YEAR 11

MATHEMATICS EXTENTION 1 ASSESSMENT

JUNE 2008

TIME : 70 minutes

Directions:
Full working should be shown in every question.
Marks may be deducted for careless or badly arranged work.
Use black or blue pen only (not pencils) to write your solutions.
No liquid paper is to be used.
If a correction is made, one line is to be ruled through the incorrect answer.

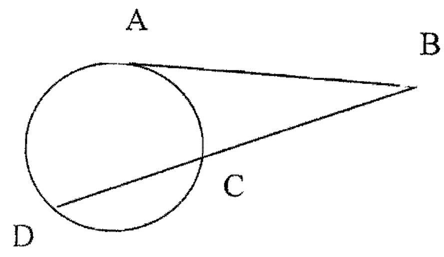
Marks

1. Solve $\frac{2x+3}{x-2} \leq 1$ **3**

2. Find, as an exact value sin 75°, showing working. **3**

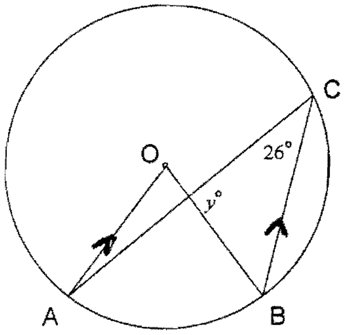
3. Find the coordinates of the point which divides the interval joining A(3, -4) And B(-2, 3) externally in the ratio 2 : 3 **2**

4. a) AB is a tangent to the circle.
DC is 5cm and CB is 4cm. Find the length of AB. **1**



NOT TO SCALE

b) Find the size of y **1**



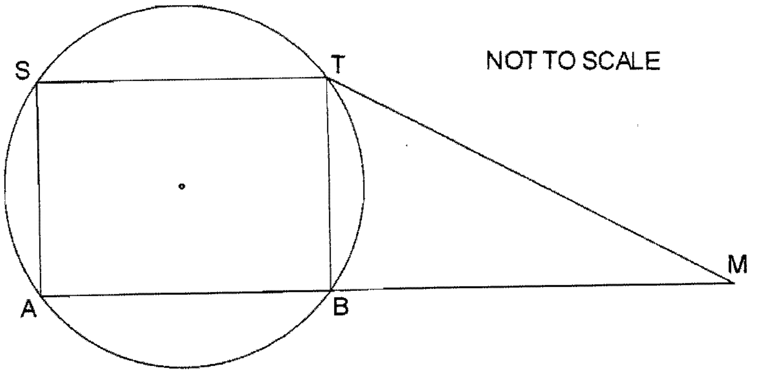
NOT TO SCALE

Marks

5. Find the acute angle between the lines $x - 2y = 0$ and $3x - y - 15 = 0$ **3**

6. Solve $\sin^2 \theta + 2 \sin\theta \cos \theta - 3 \cos^2 \theta = 0$ for $0^\circ \le \theta \le 360^\circ$ **4**

7.



Given ST || AB and TM is a tangent to the circle

i) Prove Δ TMB ||| Δ TAS **3**

ii) If BM = 4, SA = 5 and TB = 7, find the length of ST **2**

8. α and β are acute angles such that $\cos \alpha = \frac{3}{5}$ and $\sin \beta = \frac{1}{\sqrt{5}}$ **3**

Without finding the size of either angle, show that $\alpha = 2\beta$

9. Prove the trig identity **3**

$$\frac{\sin x}{\sin 2x} + \frac{\cos x}{1 + \cos 2x} = \sec x$$

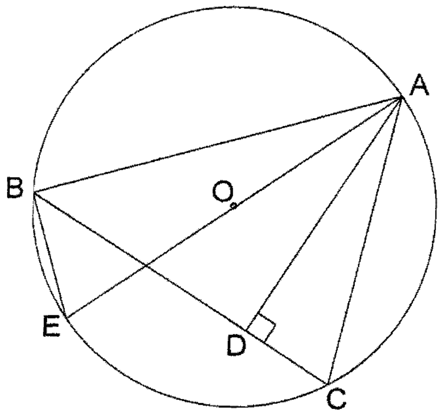
10. Express $\cos x - \sqrt{3} \sin x$ in the form

i) $A \cos (x + \alpha)$ where $A > 0$, $0 < \alpha < 90^\circ$ **2**

ii) Hence solve $\cos x - \sqrt{3} \sin x = 1$ for $0^\circ \le x \le 360^\circ$ **2**

Marks

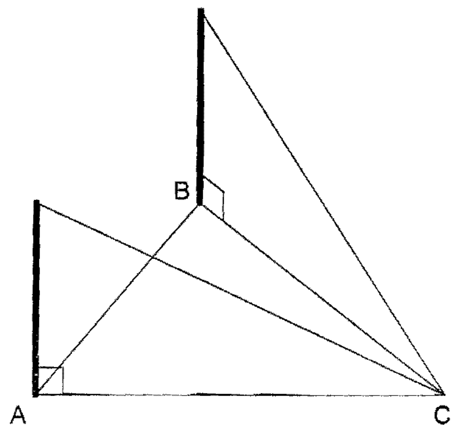
11. ABC is a triangle inscribed in a circle, centre 0, and AD is drawn perpendicular to BC. AE is the diameter. **4**



NOT TO SCALE

Prove $\angle BAE = \angle DAC$

12. A and B are the feet of two towers of equal heights. B lies due North of A. From a point C, 40m East of A and in the same horizontal plane, the angle of elevation at the top of the tower A is 53°.
From the same point, the angle of elevation of tower B is 35°.



NOT TO SCALE

i) Show that $AB = \dfrac{40\sqrt{\tan^2 53 - \tan^2 35}}{\tan 35}$ **3**

ii) Hence find the distance AB to the nearest metre. **1**

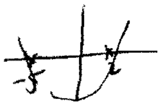
13. Prove the identity $\dfrac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$ **3**

Solutions yr11 Ext1. Exam June 2008.

1. $\dfrac{2x+3-x+2}{x-2} \le 0$

$(x-2)^2 \dfrac{(x+5)}{(x-2)} \le 0 \times (x-2)^2$ ① for working

$\therefore (x-2)(x+5) \le 0$



$-5 \le x \le 2$

but $x \ne 2$ ①

$\therefore \boxed{-5 \le x < 2}$ ①

[3]

2. $\sin 75 = \sin(45+30)$

$= \sin(45)\cos 30 + \cos 45 \sin 30$ ①

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$ ①

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$=$ ① $\frac{\sqrt{3}+1}{2\sqrt{2}}$ or $\frac{\sqrt{6}+\sqrt{2}}{4}$.

[3]

3. $(3,-4)$ (x_1, y_1); $(-2,3)$ (x_2, y_2); $m:n = 2:3$

$x = \frac{mx_2 - nx_1}{m-n}$ $\qquad$ $y = \frac{my_2 - ny_1}{m-n}$

$= \frac{2\times -2 - 3\times 3}{2-3}$ $\qquad$ $= \frac{2\times 3 - 3\times -4}{2-3}$

$= \frac{-13}{-1}$ $\qquad$ $= \frac{18}{-1}$

$\therefore (13, -18)$ ① ①

[2]

4. a) $AB^2 = DB \times CB$

$= 9 \times 4$

$= 36$

$AB = 6\text{cm}$ ①

b) $y = 180 - (26+52)$

$= 102^\circ$ ①

[2]

5) $l_1: y = \frac{1}{2}x$ ① $\qquad$ $l_2: y = 3x - 15$

$m_1 = \frac{1}{2}$ $\qquad$ $m_2 = 3$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{1}{2} - 3}{1 + \frac{1}{2}\times 3}\right|$ ①

$= 1$

$\theta = 45^\circ$ ①

[3]

6) $(\sin\theta + 3\cos\theta)(\sin\theta - \cos\theta) = 0$ ①

$\sin\theta = -3\cos\theta$ ① $\qquad$ $\sin\theta = \cos\theta$

$\tan\theta = -3$ $\qquad$ $\tan\theta = 1$ ①

$\theta = 108^\circ 26', 288^\circ 26'$ ① $\qquad$ $\theta = 45, 225$

$\therefore \theta = 45^\circ, 108^\circ 26', 225, 288^\circ 26'$

[4]

7) In $\triangle TMB$ and $\triangle TAS$

i) $A\hat{S}T = T\hat{B}M$ (ext ∠ of cyclic quad equal to inter. opp ∠) ①

$M\hat{T}B = T\hat{A}B$ (∠ between tangent and chord at pt of contact equal to ∠ in alternate segment) ①

since $ST \parallel AB$ (given)

then $S\hat{T}A = T\hat{A}B$ (alternate ∠s, equal)

$= M\hat{T}B$ ①

$\therefore \triangle TMB \sim \triangle TAS$ (matching angles equal)

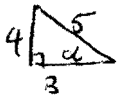
ii) $\frac{ST}{TB} = \frac{SA}{BM}$ (matching sides in ratio)

$\therefore \frac{ST}{7} = \frac{5}{4}$ ①

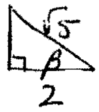
$ST = \frac{35}{4}$ ①

[5]

8. $\cos\alpha = \frac{3}{5}$



$\sin\beta = \frac{1}{\sqrt{5}}$ ①



$\sin\alpha = \frac{4}{5}$

$\sin 2\beta = 2\sin\beta\cos\beta$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ ①

$= \frac{4}{5}$ ①

[3]

since $\sin\alpha = \sin 2\beta$ then $\alpha = 2\beta$

9. $\dfrac{\sin x}{\sin 2x} + \dfrac{\cos x}{1+\cos 2x} = \sec x$

$LHS = \dfrac{\sin x}{2\sin x\cos x}$ ① $+ \dfrac{\cos x}{1+2\cos^2 x - 1}$ ①

$= \dfrac{1}{2\cos x} + \dfrac{\cos x}{2\cos^2 x}$

$= \dfrac{1}{2\cos x} + \dfrac{1}{2\cos x}$

$= \dfrac{2}{2\cos x} = \dfrac{1}{\cos x}$

$= \sec x = RHS$ ① (must have $= RHS$)

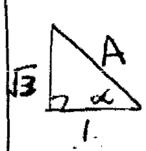
[3]

10. $\cos x - \sqrt{3}\sin x$

i) $A\cos x\cos\alpha - A\sin x\sin\alpha$

$1 = A\cos\alpha \qquad \sqrt{3} = A\sin\alpha$

$\cos\alpha = \frac{1}{A} \qquad \sin\alpha = \frac{\sqrt{3}}{A}$



$A = 2$ ① $\qquad \tan\alpha = \sqrt{3}$

$\alpha = 60°$ ①

$\therefore \cos x - \sqrt{3}\sin x = 2\cos(x+60°)$

ii) $2\cos(x+60) = 1 \qquad 60 \le x+60 \le 420$

$\cos(x+60) = \frac{1}{2}$ ①

$x+60 = 60, 300, 420$

$x = 0, 240, 360$ ①

[4]

11) Let $B\hat{A}E = x$ ①

$A\hat{B}E = 90°$ (∠ in a semi circle)

$\therefore B\hat{E}A = 90 - x$ (∠ sum of △ ABE) ①

$B\hat{C}A = 90 - x$ (∠s at circum, standing on same arc are equal) ①

$A\hat{B}C = 90$ (given)

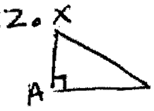
$\therefore D\hat{A}C = 180 - 90 - (90 - x)$ (∠ sum △ DAC) ①

$= x$

$\therefore B\hat{A}E = D\hat{A}C$

[4]

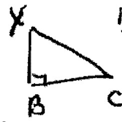
12.



In △ACX

$\tan 53 = \frac{h}{40}$

① $h = 40\tan 53$



In △BCY

$\tan 35 = \frac{h}{BC}$

$BC = \frac{h}{\tan 35}$

① $BC = \dfrac{40\tan 53}{\tan 35}$

$\therefore$ In △ABC $B\hat{A}C = 90°$

$\therefore BC^2 = AB^2 + AC^2$

$AB^2 = BC^2 - AC^2$

$= \left(\dfrac{40\tan 53}{\tan 35}\right)^2 - (40)^2$

$= \dfrac{40^2\tan^2 53 - 40^2\tan^2 35}{\tan^2 35}$ ①

$AB = \dfrac{40\sqrt{\tan^2 53 - \tan^2 35}}{\tan 35}$

ii) $AB = 64.396\ldots$

$= 64\,m$ ①

[4]

13. $\dfrac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$

$LHS = \left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right.$ ① $\left. - \tan\theta\right) \div \left(\dfrac{2\tan\theta}{1-\tan^2\theta} + \dfrac{1}{\tan\theta}\right)$

$= \dfrac{2\tan\theta - \tan\theta + \tan^3\theta}{(1-\tan^2\theta)}$ ① $\times \dfrac{\tan\theta(1-\tan^2\theta)}{(2\tan^2\theta + 1 - \tan^2\theta)}$

$= \dfrac{\tan\theta(1+\tan^2\theta) \times \tan\theta}{(\tan^2\theta + 1)}$ ①

$= \tan^2\theta$

$= RHS$ — no mark here refer (9)

(may be easier ① off each error)

[3]