sup>2</sup> = 61 - k $\Rightarrow$ centre is (6,5)		
$(x-6) + (y-5) - 61 - k \Rightarrow \text{ centre is } (6,5)$		
For exactly three intercepts the radius $= 6$	1	
61 - k = 36 (6,5)	-	
k = 25		
$\mathbf{10.B} - \frac{6^{r+s} \times 12^{r-s}}{8^{r} \times 9^{r+2s}} = \frac{2^{r+s} \times 3^{r+s} \times 2^{2r-2s} \times 3^{r-s}}{2^{3r} \times 3^{2r+4s}}$ $- \frac{2^{3r-s} \times 3^{2r}}{2^{3r-s} \times 3^{2r}}$		
10. B - $\frac{1}{9^{r} \times 9^{r+2s}} = \frac{1}{29^{3r} \times 9^{2r+4s}}$		
$2^{3r-s} \times 3^{2r}$		
$=\frac{2}{2^{3r}\times 3^{2r+4s}}$	1	
$=2^{-s}\times 3^{-4s}$		
$= 2^{-s} \times 3^{-s}$ Thus $s \le 0$ in order for expression to be an integer		
Thus 5 2 0 m older for expression to be an integer		

SECTION II			
QUESTION 11 Solution	Marks	Comments	
11(a) $\sqrt[4]{x^5} = x^{\frac{5}{4}}$	1	1 mark • Correct answer	
11 (b) $\sqrt{\frac{4.81 \times 10^5}{7.36 \times 10^9}} = 0.00808413637$ = 0.0081 correct to two significant figures	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Performs the correct calculation</li> </ul>	
11(c) $f(-1) = 7 - 2(-1)^2$ = 7 - 2 = 5	1	<ul><li>1 mark</li><li>• Correct answer</li></ul>	
$= 5$ 11 (d) $\sqrt{75} - 2\sqrt{27} = 5\sqrt{3} - 6\sqrt{3}$ $= -\sqrt{3}$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Simplifies at least one surd correctly</li> </ul>	
11 (e) $(3x - 4)(x - 2)(x + 2) = (3x - 4)(x^{2} - 4) = 3x^{3} - 12x - 4x^{2} + 16 = 3x^{3} - 4x^{2} - 12x + 16$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Performs a binomial product expansion</li> </ul>	
$= (3x - 4)(x^{2} - 4)$ $= 3x^{3} - 12x - 4x^{2} + 16$ $= 3x^{3} - 4x^{2} - 12x + 16$ 11 (f) $\frac{2}{x - 1} \le 1$ $x - 1 \ne 0$ $x \ne 1$ $x = 3$ $1$ $x < 1 \text{ or } x \ge 3$	3	<ul> <li>3 marks</li> <li>Correct graphical solution on number line or algebraic solution, with correct working</li> <li>2 marks</li> <li>Bald answer</li> <li>Identifies the two correct critical points via a correct method</li> <li>Correct conclusion to their critical points obtained using a correct method</li> <li>1 mark</li> <li>Uses a correct method</li> <li>Acknowledges a problem with the denominator.</li> <li>0 marks</li> <li>Solves like a normal equation , with no consideration of the denominator.</li> </ul>	
<b>11 (g) (i)</b> $2x^2 + 3x - 2$ = $(2x - 1)(x + 2)$	1	1 mark • Correct answer	
11 (g) (ii) $x^3 + 5x^2 + x + 5$ = $x^2(x + 5) + 1(x + 5)$ = $(x + 5)(x^2 + 1)$	1	<ul><li>1 mark</li><li>• Correct answer</li></ul>	
11 (g) (iii) $4a^{2}(x^{3} + 18ab^{2}) - (32a^{5} + 9b^{2}x^{3}) = x^{3}(4a^{2} - 9b^{2}) - 8a^{3}(4a^{2} - 9b^{2}) = (4a^{2} - 9b^{2})(x^{3} - 8a^{3}) = (2a - 3b)(2a + 3b)(x - 2a)(x^{2} + 2ax + 4a^{2})$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Correctly uses a<sup>2</sup> - b<sup>2</sup> or a<sup>3</sup> - b<sup>3</sup> factorisation</li> </ul>	
QUESTION 12 12 (a) (i) $\frac{125a^3 - 8}{a^2 - 7a + 10} \times \frac{a - 5}{25a^2 - 4}$ $= \frac{(5a - 2)(25a^2 + 10a + 4)}{(a - 5)(a - 2)} \times \frac{a - 5}{(5a - 2)(5a + 2)}$ $= \frac{25a^2 + 10a + 4}{(a - 2)(5a + 2)}$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Factorises 2 out of the 3 non-linear expressions</li> </ul>	

QUESTION 12continu		
Solution	Marks	Comments 2 membra
12 (a) (ii) $\frac{2}{x^2 - 1} - \frac{1}{x^2 - x} + \frac{x - 1}{x^2 + x}$ $= \frac{2}{(x - 1)(x + 1)} - \frac{1}{x(x - 1)} + \frac{x - 1}{x(x + 1)}$ $= \frac{2x - (x + 1) + (x - 1)^2}{x(x + 1)(x - 1)}$ $= \frac{2x - x - 1 + x^2 - 2x + 1}{x(x + 1)(x - 1)}$ $= \frac{x^2 - x}{x(x + 1)(x - 1)}$ $= \frac{x(x - 1)}{x(x + 1)(x - 1)}$ $= \frac{1}{x(x + 1)(x - 1)}$	3	<ul> <li>3 marks</li> <li>Correct solution</li> <li>2 marks</li> <li>Rewrites as a single fraction</li> <li>1 mark</li> <li>Finds the LCD</li> </ul>
$x(x + 1)(x - 1)$ $= \frac{1}{x + 1}$ <b>12 (b) (i)</b> $\frac{3x - 2}{4} - \frac{2x - 1}{8} = 5$ $2(3x - 2) - (2x - 1) = 40$ $6x - 4 - 2x + 1 = 40$ $4x = 43$ $x = \frac{43}{4}$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Removes the fractions by multiplying by the LCD, or equivalent</li> </ul>
<b>12 (b) (ii)</b>  2x + 6  = 3x - 1 2x + 6 = 3x - 1 x = 7 OR x = -1 x = 7 OR x = -1 NOT A SOLUTION $\therefore x = 7$	3	<ul> <li>3 marks</li> <li>Correct solution</li> <li>2 marks</li> <li>Finds two "possible" answers</li> <li>1 mark</li> <li>Finds an answer without considering cases.</li> </ul>
<b>12 (b) (iii)</b> $2x^2 - 6x \le 0$ $2x(x-3) \le 0$ $0 \le x \le 3$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Establishes the two critical points of the solution.</li> </ul>
12(b) (iv) $\frac{x+1}{x^2-4} \ge 0$ $x^2-4 \ne 0$ $x \ne \pm 2$ $-2 -1$ $2$ $-2 < x \le -1 \text{ or } x > 2$ OUESTION 12	3	<ul> <li>3 marks</li> <li>Correct graphical solution on number line or algebraic solution</li> <li>2 marks</li> <li>Bald answer</li> <li>Identifies the three correct critical points via a correct method</li> <li>Correct conclusion to their critical points obtained using a correct method</li> <li>1 mark</li> <li>Uses a correct method</li> <li>Acknowledges a problem with denominator.</li> </ul>
QUESTION 1313 (a) Using the graph, the question becomes; "whenis the linebelow the absolute value graph?" As we are only concerned with the x values, the solution is $x \le -1$ or $x \ge 3$	1	1 mark • Correct solution
13 (b) $\frac{10}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{10(\sqrt{5} + \sqrt{3})}{5 - 3}$ $= \frac{10(\sqrt{5} + \sqrt{3})}{2}$ $= 5(\sqrt{5} + \sqrt{3})$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Attempts to multiply by the conjugate of the denominator</li> </ul>

QUESTION 13continued.			
Solution 122 1	Marks Comments		
<b>13 (c)</b> let $x = 0.01232323$ <b>OR</b> $0.01\dot{2}\dot{3} = \frac{123 - 1}{9900}$ 100x = 1.23232323 $99x = 1.22$ $= \frac{122}{9900}$ $x = \frac{122}{9900}$ $= \frac{61}{4950}$ $\dot{x} 0.01\dot{2}\dot{3} = \frac{61}{4950}$	2 marks • Correct solution 1 mark • Evidence of a valid manual calculation 2		
13 (d) $F = \frac{mv^{2}}{gr}$ $mv^{2} = Fgr$ $v^{2} = \frac{Fgr}{m}$ $v = \pm \sqrt{\frac{Fgr}{m}}$	2 marks • Correct solution 1 mark • makes $v^2$ the subject of the formula		
<b>13 (e)</b> $x^{2} - 8x + 9 = 0$ $x^{2} - 8x = -9$ $x^{2} - 8x + 4^{2} = 7$ $(x - 4)^{2} = 7$ $x - 4 = \pm\sqrt{7}$ $x = 4\pm\sqrt{7}$ <b>OR</b> $x^{2} - 8x + 9 = 0$ $(x - 4)^{2} - 7 = 0$ $(x - 4)^{2} - 7 = 0$ $(x - 4 - \sqrt{7})(x - 4 + \sqrt{7}) = 0$ $x = 4\pm\sqrt{7}$	2 marks • Correct solution 1 mark 2 • Completes the square to obtain $(x-4)^2$ • Solves the equation for a different completed square.		
<b>13 (f) (i)</b> when $x = 2$ and $y = 1$ (x + 2y)(2x - y) + (x - y)(3x + 4y) = (2 + 2)(4 - 1) + (2 - 1)(6 + 4) = 12 + 10 = 22	) 1 <b>mark</b> • Correct answer		
<b>13 (f) (ii)</b> when $y = 1$ (x + 2)(2x - 1) + (x - 1)(3x + 4) = 22 $2x^2 + 3x - 2 + 3x^2 + x - 4 = 22$ $5x^2 + 4x - 28 = 0$ (x - 2)(5x + 14) = 0 $x = 2$ or $x = -\frac{14}{5}$ $\therefore$ the only other posible value of x is $-\frac{14}{5}$	2 marks • Correct solution 1 mark • Establishes quadratic in terms of x 2		
<b>13 (g) (i)</b> $(x - y + 2)(x + y - 1)$ = $x^{2} + xy - x - xy - y^{2} + y + 2x + 2y - 2$ = $x^{2} - y^{2} + x + 3y - 2$ = $f(x)$	1 mark • Correct solution		
<b>13 (g) (ii)</b> $x^{2} - y^{2} + x + 3y > 2$ $x^{2} - y^{2} + x + 3y - 2 > 0$ (x - y + 2)(x + y - 1) > 0 $\therefore$ either $x - y + 2 > 0$ and $x + y - 1 > 0$ OR $x - y + 2 < 0$ and $x + y - 1$ <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b>	<ul> <li>&lt; 0</li> <li>2 marks</li> <li>• Correct solution</li> <li>1 mark</li> <li>• Some correct regions indicated, with no more than one incorrect region.</li> </ul>		

QUESTION 14			
Solution	Marks	Comments	
<ul> <li>14 (a) some possibilities would include;</li> <li>A divides BP externally in the ratio 1:4</li> <li>B divides PA in the ratio 3:1</li> <li>P divides BA externally in the ratio 3:4</li> </ul>	1	<ul><li>1 mark</li><li>• Two different statements</li></ul>	
<b>14 (b) (i)</b> $f(x)$ is even if $f(-x) = f(x)$	1	1 mark • Correct condition	
14 (b) (ii) some examples would include; $y = x^{2}$ y =  x  $y = \cos x$	1	1 mark • Correct example	
<b>14 (c)</b> $f(2) + f(5) - f(-2) = 3(2) + (5^2 + 1) - 3(-2)$ = 6 + 26 + 6 = 38	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>finds at least two function values</li> </ul>	
<b>14 (d) (i)</b> $x^2 - 16 \ge 0$ $x \le -4$ or $x \ge 4$	1	1 mark • Correct answer	
$x \le -4 \text{ or } x \ge 4$ 14 (d) (ii) $y \ge 0$	1	1 mark • Correct answer	
<b>14 (e) (i)</b> <i>P</i> (0,1)	1	1 mark • Correct answer	
<b>14 (e) (ii)</b> $x^{2} + y^{2} = 2^{2} + 4^{2}$ = 20 $\therefore$ circle has equation $x^{2} + y^{2} = 20$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Recognises that a circle's equation is of the form x<sup>2</sup> + y<sup>2</sup> = k</li> </ul>	
<b>14 (e) (iii)</b> (2,4) : $4 = a^2$ a = 2 $\therefore$ exponential has the equation $y = 2^x$	1	1 mark • Correct answer	
<b>14 (e) (iv)</b> $x \ge 0$ $x^2 + y^2 \le 20$ $y \ge 2^x$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>At least two correct inequations</li> </ul>	
14 (f) $f(-x) =  -x-2  -  -x+2 $ =  -1   x+2  -  -1   x-2  =  x+2  -  x-2  = - f(x) $\therefore$ function is odd	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Recognises the condition for an odd function</li> </ul>	
QUESTION 15		•	
<b>15 (a) (i)</b> $\cos 75^\circ = \sin(90^\circ - 75^\circ)$ $= \sin 15^\circ$ $= \frac{\sqrt{3} - 1}{2\sqrt{2}}$ <b>15 (a) (ii)</b> $\operatorname{cosec} 15^\circ = \frac{1}{\sin 15^\circ}$	1	1 mark • Correct solution	
<b>15 (a) (ii)</b> $\operatorname{cosec} 15^\circ = \frac{1}{\sin 15^\circ}$ $= \frac{2\sqrt{2}}{\sqrt{3} - 1}$	1	1 mark • Correct solution	
<b>15 (a) (iii)</b> $\sin 195^\circ = \sin(180^\circ + 15^\circ)$ = $-\sin 15^\circ$ = $\frac{1 - \sqrt{3}}{2\sqrt{2}}$	1	1 mark • Correct solution	
<b>15 (b)</b> $2x + y = 4$ 5x + 2y = 9 x = 1 $\therefore y = 2$ 2x + y = 4 5x + 2y = 9 x = 1	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Finds the correct value for one pronumeral</li> </ul>	

	QUESTION 15contin		1
	Solution	Marks	Comments
15 (c)	$\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$		<ul><li>2 marks</li><li>• Correct solution</li></ul>
	$=\sin^2 x - \cos^2 x$	2	1 mark
	$=(1-\cos^{2}x)-\cos^{2}x$		• Uses a valid trig identity in a relevant
	$= 1 - 2\cos^2 x$		manner.
	The student divided out a possible solution when canceling $\sin x$ .	1	1 mark
	When solving equations you can only divide by an unknown if there is no possibility that the unknown could equal zero.	1	• Correct explanation
	The missing answer comes from the possibility that;		1 mark
()()	$\sin x = 0$	1	• Correct solution
	$x = 0^{\circ}, 180^{\circ}, 360^{\circ}$	1	
	So the missing answer is $x = 180^{\circ}$ .		
15 (e) (i)	$\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta$		3 marks • Correct solution
	$\frac{\sin^2\theta}{\cos\theta} + 2\sin\theta = 3\cos\theta$		2 marks
			• Finds the two possibilities for $\tan \theta$
	$\sin^2\theta + 2\sin\theta\cos\theta = 3\cos^2\theta$	2	1 mark
	$3\cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta = 0$	3	• Correctly manipulates terms into a
	$(3\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 0$		quadratic equation
	$\tan \theta = -3 \qquad \text{OR} \qquad \tan \theta = 1 \\ \theta = 108^\circ, 288^\circ \qquad \qquad \theta = 45^\circ, 225^\circ$		
	$\theta = 108$ , 288 $\theta = 45^{\circ}$ , 225 $\theta = 45^{\circ}$ ,		
	c = 103, 233 $c = 45^{\circ}, 108^{\circ}, 225^{\circ}, 288^{\circ}$ $sin(20^{\circ} - 2\theta) = \frac{1}{7}$		3 marks
15 (e) (ii)	$\sin(20^\circ - 2\theta) = \frac{1}{7}$		• Correct solution
			2 marks
	$\sin \alpha = \frac{1}{7}$		• Finds four consecutive answers for
	$\alpha = 8^{\circ}$		$(20^\circ - 2\theta)$
	$(20^{\circ} - 2\theta) = 8^{\circ}, 172^{\circ}, 368^{\circ}, 532^{\circ}$	3	• Finds two answers for $\theta$
	$-2\theta = -12^{\circ}, 152^{\circ}, 348^{\circ}, 512^{\circ}$		1 mark
	$\theta = 6^{\circ}, -76^{\circ}, -174^{\circ}, -256^{\circ}$		• Finds four consecutive answers for $(20^\circ - 2\theta)$
	However solutions need to be $0^{\circ} \le \theta \le 360^{\circ}$ , so add $\pm 360^{\circ}$ to any answers not I required range		• Finds two answers for $\pm 2\theta$
	$\therefore \theta = 6^\circ, 104^\circ, 186^\circ, 284^\circ$		
	QUESTION 16	1	1
16(a) (i)	$\sin \angle TPS = \frac{2.5}{4}$		1 mark
		1	• Correct answer Note: no rounding penalty
	$\angle$ TPS = 36.6821875 = 36° to nearest degree		Note. no rounding penalty
	$( )^2$		1 mark
16 (a) (ii)	$\left(\frac{1}{2}PS\right)^2 = 4^2 - 2.5^2$		• Correct answer
· ()			Note: no rounding penalty
	$\frac{1}{2}$ PS = 3.1224998999	1	
	PS = 6.244997998		
	PS = 6.244997998 = 6.24 metres to the nearest centimetre		
	25		1 mark
16 (a) (iii)	$\frac{2.5}{\mathrm{TQ}} = \sin 65^{\circ}$		• Correct answer
	-	1	Note: no rounding penalty
	$TQ = \frac{2.5}{\sin 65^{\circ}}$	1	
	= 2.758444797		
	= 276 metres to the nearest centimetre		1
16 (b) (i)	Area = $\frac{1}{2} \times 40 \times 80 \times \sin 130^{\circ}$		1 mark • Correct answer
	= 1225.671109	1	<i>Note: no rounding penalty</i>
	= 1225.071105 = 1226 m <sup>2</sup>		g p children and a ch
16 (b) (ii)	The largest side is always opposite the largest angle, and since		1 mark
··· (~) (••)	the angle is obtuse, it must be the largest angle.	1	• Correct explanation
16 (b) (iii)			1 mark
	x = 110.0628943	1	• Correct answer
	x = 110 metres to the nearest metre		Note: no rounding penalty

QUESTION 16continued.	1	1
Solution	Marks	Comments
16 (c) (i) G W B 302° 1500 metres 296°	1	1 mark • Correct diagram with all information labelled
16 (c) (ii) $\angle GBW = 302^{\circ} - 270^{\circ} = 32^{\circ}$ $\angle GAB = 296^{\circ} - 270^{\circ} = 26^{\circ}$ $\angle GBW = \angle AGB + \angle GAB$ (exterior $\angle ABAG$ ) $32^{\circ} = \angle AGB + 26^{\circ}$ $\angle AGB = 6^{\circ}$	1	1 mark • Correct explanation Note: formal geometric explanation not required
16 (c) (iii) $\frac{BG}{\sin \angle GAB} = \frac{AB}{\sin \angle AGB}$ $\frac{BG}{\sin 26^{\circ}} = \frac{\frac{1500}{\sin 6^{\circ}}}{BG}$ $BG = \frac{1500 \sin 26^{\circ}}{\sin 6^{\circ}}$	1	<ul> <li>1 mark</li> <li>Evidence of using sine rule in finding the correct expression, or similar merit.</li> </ul>
16 (c) (iv) G 5000 metres W R B B B B B B B B	3	<ul> <li>3 marks</li> <li>Correct solution</li> <li>2 marks</li> <li>Establishes a quadratic equation in terms of BR or similar merit</li> <li>1 mark</li> <li>Attempts to find RG using the cosine rule or similar merit.</li> <li>Notes:</li> <li>no rounding penalty</li> <li>OK for approximate BG value (6290.695373) to be used in working and calculations</li> </ul>
<b>16 (d)</b> In $\Delta OPQ$ ; $\cos \theta = \frac{OP^2 + OQ^2 - PQ^2}{2 \times OP \times OQ}$ $= \frac{\left(\frac{R}{2}\right)^2 + (R - r)^2 - \left(\frac{R}{2} + r\right)^2}{2\left(\frac{R}{2}\right)(R - r)}$ $= \frac{\frac{R^2}{4} + R^2 - 2rR + r^2 - \frac{R^2}{4} - rR - r^2}{R(R - r)}$ $= \frac{\frac{R^2 - 3rR}{R(R - r)}}{R(R - r)}$	3	3 marks • Correct solution 2 marks • Establishes $\cos \theta = \frac{R-3r}{R-r}$ , or equivalent 1 mark • Finds a relationship between $\theta$ , R and r.

QUESTION 17		
Solution	Marks	Comments
<b>17 (a)</b> $A(-2,4)$ $B(3,-11)$ $P\left(\frac{6-6}{5},\frac{12-22}{5}\right)$ = (0,-2)	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Finds correct x or y value</li> </ul>
<b>17 (b) (i)</b> $PX^{2} = PR^{2} - RX^{2}$ $\tan \alpha = \frac{3\sqrt{3}}{8}$ $= 6^{2} - 3^{2}$ $\alpha = 33.0044916$ $= 27$ $\alpha = 33^{\circ}$ $PX = 3\sqrt{3}$ $B \ 6 \ cm \ C \ 8 \ cm$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Calculates <i>PX</i></li> <li>Correctly identifies the angle to be found <i>Note: no rounding penalty</i></li> </ul>
<b>17 (b) (ii)</b> $PC^{2} = PR^{2} + PR^{2}$ $= 6^{2} + 8^{2}$ = 100 PC = 10 $\sin\beta = \frac{3\sqrt{3}}{10}$ $\beta = 31.30644625$ $\beta = 31^{\circ}$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Calculates <i>PC</i> or <i>CX</i></li> <li>Correctly identifies the angle to be found <i>Note: no rounding penalty</i></li> </ul>
17 (c) $y = \frac{x^2}{x^2 + 5x + 6}$ $= \frac{x^2 + 5x + 6}{x^2 + 5x + 6} - \frac{5x + 6}{x^2 + 5x + 6}$ Thus; vertical asymptotes are $x = -3$ and $x = -2$ horizontal asymptote is $y = 1$ $= 1 - \frac{5x + 6}{(x + 3)(x + 2)}$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Finds both vertical asymptotes</li> <li>Finds the horizontal asymptote</li> </ul>
<b>17 (d)</b> $3^{2018} - 2^{2018} = (3^2)^{1009} - (2^2)^{1009}$ $= (3^2 - 2^2)(3^{2016} + 3^{2014} \times 2^2 + 3^{2012} \times 2^4 + + 2^{2016})$ As neither factor is equal to 1, then $3^{2018} - 2^{2018}$ is <b>not</b> prime	1	1 mark • Correct explanation
<b>17 (e) (i)</b> $M(1,2)$ $N(-1,-4)$ $x = \frac{1-k}{k+1}$ $y = \frac{2-4k}{k+1}$	1	1 mark • Correct answer
<b>17 (e) (ii)</b> $3\left(\frac{1-k}{k+1}\right) - 4\left(\frac{2-4k}{k+1}\right) - 5 = 0$ 3(1-k) - 4(2-4k) - 5(k+1) = 0 3-3k - 8 + 16k - 5k - 5 = 0 8k - 10 = 0 $k = \frac{10}{8}$ $k = \frac{5}{4}$	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Substitutes the found coordinates of <i>K</i> into the equation if the line.</li> </ul>

QUESTION 17continued.			
Solution	Marks	Comments	
17 (f) $\frac{AC}{h} = \tan 56^{\circ} \qquad \angle ACB = 72^{\circ} \qquad (\text{alternate } \angle \text{'s} = \ \ \text{lines}) \\ AC = h \tan 56^{\circ} \qquad \angle ABC + 45^{\circ} + \angle ACB = 180^{\circ} \\ \angle ABC + 45^{\circ} + 72^{\circ} = 180^{\circ} \\ \angle ABC + 45^{\circ} + 72^{\circ} = 180^{\circ} \\ \angle ABC = 63^{\circ} \\ \frac{AC}{\sin 63^{\circ}} = \frac{1}{\sin 72^{\circ}} \\ \text{htan } 56^{\circ} = \frac{\sin 63^{\circ}}{\sin 72^{\circ}} \\ \text{htan } 56^{\circ} = \frac{\sin 63^{\circ}}{\sin 72^{\circ}} \\ h = \frac{\sin 63^{\circ}}{\sin 72^{\circ} \tan 56^{\circ}} \\ h = 0.63199198479 \\ h = 632 \text{ m}, \text{ correct to the nearest metre} \end{cases}$	3	<ul> <li>3 marks</li> <li>Correct solution</li> <li>2 marks</li> <li>Finds sufficient sides and angles in Δ<i>ABC</i> in order to solve th eproblem</li> <li>1 mark</li> <li>Finds an expression for AC in terms of <i>h</i>.</li> <li>Finds the missing two angles in Δ<i>ABC</i></li> <li>Note: full marks awarded for the "exact value" for h</li> </ul>	
QUESTION 18	1		
18 (a) (i)       Ways = 8!         = 40320         18(a) (ii)       Ways = 2! × $(3 × 2) × 6!$	1	1 mark • Correct answer 2 marks	
= 8640	2	<ul> <li>Correct solution</li> <li><b>1 mark</b></li> <li>Handles restriction in a logical manner</li> </ul>	
<b>18 (b) (i)</b> Possibilities $= {}^{22}C_5$ = 26334	1	1 mark • Correct answer	
<b>18(b) (ii)</b> Committees $= {}^{22}C_5 - {}^{13}C_5 - {}^{9}C_5$ = 24921	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Handles restriction in a logical manner</li> </ul>	
<b>18 (b) (iii)</b> Ways = 4! = 24	1	1 mark • Correct answer	
<b>18 (c) (i)</b> Arrangements $=\frac{10!}{3!2!}$ = 302400	1	<ul><li>1 mark</li><li>• Correct answer</li></ul>	
<b>18 (c) (ii)</b> Arrangements $=\frac{7!}{21} \times {}^{8}C_{3}$ = 141120	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Approaches the restriction in a logical manner</li> </ul>	
<b>18 (d)</b> # triangles = ${}^{10}C_3 - {}^{4}C_3$ = 116	2	<ul> <li>2 marks</li> <li>Correct solution</li> <li>1 mark</li> <li>Selects three points from ten, without considering the implication of the collinear points</li> </ul>	
18 (e) Case 1: 3 red plus 1 other colour $= 2 \times \frac{4!}{3!}$ = 8 Case 2: 2 red plus 2 blue $= \frac{4!}{2!2!}$ = 6 Case 3: 2 of one colour plus 2 other colours $= 2 \times \frac{4!}{2!}$ = 24 Total # signals $= 8 + 6 + 24$ = 38	3	<ul> <li>3 marks</li> <li>Correct solution</li> <li>2 marks</li> <li>Finds the correct number of ways in their different cases</li> <li>1 mark</li> <li>Considers different cases in an attempt to solve the problem</li> </ul>	