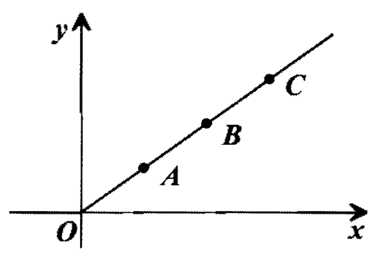
Year 11 Mathematics Extension Task 2
Friday 24th June 2011
Time Allowed: 70 minutes

Instructions:
a) Write all your answers on the paper provided.
b) Show all necessary working.
c) Marks may be deducted for careless or badly arranged work.
d) Use black or blue pen to write your solutions.
e) No liquid paper is to be used. If a correction is to be made, rule one line through the incorrect solution.

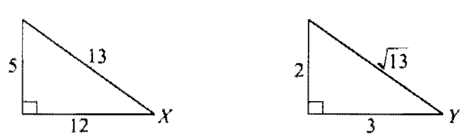
1.	In the diagram the points $A(1,1)$, $B(2,2)$, $C(3,3)$ and the Origin are illustrated.  Indicate whether each of the following is Correct or Incorrect. a. A divides the interval OC internally in the ratio 1:2 b. A divides the interval BC externally in the ratio 1:2 c. B divides the interval CO internally in the ratio 1:2 d. C divides the interval AB externally in the ratio 1:2	2
2.	Find the acute angle between the lines $x - y + 3 = 0$ and $2x + y + 1 = 0$. Give your answer correct to the nearest minute.	3
3.	Without using your calculator, and showing all of your working, find the exact values of;	
	a) $\sin 105^\circ$	2
	b) $\dfrac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$	2
4.	If X and Y are acute angles such that $\tan X = \dfrac{5}{12}$ and $\cos Y = \dfrac{3}{\sqrt{13}}$, find the exact values of;	
	a) $\cos(X - Y)$	2
	b) $\tan \dfrac{X}{2}$	3
5.	Let $A(-1,2)$ and $B(3,5)$ be points on the number plane. Find the coordinates of the point C that divides the interval AB internally in the ratio 1:5	2
6.	The lines $3x - y + 2 = 0$ and $mx - y - 1 = 0$ intersect at 60°. Find the possible values of m.	3

7.	Prove the following; a) $\dfrac{\cos 2x}{\cos x + \sin x} = \cos x - \sin x$ b) $2 - 2\tan x - \dfrac{2\tan x}{\tan 2x} = (1 - \tan x)^2$ c) $\dfrac{\tan A - \tan B}{\tan A + \tan B} = \dfrac{\sin(A - B)}{\sin(A + B)}$	*2* *2* *3*
8.	If $t = \tan\frac{x}{2}$, express $\sqrt{\dfrac{1 - \sin x}{1 + \sin x}}$ in terms of t.	*3*
9.	The point $P(-12, -2)$ divides the interval joining $A(-2, 4)$ and $B(3, 7)$, externally in the ratio $k : 1$. Find the value of k.	*2*
10.	Solve the following, correct to the nearest degree, for $0^\circ \le x \le 360^\circ$ a) $2\cos x + 5\sin x = 3$, by first rewriting as a single trigonometric function. b) $3\cos x + 4\sin x = 4$, using the t-results.	*3* *3*
11.	Explain, in your own words, why it is important to check whether $x = 180^\circ$ is a solution when using the t-results to solve a trigonometric equation.	*1*
12.	a) Express the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$, in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants. b) Hence solve for $0^\circ \le \theta \le 360^\circ$, the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$, correct to the nearest degree.	*2* *2*
13.	a) Show that $\tan rx \tan(r + 1)x = \dfrac{\tan(r + 1)x}{\tan x} - \dfrac{\tan rx}{\tan x} - 1$ b) Hence, or otherwise, show that $\tan 20^\circ \tan 40^\circ + \tan 40^\circ \tan 60^\circ + \ldots + \tan 180^\circ \tan 200^\circ = -9$	*2* *2*

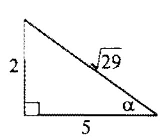
END OF EXAM

BAULKHAM HILLS HIGH SCHOOL

YEAR 11 EXTENSION TASK 2 2011 SOLUTIONS

Solution	Marks	Comments
1. a) Correct b) Correct c) Correct d) Incorrect	**2**	**1 mark** • 3 out of 4 correct
2. $x - y + 3 = 0 \Rightarrow m = 1$ $2x + y + 1 = 0 \Rightarrow m = -2$ $\tan\alpha = \left\lvert \frac{1-(-2)}{1+(1)\times(-2)} \right\rvert$ $= \left\lvert \frac{3}{-1} \right\rvert$ $= 3$ $\alpha = 71^\circ 34'$	**3**	**2 marks** • $\tan\alpha = 3$ • Correct solution for their slopes **1 mark** • Correctly finds the two required slopes • Correctly substitutes their slopes into the formula *Note: no penalty for rounding error*
3a. $\sin 105^\circ = \sin(60+45)^\circ$ $= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{\sqrt{3}+1}{2\sqrt{2}}$	**2**	**1 mark** • Correctly applies $\sin(\alpha + \beta)$ *disregarding actual values substituted*
3b. $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ} = \cos(2 \times 15)^\circ$ $= \cos 30^\circ$ $= \frac{\sqrt{3}}{2}$	**2**	**1 mark** • Recognises t result for $\cos\theta$
4a.  $\cos(X - Y) = \cos X \cos Y + \sin X \sin Y$ $= \left(\frac{12}{13}\right)\left(\frac{3}{\sqrt{13}}\right) + \left(\frac{5}{13}\right)\left(\frac{2}{\sqrt{13}}\right)$ $= \frac{46}{13\sqrt{13}}$	**2**	**1 mark** • Correctly applies $\cos(\alpha - \beta)$

Solution	Marks	Comments
4b. $\tan X = \dfrac{2\tan\frac{X}{2}}{1-\tan^2\frac{X}{2}}$ $\dfrac{5}{12} = \dfrac{2\tan\frac{X}{2}}{1-\tan^2\frac{X}{2}}$ $5 - 5\tan^2\frac{X}{2} = 24\tan\frac{X}{2}$ $5\tan^2\frac{X}{2} + 24\tan\frac{X}{2} - 5 = 0$ $\left(5\tan\frac{X}{2} - 1\right)\left(\tan\frac{X}{2} + 5\right) = 0$ $\tan\frac{X}{2} = \frac{1}{5}$ or $\tan\frac{X}{2} = -5$ However $\frac{X}{2}$ is acute $\therefore \tan\frac{X}{2} = \frac{1}{5}$	3	**2 marks** • Correct solution, however, does not justify why negative solution is discounted • Solves the quadratic correctly • Solves a quadratic, gaining two solutions and justifies the discounting of one solution. **1 mark** • Uses a correct method
5. $A(-1,2)$ $B(3,5)$ $1 : 5$ $C = \left(\dfrac{-1\times 5 + 3\times 1}{1+5}, \dfrac{2\times 5 + 5\times 1}{1+5}\right)$ $= \left(-\frac{1}{3}, \frac{5}{2}\right)$	2	**1 mark** • Uses a correct method
6. $3x - y + 2 = 0 \Rightarrow m = 3$ $\tan 60^\circ = \left\lvert \dfrac{m-3}{1+3m} \right\rvert$ $\sqrt{3} = \left\lvert \dfrac{m-3}{1+3m} \right\rvert$ $\sqrt{3}\lvert 1+3m\rvert = \lvert m-3\rvert$ $\sqrt{3}(1+3m) = m-3$ $\quad$ $-\sqrt{3}(1+3m) = m-3$ $(1-3\sqrt{3})m = 3+\sqrt{3}$ $\quad$ $(1+3\sqrt{3})m = 3-\sqrt{3}$ $m = \dfrac{3+\sqrt{3}}{1-3\sqrt{3}}$ or $m = \dfrac{3-\sqrt{3}}{1+3\sqrt{3}}$	3	**2 marks** • Correctly finds one possible value of m • Finds two values for m, correct for their value of tan60. • Establishes a correct absolute value equation **1 mark** • Uses a correct method
7a. $\dfrac{\cos 2x}{\cos x + \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x + \sin x}$ $= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)}$ $= \cos x - \sin x$	2	**1 mark** • Progress towards solution involving correct substitution of $\cos 2x$ or equivalent merit
7b. $2 - 2\tan x - \dfrac{2\tan x}{\tan 2x} = 2 - 2\tan x - \dfrac{2\tan x}{1} \times \dfrac{1-\tan^2 x}{2\tan x}$ $= 2 - 2\tan x - 1 + \tan^2 x$ $= 1 - 2\tan x + \tan^2 x$ $= (1-\tan x)^2$	2	**1 mark** • Progress towards solution involving correct substitution of $\tan 2x$ or equivalent merit

Solution	Marks	Comments
7c. $\dfrac{\tan A - \tan B}{\tan A + \tan B} = \dfrac{\dfrac{\sin A}{\cos A} - \dfrac{\sin B}{\cos B}}{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}$ $= \dfrac{\dfrac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}}{\dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B}}$ $= \dfrac{\sin(A-B)}{\sin(A+B)}$	3	**2 marks** • Significant progress towards correct solution **1 mark** • Progress towards solution that involves either correct substation for tan's or sin $(A+B)$ or equivalent merit *Note: in parts a to c, any attempt to solve like an equation should only be marked up to that point.*
8. $\sqrt{\dfrac{1-\sin x}{1+\sin x}} = \sqrt{\dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}}$ $= \sqrt{\dfrac{1+t^2-2t}{1+t^2+2t}}$ $= \sqrt{\dfrac{(1-t)^2}{(1+t)^2}}$ $= \left\lvert \dfrac{1-t}{1+t} \right\rvert$	3	**2 marks** • Significant progress towards correct solution **1 mark** • Progress towards solution that involves correct t result for $\sin x$ *Note: no penalty incurred for the lack of absolute value signs.*
9. $A(-2,4)$ $B(3,7)$ $-k : 1$ Using x-values: $\dfrac{-2\times 1 + 3\times -k}{-k+1} = -12$ $-2-3k = 12k-12$ $15k = 10$ $k = \dfrac{2}{3}$	2	**1 mark** • Uses a correct method • $k = -\frac{2}{3}$
10a.  $\alpha = \tan^{-1}\left(\dfrac{2}{5}\right)$ $= 21.8^\circ$ $2\cos x + 5\sin x = 3$ $\sqrt{29}\sin(x+21.8)^\circ = 3$ $\sin(x+21.8)^\circ = \dfrac{3}{\sqrt{29}}$ Q1 & Q2 $x + 21.8^\circ = 33.9^\circ, 146.1^\circ$ $\therefore x = 12^\circ, 124^\circ$	3	**2 marks** • Correctly finds two possible answers for the "compound" angle • Correctly finds two possible values of x, considering their answers for the compound angle **1 mark** • Correctly transforms into a single trig expression

Solution	Marks	Comments
10. $3\cos x + 4\sin x = 4$ $3\left(\frac{1-t^2}{1+t^2}\right) + 4\left(\frac{2t}{1+t^2}\right) = 4$ $3 - 3t^2 + 8t = 4 + 4t^2$ $7t^2 - 8t + 1 = 0$ $(7t-1)(t-1) = 0$ $t = \frac{1}{7}$ $\quad$ $t = 1$ $\tan\frac{x}{2} = \frac{1}{7}$ $\quad$ $\tan\frac{x}{2} = 1$ Q1 $\quad$ Q1 $\frac{x}{2} = 8.1^\circ$ $\quad$ $\frac{x}{2} = 45^\circ$ $x = 16.2^\circ$ $\quad$ $x = 90^\circ$ or $\therefore x = 16^\circ, 90^\circ$	3	**2 marks** • Finds more than two values for x • Correctly evaluates one possible value for x • Correctly finds two possibilities for $\tan\frac{x}{2}$ • Correctly finds two possible values for x from their quadratic **1 mark** • Correctly substitutes the t-results
11. As $\tan 90^\circ$ is undefined, which is a solution that cannot be found when solving a polynomial. i.e. the t-results method will never discover the solution $x = 180^\circ$, so its possibility must be checked.	1	
12a. $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$ $\frac{3(1-2\sin^2\theta)+7}{\sin\theta} + 11 = 0$ $\frac{6\sin^2\theta - 10}{\sin\theta} - 11 = 0$ $6\sin^2\theta - 11\sin\theta - 10 = 0$	2	**1 mark** • Uses $\cos 2\theta = 1 - 2\sin^2\theta$
12b. $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$ $6\sin^2\theta - 11\sin\theta - 10 = 0$ $(3\sin\theta + 2)(2\sin\theta - 5) = 0$ $\sin\theta = -\frac{2}{3}$ $\quad$ $\sin\theta = \frac{5}{2}$ Q3 & Q4 $\quad$ no solutions $\theta = 222^\circ, 318^\circ$ or $\therefore \theta = 222^\circ, 318^\circ$	2	**1 mark** • Correctly finds two possible values for $\sin x$ • Correct two values for θ, without evidence that $\sin\theta = \frac{5}{2}$ has been considered
13a. $\tan x = \tan\{(r+1)x - rx\}$ $= \frac{\tan(r+1)x - \tan rx}{1 + \tan(r+1)x\tan rx}$ $\therefore 1 + \tan(r+1)x\tan rx = \frac{\tan(r+1)x - \tan rx}{\tan x}$ $\tan(r+1)x\tan rx = \frac{\tan(r+1)x}{\tan x} - \frac{\tan rx}{\tan x} - 1$	2	**1 mark** • Progress towards a solution, involving the use of tan(A – B) expansion
13b. $\tan 20^\circ\tan 40^\circ + \tan 40^\circ\tan 60^\circ + \ldots + \tan 180^\circ\tan 200^\circ$ $= \frac{\tan 40^\circ}{\tan 20^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1 + \frac{\tan 60^\circ}{\tan 20^\circ} - \frac{\tan 40^\circ}{\tan 20^\circ} - 1 + \ldots + \frac{\tan 200^\circ}{\tan 20^\circ} - \frac{\tan 180^\circ}{\tan 20^\circ} - 1$ $= \frac{\tan 200^\circ}{\tan 20^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 9 \times 1$ $= \frac{\tan 20^\circ}{\tan 20^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 9$ $= 1 - 1 - 9$ $= -9$	2	**1 mark** • Correctly simplifying the sum to three terms.