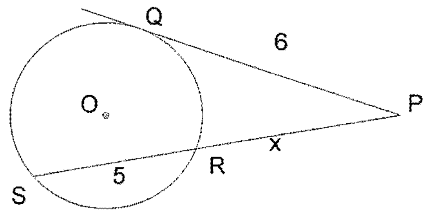
Name: ______________________________ **Teacher:** ______________________

YEAR 11 MATHEMATICS EXTENSION 1
ASSESSMENT TASK – JUNE 2009

Time Allowed: 70 minutes
Full working should be shown in every question.
Marks may be deducted for careless or badly arranged work.
No liquid paper is to be used.
If a correction is to be made, one line is to be ruled through the incorrect answer.

Marks

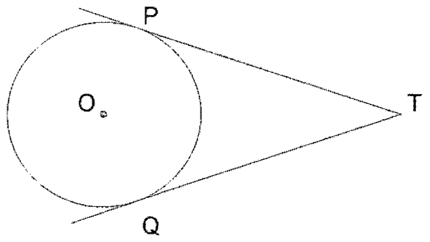
1. PQ is a tangent to the circle below. Find x giving reasons where necessary. 3



2. Find the acute angle between the lines $x-2y+1=0$ and $x+3y+2=0$. 3

3. Solve $2Tan\theta - 3Cot\theta = 5$ for $0^0 \le \theta \le 360^0$. 3

4. PT and QT are tangents to a circle with centre O. Prove $\angle PTQ = 2\angle PQO$ 3

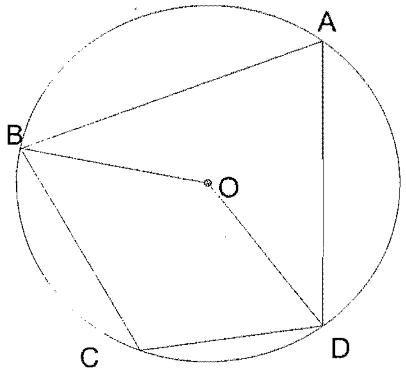


5. Express $2\cos\theta + 2\sqrt{3}\sin\theta$ in the form $R\cos(\theta-\alpha)$ and hence find the minimum value of the expression. 3

6. Prove, using $t=\tan\frac{\theta}{2}$, that $\dfrac{1+\cos ec\theta}{\cot\theta} = \dfrac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}$ 3

7. The point P(-2,9) divides the interval AB in the ratio k:1, where A is (7,-3) and B is (1,5). Find the value of k. 2

8. $\angle BAD = \alpha$ and $\angle BOD = \angle BCD$. Find α giving reasons. 3



9. Prove $\dfrac{SinA}{CosA + SinA} + \dfrac{SinA}{CosA - SinA} = Tan2A$ 3

10. The angle between two lines $y = mx$ and $y = \frac{1}{2}x$ is 45^0. Find the possible values of m. 3

11. Solve $\sqrt{2}Sin\theta - Cos\theta = 1$ for $0^0 \le \theta \le 360^0$ 3

12. AB and CD are two intersecting chords of a circle and CD is parallel to the tangent to the circle at B.

(a) Draw a neat sketch of the above information.

(b) Prove that AB bisects $\angle CAD$. 3

END OF TEST

YR 11 MATHS EXT 1 JUNE 09 SOLUTIONS /35

Q1. $x(x+5)=6^2$ (sq. of tangent = product of intercepts) — 1

$x^2+5x-36=0$

$(x+9)(x-4)=0$

$x=-9$ or 4 — 1

since $x>0$, $\underline{x=4}$ — 1 (3)

Q2 $x-2y+1=0 \therefore y=\frac{1}{2}x+\frac{1}{2} \therefore m_1=\frac{1}{2}$

$x+3y+2=0 \therefore y=-\frac{1}{3}x-\frac{2}{3} \therefore m_2=-\frac{1}{3}$ — 1

$\tan\alpha=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$=\left|\frac{\frac{1}{2}--\frac{1}{3}}{1+\frac{1}{2}\times-\frac{1}{3}}\right|$ — 1

$=\left|\frac{\frac{5}{6}}{\frac{5}{6}}\right|$

$=1$ (3)

$\therefore \underline{\alpha=45^\circ}$ — 1

Q3 $2\tan\theta-\frac{3}{\tan\theta}=5$

$2\tan^2\theta-3=5\tan\theta$

$2\tan^2\theta-5\tan\theta-3=0$ — 1

$2\tan^2\theta-6\tan\theta+\tan\theta-3=0$

$2\tan\theta(\tan\theta-3)+1(\tan\theta-3)=0$

$(2\tan\theta+1)(\tan\theta-3)=0$

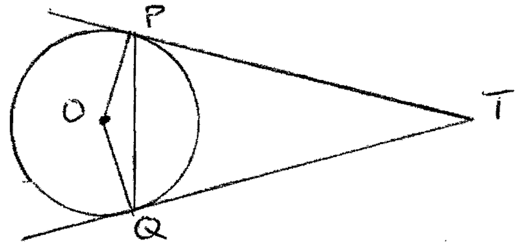
$\therefore \tan\theta=-\frac{1}{2}$ or $\tan\theta=3$ — 1

$\therefore \theta=153^\circ26', 333^\circ26', 71^\circ34', 251^\circ34'$ — 1 (3)

In order:

$\underline{\theta=71^\circ34', 153^\circ26', 251^\circ34', 333^\circ26'}$

Q4.



Let $\angle PQO=\alpha$

$\therefore \angle PQT=90^\circ-\alpha$ ($\angle$ bet. tang & radius $=90^\circ$) — 1

ΔPQT is isosceles (Tangs from ext. pt. are equal) — 1

$\therefore \angle QPT=90^\circ-\alpha$ (eq. $\angle$'s of isos. Δ)

$\therefore \angle PTQ=180^\circ-2(90^\circ-\alpha)$ ($\angle$ sum of Δ) — 1

$=2\alpha$

$\therefore \underline{\angle PTQ=2\times\angle PQO}$ (3)

Q5. $2\cos\theta+2\sqrt{3}\sin\theta\equiv R\cos(\theta-\alpha)$

$\equiv R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$

$\therefore R\cos\alpha=2$, $R\sin\alpha=2\sqrt{3}$

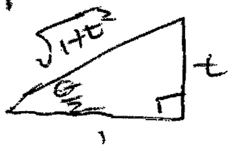
$\therefore R=\sqrt{2^2+(2\sqrt{3})^2}=4$

and $\tan\alpha=\frac{2\sqrt{3}}{2}=\sqrt{3} \therefore \alpha=60^\circ$

$\therefore \underline{\text{Exp.}=4\cos(\theta-60^\circ)}$ — 1-R, 1-α

$\underline{\text{Min. Value}=-4}$ — 1 (3)

Q6. If $\tan\frac{\theta}{2}=\frac{t}{1}$



then $\csc\theta=\frac{1}{\sin\theta}$

$=\frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$=\frac{1}{2\frac{t}{\sqrt{1+t^2}}\cdot\frac{1}{\sqrt{1+t^2}}}$

$=\frac{1+t^2}{2t}$ } — 1

and $\cot\theta=\frac{1}{\tan\theta}$

$=\frac{1}{\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}}$

$=\frac{1-t^2}{2t}$

$\therefore \text{LHS}=\frac{1+\frac{1+t^2}{2t}}{\frac{1-t^2}{2t}}$ — 1

$=\frac{2t+1+t^2}{1-t^2}$

$=\frac{(1+t)^2}{(1+t)(1-t)}$ — 1

$=\frac{1+t}{1-t}$ (3)

$=\text{RHS}$

Q7. Using x-values and $m=k, n=1$

$x_P=\frac{mx_2+nx_1}{m+n}$

$-2=\frac{k\times1+1\times7}{k+1}$ — 1

$-2k-2=k+7$

$-3k=9$

$\underline{k=-3}$ — 1 (2)

Q8. $\angle BAD = \alpha$ — 1

$\therefore \angle BOD = 2\alpha$ ($\angle$ at centre $= 2 \times \angle$ at circum. stand. on same arc)

$\angle BCD = 180^\circ - \alpha$ (opp. $\angle$'s of a cyclic quad. are suppl.)

But $\angle BOD = \angle BCD$ (given) — 1

$\therefore 2\alpha = 180^\circ - \alpha$

$3\alpha = 180^\circ$

$\underline{\alpha = 60^\circ}$ — 1 (3)

Q9. $\text{LHS} = \dfrac{\sin A\cos A - \sin^2 A + \cos A\sin A + \sin^2 A}{\cos^2 A - \sin^2 A}$ — 1

$= \dfrac{2\sin A\cos A}{\cos^2 A - \sin^2 A}$ — 1

$= \dfrac{\sin 2A}{\cos 2A}$ — 1

$= \tan 2A$

$\underline{= \text{RHS}}$ (3)

Q10. $\left|\dfrac{m - \frac{1}{2}}{1 + m\times\frac{1}{2}}\right| = \tan 45^\circ = 1$ — 1

$\dfrac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = \pm 1$

$\therefore m - \frac{1}{2} = 1 + \frac{1}{2}m$ or $m - \frac{1}{2} = -1 - \frac{1}{2}m$

$\frac{1}{2}m = \frac{3}{2}$ — 1 $\quad \frac{3}{2}m = -\frac{1}{2}$ — 1

$\underline{m = 3}$ or $\underline{m = -\frac{1}{3}}$ (3)

Q11. let $\sqrt{2}\sin\theta - \cos\theta \equiv R\sin(\theta - \alpha)$

$\equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$\therefore R\cos\alpha = \sqrt{2}$, $R\sin\alpha = 1$

$\therefore R = \sqrt{2+1} = \sqrt{3}$

and $\tan\alpha = \frac{1}{\sqrt{2}}$ $\therefore \alpha = 35^\circ 16'$ $\}$ — 1

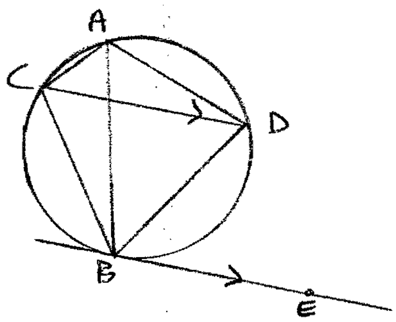
$\therefore \sqrt{3}\sin(\theta - 35^\circ 16') = 1$

$\sin(\theta - 35^\circ 16') = \frac{1}{\sqrt{3}}$ — 1

$\therefore \theta - 35^\circ 16' = 35^\circ 16', 144^\circ 44'$

$\therefore \underline{\theta = 70^\circ 32', 180^\circ}$ — 1 (3)

Q12. (a)



(b) Draw AC, AD, BC, BD

let $\angle DBE = \alpha$ — 1

$\therefore \angle BAD = \alpha$ ($\angle$ bet. tang & chord $= \angle$ in altern. seg.)

$\angle BDC = \alpha$ (Altern. $\angle$'s, $CD \parallel BE$) — 1

$\angle BAC = \alpha$ ($\angle$'s on circum. stand on the same arc B) — 1

$\therefore \underline{\angle BAD = \angle BAC}$ (3)

<u>Alternative Q11 soln</u>

If $\sqrt{2}\sin\theta - \cos\theta = 1$

let $t = \tan\frac{\theta}{2}$

$\therefore \sin\theta = \frac{2t}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$

$\therefore \sqrt{2}\times\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = 1$ — 1

$2\sqrt{2}t - 1 + t^2 = 1 + t^2$

$2\sqrt{2}t = 2$

$\tan\frac{\theta}{2} = \frac{1}{\sqrt{2}}$

$\therefore \frac{\theta}{2} = 35^\circ 16'$

$\theta = 70^\circ 32'$ — 1

Check $\theta = 180^\circ$: — 1

$\text{LHS} = \sqrt{2}\times 0 - -1 = 1 = \text{RHS}$

$\therefore \underline{\theta = 70^\circ 32', 180^\circ}$ (3)