Year 11 Mathematics

Extension 1 - June 2010

Time: 60 Minutes

DIRECTIONS

- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (*not pencils*) to write your solutions.
- No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.

1. Given that $\tan\alpha = -\frac{2}{5}$ when $\sin\alpha > 0$ and $\cos\beta = -\frac{3}{7}$ when $\tan\beta > 0$,
find the exact value of $\tan(\alpha - \beta)$ **3**

2. Solve for $0° \leq x \leq 360°$

a) $\cos 2x = \sin x$ **3**

b) $\sin 2x + \sqrt{3}\sin x = 0$ **3**

3. a) Express $2\cos x - 3\sin x$ in the form $A\cos(x + \alpha)$
where α is acute (correct to the nearest minute).
Show all working. **3**

b) Hence solve $2\cos x - 3\sin x = \sqrt{13}$ for $0° \leq x \leq 360°$ **2**

4. Find the point P which divides the interval AB externally in the ratio 3: 2,
where $A(-4,-9)$ and $B(5,9)$ **2**

5. Given $A(-1,4)$ and $B(2,-2)$. $P(x,y)$ divides AB internally in the ratio k: 1 and P lies on $-2y - 1 = 0$. Find k. **3**

6. Given two straight lines $kx - y - 5 = 0$ and $3x + y + 1 = 0$ which intersect at 45°.
Find all possible values of k. **3**

7. A committee of 5 people be selected from a group of 10 people.
What is the probability that

a) 2 particular members are included. **2**

b) 2 particular members are not on the committee together. **1**

8. a) Find how many arrangements of the letters of the word *PERMUTATION* can be formed using all letters. **1**

b) How many of these arrangements have

(i) the 5 vowels together. **1**

(ii) the *T*'s are together. **1**

(iii) the consonants and vowels alternate. **1**

9. a) Write down the formula for $\cos 2A$ in terms of $\tan A$ **1**

b) Hence prove that if $\tan^2 y = \cos 2x$, then $\cos 2y = \tan^2 x$ **2**

10. There are 5 boys and 4 girls to be seated at the round table.
In how many ways can they be arranged so that

a) Two particular girls are not together. **1**

b) no two girls are together. **1**

11. a) Show that the exact value of $\sin 165°$ is $\frac{\sqrt{3}-1}{2\sqrt{2}}$. **2**

b) Hence, find the exact value of $\sin 82\frac{1}{2}°$ **3**

~ END OF EXAM ~

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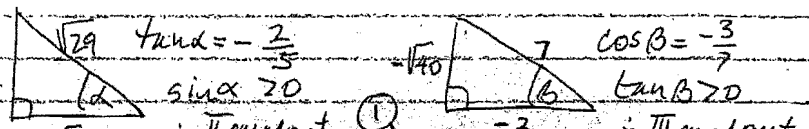
Answers /39

Question 1



$\tan\alpha = -\frac{2}{5}$, $\sin\alpha > 0$, $\therefore$ II quadrant

$\cos\beta = -\frac{3}{7}$, $\tan\beta > 0$, $\therefore$ III quadrant (1)

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{-\frac{2}{5} - \frac{\sqrt{40}}{3}}{1+\frac{2}{5}\cdot\frac{\sqrt{40}}{3}} \text{ (1)} = \frac{\frac{-6-5\sqrt{40}}{15}}{\frac{15-2\sqrt{40}}{15}} = \frac{-6-10\sqrt{10}}{15-4\sqrt{10}} \text{ (1)}$$

Question 2

a) $\cos 2x = \sin x$

$\cos^2 x - \sin^2 x - \sin x = 0$

$1 - 2\sin^2 x - \sin x = 0$ (1)

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$

$x = 30°, 150°$ (1) $\quad x = 270°$ (1)

b) $\sin 2x + \sqrt{3}\sin x = 0$

$2\sin x\cdot\cos x + \sqrt{3}\sin x = 0$

$\sin x(2\cos x + \sqrt{3}) = 0$ (1)

$\sin x = 0$ $\quad \cos x = -\frac{\sqrt{3}}{2}$

$x = 0°, 180°, 360°$ (1) $\quad x = 150°, 210°$ (1)

Question 3

a) $2\cos x - 3\sin x = A\cos(x+\alpha) = A\cos x\cdot\cos\alpha - A\sin x\cdot\sin\alpha$

$\therefore 2 = A\cos\alpha$ $\quad A = \sqrt{2^2+3^2} = \sqrt{13}$ (1)

$3 = A\sin\alpha$ $\quad \tan\alpha = \frac{3}{2}$ (1) $\therefore \alpha = 56°19'$ (1)

$\therefore 2\cos x - 3\sin x = \sqrt{13}\cos(x + 56°19')$

b) $\therefore \sqrt{13}\cos(x+56°19') = \sqrt{13}$

$\cos(x+56°19') = 1$ (1)

$\therefore x + 56°19' = 0°$ or $360°$

$\therefore x = -56°19'$ or $303°41'$ (1)

Question 4

$A(-4,-9)$ $\quad B(5,9)$

$-3 \quad : \quad 2$

$$\therefore P\left(\frac{-4\times2+5\times-3}{3+2}, \frac{-9\times2+9\times-3}{-1}\right) = (23, 45)$$

(1) (1)

Question 5

$A(-1,4)$ $\quad B(2,-2)$

$K \quad : \quad 1$

$$P\left(\frac{-1\times1+2K}{K+1}, \frac{4-2K}{K+1}\right) \text{ (1)}$$

~~P lies on~~ $x - 2y - 1 = 0$

$$\therefore \frac{-1+2K}{K+1} - 2\cdot\frac{4-2K}{K+1} - 1 = 0 \text{ (1)}$$

$-1+2K-8+4K-K-1=0$ $\therefore K = 2$ (1)

Question 6

$kx - y - 5 = 0$ $\therefore m_1 = k$

$3x + y + 1 = 0$ $\therefore m_2 = -3$

$$\therefore \tan 45° = 1 = \left|\frac{k - -3}{1 + k\times-3}\right| \text{ (1)}$$

$|1-3K| = |K+3|$

$1 - 3K = K + 3$ or $-1 + 3K = K + 3$

$K = -\frac{1}{2}$ (1) $\quad K = 2$ (1)

Question 7

a) $P = \frac{{}^8C_3}{{}^{10}C_5}$ (1) $= \frac{2}{9}$ (1)

b) $P = 1 - \frac{2}{9} = \frac{7}{9}$ ✓ (1)

(or) A ✗ ⌣ ⌣ ⌣ ${}^8C_4$

B ✗ ⌣ ⌣ ⌣ ${}^8C_4$

✗ ✗ ⌣ ⌣ ⌣ ${}^8C_5$

$\therefore P = \frac{2\times{}^8C_4 + {}^8C_5}{{}^{10}C_5} = \frac{7}{9}$

Question 8

a) $\frac{11!}{2!}$ (1) $= 19958400$

b) i) (EUAIO) _ _ _ _ _ 5! $\frac{7!}{2!}$ (1) $= 302400$

ii) (IT) _ _ _ _ _ _ _ _ _ _ 10! (1) $= 3628800$

iii) - E - U - A - I - O - 5! $\times \frac{6!}{2!}$ (1) $= 43200$ (1)

Question 9

a) $\cos 2A = \frac{1-2\sin^2 A}{\sin^2 A+\cos^2 A}\left(\div \cos^2 A\right) = \frac{\sec^2 A - 2\tan^2 A}{\tan^2 A+1} = \frac{1+\tan^2 A - 2\tan^2 A}{\tan^2 A+1}$

$= \frac{1-\tan^2 A}{1+\tan^2 A}$ (1)

(or) t-formula $\tan A = t$ $\therefore \cos 2A = \frac{1-t^2}{1+t^2} = \frac{1-\tan^2 A}{1+\tan^2 A}$ (1)

(or) $\cos 2A = \frac{\cos^2 A - \sin^2 A}{\cos^2 A+\sin^2 A}\left(\div \cos^2 A\right) = \frac{1-\tan^2 A}{1+\tan^2 A}$ (1)

b) $\therefore \cos 2y = \frac{1-\tan^2 y}{1+\tan^2 y} = \frac{1-\cos 2x}{1+\cos 2x}$ (1) $= \frac{1-(\cos^2 x - \sin^2 x)}{1+\cos^2 x - \sin^2 x}$

$= \frac{1-\cos^2 x + \sin^2 x}{1-\sin^2 x + \cos^2 x}$ (1) $\frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x$ ∎

Question 10

a) $(G_1 G_2)$ are together $\therefore 2 \times 7!$

$\therefore$ not together $\therefore 8! - 2\times 7! = 30240$ (1)

b) $4! \times 5!$ (1) $= 2880$

$4!$ ↓ 5 boys around table

$5!$ ↓ 6 girls in between

Question 11

a) $\sin 165^\circ = \sin(150^\circ + 15^\circ) = \ldots$ (1)

or $= \sin(120^\circ + 45^\circ) = \sin 120^\circ \cdot \cos 45^\circ + \cos 120^\circ \cdot \sin 45^\circ$

$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{-1}{2}\cdot\frac{1}{\sqrt{2}}$ (1) $= \frac{\sqrt{3}-1}{2\sqrt{2}}$ ∴ shown

(b) $\sin 165^\circ = 2\sin 82\frac{1}{2}^\circ \cos 82\frac{1}{2}^\circ$

Now $\cos 165^\circ = 2\cos^2 82\frac{1}{2}^\circ - 1$

and $\cos 165^\circ = -\cos(45^\circ - 30^\circ)$

$= -\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= -\frac{(\sqrt{3}+1)}{2\sqrt{2}}$ — 1

$\therefore \cos^2 82\frac{1}{2}^\circ = \frac{1}{2}(\cos 165 + 1)$

$= \frac{1}{2}\left(\frac{-\sqrt{3}-1}{2\sqrt{2}} + \frac{2\sqrt{2}}{2\sqrt{2}}\right)$

$= \frac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}$

$\cos 82\frac{1}{2}^\circ = \left(\frac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}\right)^{\frac{1}{2}}$ — 1

since $\sin 82\frac{1}{2}^\circ > 0$

So, $\sin 82\frac{1}{2}^\circ = \frac{\sin 165^\circ}{2\cos 82\frac{1}{2}^\circ}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \div 2\left(\frac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}\right)^{\frac{1}{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \left(\frac{\sqrt{2}}{2\sqrt{2}-\sqrt{3}-1}\right)^{\frac{1}{2}}$ — 1

or equivalent.

$(\doteq 0.99144\ldots)$