BAULKHAM HILLS HIGH SCHOOL

Assessment Task 2 2013

YEAR 11

Mathematics Extension 1

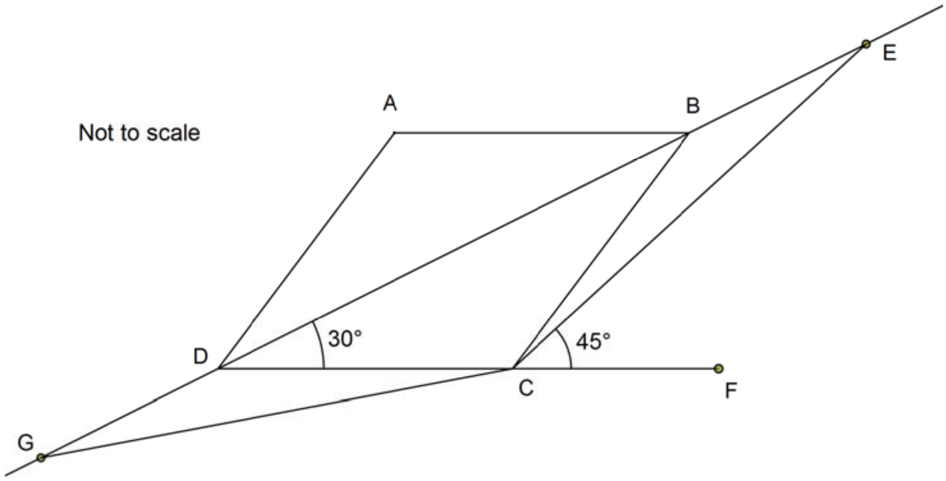
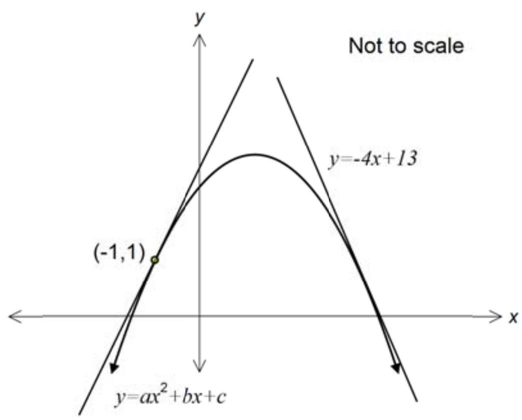
General Instructions

- Reading time – 5 minutes
- Working time – 1 hour and 10 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 1-14
- Marks may be deducted for careless or badly arranged work

Total marks – 48
Exam consists of 2 pages.

This paper consists of 14 Questions.

1.	If A and B have coordinates $(1,-2)$ and $(-3,4)$ respectively, find the point $P(x,y)$ which divides the interval AB externally in the ratio $2:3$	**2**
2.	Without solving simultaneously, find the number of times $y = x - 4$ cuts the circle $(x-1)^2 + (y-2)^2 = 18$	**3**
3.	Find the exact value of $\sin 105°$	**3**
4.	Find the acute angle between the lines $y = \frac{x}{3} - 2$ and $2x + y + 1 = 0$ to the nearest degree	**3**
5.	(i) Prove $\frac{\sin 2x}{1-\cos 2x} = \cot x$ (ii) Hence find the exact value of $\cot 15°$	**2** **2**
6.	Show $\frac{d}{dx}\left(\frac{(2x-1)^3}{\sqrt{2x-1}}\right) = 5\sqrt{(2x-1)^3}$	**2**
7.	Solve $2\sin\theta - \cos\theta = 1$ for $0° \leq \theta \leq 360°$ using the t-method where $t = \tan\left(\frac{\theta}{2}\right)$	**4**
8.	(i) Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $A\sin(\theta + \alpha)$ (ii) Hence solve $\sqrt{3}\sin\theta + \cos\theta = 1$ for $0° \leq \theta \leq 360°$	**2** **2**
9.	Differentiate $f(x) = \frac{2}{x^2}$ from first principles.	**3**
10.	Solve for $0° \leq x \leq 360°$ $2\sin 2x = \cos x$	**3**

11.	 $ABCD$ is a rhombus. DB is produced to E and BD is produced to G. $\angle BDC = 30°$ and $\angle ECF = 45°$ (i) Prove ΔBCE is isosceles (ii) If $DG = 2BD$ what is the ratio of the area of ΔBGC to the area of the rhombus $ABCD$?	**4** **2**
12.	Find the point(s) on the curve $y = \sqrt{1 - x^2}$ where the normal is inclined at $135°$ to the positive direction of the x-axis.	**4**
13.	Solve for $0° \leq x \leq 360°$ $$\frac{6^{\sin x} + 3^{\sin x}}{2^{1+\sin x} + 2} = \frac{\sqrt[3]{3}}{2}$$	**3**
14.	 In the diagram above, the tangent at the point $(-1,1)$ on the parabola $y = ax^2 + bx + c$ has a gradient of 4. The line $y = -4x + 13$ is also a tangent to the parabola. Find the values of a, b and c.	**4**

End of Exam

Solutions. /48

1. $A(1,-2)$ $B(-3,4)$ [2]

$-2:3$

$P\left(\frac{-2(-3)+3(1)}{-2+3}, \frac{-2(4)+3(-2)}{-2+3}\right)$ (1) (1)

$P=(9,-14)$

Award 1 for internal division (3,2)

2. [3] If $y=x-4$ cuts circle at 2 places then the ⊥ distance from (1,2) to the line must be less than the radius of the circle ie $\sqrt{18}$ (1)

$x-y-4=0$ (1,2)

$d=\frac{|1(1)+2(-1)-4|}{\sqrt{(1)^2+(-1)^2}}$ (1)

$=\frac{5}{\sqrt{2}}$

$=3.535..$

now $\sqrt{18}=4.24..$ (1)

since $3.535 < 4.24..$ it cuts the circle at 2 places

Bold answer → 2 times 1 mark
- Using Δ function with working → 3 marks
- Proof diagramatically showing radius centre & correct evidence → 3 marks

3. [3] $\sin 105° = \sin(60+45)$

$=\sin 60\cos 45+\cos 60\sin 45$ (1)

$=\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1}{2}\cdot\frac{1}{\sqrt{2}}$ (1)

$=\frac{\sqrt{3}+1}{2\sqrt{2}}$ or $\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$ (1)

4. [3] $y=\frac{x}{3}-2 \rightarrow m_1=\frac{1}{3}$ (1)

$2x+y+1=0$

$y=-2x-1 \rightarrow m_2=-2$

$\tan\alpha=\frac{\frac{1}{3}+2}{1+(\frac{1}{3})(-2)}$ (1)

$=\frac{2\frac{1}{3}}{\frac{1}{3}}$

$=7$

$\theta=82°$ (1)

5 (i) [4] Prove $\frac{\sin 2x}{1-\cos 2x}=\cot x$

LHS $=\frac{2\sin x\cos x}{1-(1-2\sin^2 x)}$ (1)

$=\frac{2\sin x\cos x}{2\sin^2 x}$ (1)

$=\frac{\cos x}{\sin x}$

$=\cot x$ = RHS.

(ii) $\therefore \cot 15° = \frac{\sin 30}{1-\cos 30°}$ (1)

$=\frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}}$

$=\frac{\frac{1}{2}}{\frac{2-\sqrt{3}}{2}}$

$=\frac{1}{2-\sqrt{3}}$ (1)

(or $2+\sqrt{3}$)

6. [2] $\frac{d}{dx}\left(\frac{(2x-1)^3}{\sqrt{2x-1}}\right)=\frac{d}{dx}(2x-1)^{\frac{5}{2}}$ (1)

$=\frac{5}{2}(2x-1)^{\frac{3}{2}}\times 2$ (1)

$=5\sqrt{(2x-1)^3}$

If they use quotient rule 1 mark for correct substitution then 1 mark for correct simplification.

ie $y'=\frac{6(2x-1)^2\sqrt{2x-1}-\frac{(2x-1)^3}{\sqrt{2x-1}}}{(\sqrt{2x-1})^2}$ (1)

$=\frac{6(2x-1)^2(2x-1)-(2x-1)^3}{(2x-1)\sqrt{2x-1}}$

$=\frac{5(2x-1)^3}{(2x-1)^{\frac{3}{2}}}$

$=5(2x-1)^{\frac{3}{2}}$

$=5\sqrt{(2x-1)^3}$ (1)

7. [4] $2\sin\theta-\cos\theta=1$ using t method.

$\frac{4t}{1+t^2}-\frac{1-t^2}{1+t^2}=1$ (1)

$\therefore 4t-1+t^2=1+t^2$

$4t=2$

$t=\frac{1}{2}$ (1)

$\therefore \tan\frac{\theta}{2}=\frac{1}{2}$

$\frac{\theta}{2}=26°34'$, ~~206°34'~~

$\theta=53°08'$ (1)

test $\theta=180°$

$2\sin 180-\cos 180=1$

$0-(-1)=1$

True (1)

$\therefore$ Sol'ns are $53°08'$, $180°$.

8. [4] $A\sin(\theta+\alpha)=\sqrt{3}\sin\theta+\cos\theta$

$A\sin\theta\cos\alpha+A\cos\theta\sin\alpha=$ RHS

$\therefore A\sin\alpha=1$ $A\cos\alpha=\sqrt{3}$ (1)

$\therefore \tan\alpha=\frac{1}{\sqrt{3}} \rightarrow \alpha=30°$ (1)

$A=\sqrt{(\sqrt{3})^2+1^2} \rightarrow A=2$

$\therefore 2\sin(\theta+30°)=\sqrt{3}\sin\theta+\cos\theta$

(ii) solve $2\sin(\theta+30°)=1$

$\sin(\theta+30°)=\frac{1}{2}$

(1) $\theta+30°=30°, 150°$

(1) $\therefore \theta=0, 120°, 360°$

9. [3] $f(x)=\frac{2}{x^2}$

$f'(x)=\lim_{h\to 0}\frac{\frac{2}{(x+h)^2}-\frac{2}{x^2}}{h}$ (1)

$=\lim_{h\to 0}\frac{2x^2-2(x^2+2xh+h^2)}{h\,x^2(x+h)^2}$

$=\lim_{h\to 0}\frac{2x^2-2x^2-4xh-2h^2}{h\,x^2(x+h)^2}$

(1) $=\lim_{h\to 0}\frac{h(-4x-2h)}{h\,x^2(x+h)^2}$

$=\frac{-4x-0}{x^2(x+0)^2}$

$=\frac{-4x}{x^4}$

$=\frac{-4}{x^3}$ (1)

(NB must carry $\lim_{h\to 0}$ throughout working)

10. [3] $2\sin 2x=\cos x$ $0\le x\le 360°$

$4\sin x\cos x-\cos x=0$

$\cos x(4\sin x-1)=0$ (1)

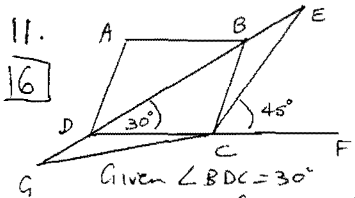
$\cos x=0$ $\sin x=\frac{1}{4}$

$x=90°, 270°$ $x=14°29', 165°31'$

$x=14°29', 165°31'$, $90°, 270°$

(1) (1)

11.
[16]



Given $\angle BDC = 30°$ (1)

$\angle ADB = 30°$ (diagonals of a rhombus bisect $\angle$ at corner) (1)

$\therefore \angle ADC = 60°$

$AD \parallel BC$ (Opposite sides of a rhombus)

$\angle ADC = \angle BCF = 60°$ (1)

(1) (Corresponding $\angle$'s $AD \parallel BC$)

$\angle ECF = 45°$ (Give)

$\therefore \angle BCE = 15°$ (Subtraction)

$\angle ADB = \angle DBC$ (Alternate $\angle$'s $AD \parallel BC$.)
$= 30°$

$\angle EBC + 30° = 180°$ (~~Straight~~ $\angle$) (1)

(1) $\therefore \angle EBC = 150°$

$\angle BEC = 180 - (150 + 15)$

(1) (~~Angle sum ΔBCE~~)

$\therefore \angle BEC = 15°$ (1)

Since $\angle BCE = \angle BEC$

ΔBEC is isosceles

(NB → other methods award marks accordingly)

Let $BD = x$

$\therefore DG = 2x$ (Give $DG = 2BD$)

Construct AC. let it intersect BD at H. let $AC = h$

$\therefore$ Area Rhombus $= \frac{1}{2} AC . BD$
$= \frac{1}{2} . x h$

Area of $\Delta = \frac{1}{2} . 3x . \frac{1}{2} h$
$= \frac{3xh}{4}$

(1) for either

$\therefore$ Ratio $\frac{3xh}{4} : \frac{1}{2} xh$
$= \frac{3}{4} : \frac{1}{2}$
$= 3 : 2.$ (1)

(NB If they calculate sides & heights if correct award 2)

12.
[4]

$y = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} . -2x$

$= \frac{-x}{\sqrt{1-x^2}}$ (1)

Normal at 135° ⇒ tangent at 45°

$\therefore$ grad. of tangent $= 1$

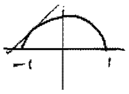
hence $\frac{-x}{\sqrt{1-x^2}} = 1$ (1)

$-x = \sqrt{1-x^2}$

$x^2 = 1 - x^2$

$2x^2 = 1$

$x^2 = \frac{1}{2}$ (1)



$x = \pm \frac{1}{\sqrt{2}}$

but since graph is a semi circle

$x = -\frac{1}{\sqrt{2}}$ is the only point where this is true

$y = \sqrt{1 - (-\frac{1}{\sqrt{2}})^2}$

$= \sqrt{1 - \frac{1}{2}}$

$= \frac{1}{\sqrt{2}}$ (1)

$\therefore$ Point is $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

13.
[3]

$\frac{6^{\sin x} + 3^{\sin x}}{2^{1+\sin x} + 2} = \frac{\sqrt[3]{3}}{2}$

let $a = \sin x$

$\frac{6^a + 3^a}{2^{a+1} + 2} = \frac{3^{\frac{1}{3}}}{2}$

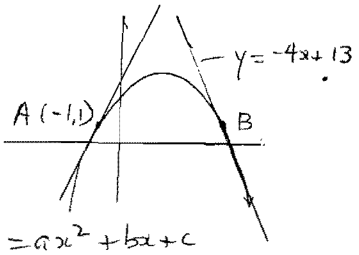
$\frac{3^a (\cancel{2^a + 1})}{2(\cancel{2^a + 1})} = \frac{3^{\frac{1}{3}}}{2}$ (1)

$\therefore 3^a = 3^{\frac{1}{3}}$

$\therefore \sin x = \frac{1}{3}$ (1)

$x = 19°28', 160°32'$ (1)

14.
[4]



$y = ax^2 + bx + c$

$(-1,1)$ satisfies the eq'n

$a - b + c = 1$ — (1)

$y' = 2ax + b$ given when $x = -1$ $y' = 4$

$\therefore 4 = -2a + b$ — (2)

Eq'n of tangent at A

$y - 1 = 4(x+1)$

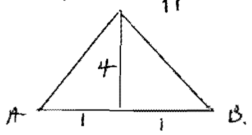
$y = 4x + 5$

Find pt. of intersection of $y = 4x+5$ & $y = -4x+13$

$4x + 5 = -4x + 13$

$8x = 8$

$x = 1$ → this must be the axis of sym. of the parabola since gradients at A & B are opposites



$\therefore$ when $x = 1$ $y' = 0$

$0 = 2a + b$ — (3)

Solve (2) & (3) simult'ly

(2) + (3) $4 = 2b$ → $b = 2$

$\therefore a = -1$

sub into (1)

$-1 - 2 + c = 1$

$c = 4$

$\therefore a = -1$, $b = 2$ and $c = 4$.

Question 14

Award 1 mark for a correct equation in a, b and C

Award 2 marks for 2 or 3 correct eq'ns in a, b and C

Award 3 marks if they work significantly towards a sol'n.

Award 4 marks — if they then solve for a, b & c.