BAULKHAM HILLS HS PERSEVERE

Year 11 Mathematics Extension 1

Task 2, June 2012

Time allowed : 60 minutes

Instructions:

- Full working should be shown in every question.
- Use black or blue pen to write your solution
- No liquid paper/tape to be used to correct your solution.

Question No	Question	Marks
1	If $\tan\alpha = \frac{4}{3}$ and $\cos\beta = \frac{12}{13}$, where $0 < \beta < \alpha < 90^0$ find the exact value of $\cos(\alpha - \beta)$.	3
2	Solve for θ where $0^\circ \le \theta \le 360^\circ$ a) $\tan\theta = \sin 2\theta$ b) $2 + \cos\theta = \cos 2\theta$	 3 3
3	a) Express $\sin x - \cos x$ in the form of $A\sin(x - y)$. b) Hence solve $\sin x - \cos x = 1$ *for* x *where* $0^\circ \le x \le 360^\circ$	2 3
4	a) Find the acute angle between the lines $x - 2y = 6$ and $y = 3x - 1$. Give your answer to the nearest minute. b) Find the values of k if the line $2x - y + 1 = 0$ and $kx - 7y + 5 = 0$ make an angle 45^0 at their intersection.	2 2
5	a) AB is an interval joining points A (1,2) and B(7,8). Point P is on AB such that AP = 2PB. Find the coordinates of the point P. b) Find the coordinates of the point which divides the interval joining (3,-2) and (-5,4) externally in the ratio 5:3.	2 2
6	a) Show that $\tan(45^o + x) = \frac{\cos x + \sin x}{\cos x - \sin x}$ b) Hence or otherwise give the smallest positive solution of $\frac{\cos 2x + \sin 2x}{\cos 2x - \sin 2x} = \sqrt{3}$	2 2
7	a) If $\tan\frac{\theta}{2} = t$, express $\frac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta}$ in terms of t. b) Using t-method solve $5\cos\theta + 6\sin\theta = -5$ for $0^\circ \le \theta \le 360^\circ$	3 3
8	Given $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, find $\cos 36^\circ$ in exact form.	2
9	a) Expand $(x + y)(x^2 - xy + y^2)$ b) Prove that $\cos^6\beta + \sin^6\beta = \frac{1}{4} + \frac{3}{4}\cos^2 2\beta$	1 3
10	a) Show that $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$ b) Hence show that $12\sin^3\alpha + 2\cos 3\alpha - 9\sin\alpha + \sin 3\alpha = 2\sqrt{2}\cos(3\alpha + 45^\circ)$ c) Hence solve $12\sin^3\alpha + 2\cos 3\alpha - 9\sin\alpha + \sin 3\alpha = 2$ for $0^\circ \le \alpha \le 360^\circ$	2 3 2

End of Task 2

Solution. Year 11 Mathematics Ext. 1 June 2012

1) $\tan\alpha = \frac{4}{3}$ $\cos\beta = \frac{12}{13}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \frac{3}{5}\checkmark \cdot \frac{12}{13} + \frac{4}{5}\cdot\frac{5}{13}\checkmark$

$= \frac{56}{65}\checkmark$

2) a) $\tan\theta = \sin 2\theta$

$\frac{\sin\theta}{\cos\theta} = 2\sin\theta\cos\theta$.

$\sin\theta(1-2\cos^2\theta) = 0 \checkmark$

$\sin\theta = 0$ $\qquad$ $\cos^2\theta = \frac{1}{2}$

$\qquad\qquad\qquad$ $\cos\theta = \pm\frac{1}{\sqrt{2}}$

$0°, 180°, 360° \checkmark$ $\qquad$ $45°, 135°, 225°, 315° \checkmark$

b) $2 + \cos\theta = \cos 2\theta$.

$2\cos^2\theta - \cos\theta - 3 = 0$

$(2\cos\theta - 3)(\cos\theta + 1) = 0 \checkmark$

$\cos\theta = 3/2$ $\qquad$ $\cos\theta = -1$

No solution $\checkmark$ $\qquad$ $180° \checkmark$

3) a) $\sin x - \cos x$.

$= \sqrt{2}\checkmark\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)$

$= \sqrt{2}(\sin x\cos y - \cos x\sin y)$

where $\tan y = \frac{1}{1}\checkmark$

$= \sqrt{2}\sin(x-y)$. where $\tan y = 1$

b) $\sqrt{2}\sin(x-y) = 1\checkmark$

$\sin(x-y) = 1/\sqrt{2}$.

acute $x - y = 45°\checkmark$

$\tan^{-1}(1) = y \Rightarrow y = 45°$.

$x - y = 45°, 135°$.

$x = 90°, 180° \checkmark$ where $y = 45°$.

4) a) $2y = x - 6$

$y = \frac{1}{2}x - 3$ — (1)

$y = 3x - 1$ — (2).

$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$ where

m_1, m_2 are the gradients of the lines.

$= \left|\frac{3 - \frac{1}{2}}{1 + 3\cdot\frac{1}{2}}\right|\checkmark = \frac{5/2}{5/2} = 1$

$\therefore \alpha = 45°0'\checkmark$

b) $y = 2x + 1$; $y = \frac{k}{7}x + \frac{5}{7}$

$m_1 = 2$ $\qquad$ $m_2 = \frac{k}{7}$

$\therefore 1 = \left|\frac{2 - \frac{k}{7}}{1 + 2\cdot\frac{k}{7}}\right|\checkmark$

$1 = \left|\frac{14 - k}{7 + 2k}\right|$.

$\frac{14 - k}{7 + 2k} = 1 \Rightarrow 3k = 7 \therefore k = 7/3$

$-\left(\frac{14 - k}{7 + 2k}\right) = 1 \Rightarrow 14 - k = -7 - 2k$

$k = -21$.

Possible values for $k = \frac{7}{3}, -21\checkmark$.

5) a) A (1,2) —— 2 —— P —— 1 —— B (7, 8)

$P\left(\frac{2\times7 + 1\times1}{3}\checkmark, \frac{2\times8 + 1\times2}{3}\checkmark\right)$.

$P(5, 6)$

b) $A(3,-2)$ $B(-5,4)$.

$\left(\frac{5\times-5 + (-3)\times3}{5-3}, \frac{5\times4 + (-3)\times-2}{5-3}\right)$

$\left(\frac{-25-9}{2}, \frac{26}{2}\right)$

$=(-17, 13)$ ✓ ✓

6) a) $\tan(45^\circ + x)$

$= \frac{\tan 45^\circ + \tan x}{1 - \tan 45^\circ \tan x}$

$= \frac{1+\tan x}{1-\tan x}$ ✓

$= \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}}$ ✓

$= \frac{\cos x + \sin x}{\cos x - \sin x}$.

= RHS.

b) $\tan(45^\circ + 2x) = \sqrt{3}$. ✓

$45^\circ + 2x = \tan^{-1}(\sqrt{3})$.

acute ∠ $= 60^\circ$

$2x = 15$

$x = 7\frac{1}{2}^\circ$ ✓

7. a) $\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$

$= \frac{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}$ ✓ ✓

$= \frac{1+\cancel{t^2}+2t+1-\cancel{t^2}}{\cancel{1}+t^2+2t-\cancel{1}+t^2}$

$= \frac{\cancel{2}(t+1)}{\cancel{2}t(t+1)} = \frac{1}{t}$ ✓

(b) $\frac{5(1-t^2)}{1+t^2} + 6\cdot\frac{2t}{1+t^2} = -5$

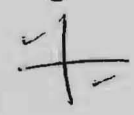
$5 - \cancel{5t^2} + 12t = -5 - \cancel{5t^2}$

$12t = -10$

$t = -\frac{10}{12}$

$= -\frac{5}{6}$ ✓

$\tan(\theta/2) = -5/6$



$\frac{\theta}{2} \doteq 140^\circ 12'$

$\theta = 280^\circ 23'$ ✓

Test for 180°:

$5\times(-1) + 6\times 0 = -5$ ✓

∴ solution are 180°, $280^\circ 23'$ ✓

8) $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

$\cos 36 = 1 - 2\sin^2 18^\circ$ ✓

$= 1 - \cancel{2}\,\frac{(\sqrt{5}-1)^2}{\cancel{16}\,8}$

$= \frac{8-(5-2\sqrt{5}+1)}{8}$

$= \frac{2+2\sqrt{5}}{8} = \frac{1+\sqrt{5}}{4}$ ✓

9) a) $(x+y)(x^2-xy+y^2)$

$= x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{yx^2} - \cancel{xy^2} + y^3$

$= x^3+y^3$ ✓

b) LHS $= \cos^6\beta + \sin^6\beta$.

$(\cos^2\beta)^3 + (\sin^2\beta)^3 = (\cos^2\beta+\sin^2\beta)(\cos^4\beta - \cos^2\beta\sin^2\beta + \sin^4\beta)$ ✓

$= 1\times(\cos^4\beta - 2\cos^2\beta\sin^2\beta + \sin^4\beta + \sin^2\beta\cos^2\beta)$

$= (\cos^2\beta - \sin^2\beta)^2 + \frac{1}{4}\sin^2 2\beta$ ✓

$= \cos^2 2\beta + \frac{1}{4}(1-\cos^2 2\beta)$ ✓

$= \frac{1}{4} + \frac{3}{4}\cos^2 2\beta$. = RHS.

10) a) $\sin 3\alpha = \sin(2\alpha + \alpha)$

$= \sin 2\alpha . \cos \alpha + \cos 2\alpha \sin \alpha$ ✓

$= 2\sin\alpha \cos^2\alpha + (1 - 2\sin^2\alpha)\sin\alpha$

✓ $= 2\sin\alpha(1 - \sin^2\alpha) + (1 - 2\sin^2\alpha)\sin\alpha$

$= 3\sin\alpha - 4\sin^3\alpha.$

b) $3(4\sin^3\alpha - 3\sin\alpha) + 2\cos 3\alpha + \sin 3\alpha$

$= 3(-1)\sin 3\alpha + 2\cos 3\alpha + \sin 3\alpha$ ✓

$= 2\cos 3\alpha - 2\sin 3\alpha$ ✓

$= 2\sqrt{2}\left(\frac{2}{2\sqrt{2}} . \cos 3\alpha - \frac{2}{2\sqrt{2}}\sin 3\alpha\right)$

$= 2\sqrt{2}\left(\frac{1}{\sqrt{2}} . \cos 3\alpha - \frac{1}{\sqrt{2}}\sin 3\alpha\right)$ ✓

$= 2\sqrt{2}\cos(3\alpha + \cancel{45})$ where

$\alpha = \tan^{-1}(1)$

$= 2\sqrt{2}\cos(3\alpha + 45^\circ)$.

c) $2\sqrt{2}\cos(3\alpha + 45^\circ) = 2$

~~$\cos(3\alpha + 45^\circ) = \frac{1}{\sqrt{2}}$~~

acute $\angle = 45^\circ$.

$45^\circ \leq 3\alpha + 45^\circ \leq 1125^\circ$

✓

$\therefore 3\alpha + 45^\circ = 45, 315^\circ, 405^\circ, 675^\circ, 765^\circ, 1035^\circ$

$\alpha = 0^\circ, 90^\circ, 120^\circ, 210^\circ, 240^\circ, 330^\circ$ ✓