BAULKHAM HILLS HIGH SCHOOL

Assessment Task 2 2014

YEAR 11

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 1-16
- Marks may be deducted for careless or badly arranged work

Total marks – 48

This paper consists of 16 Questions.

1.	A and B are the points $(-5, 12)$ and $(4, 9)$ respectively. Find the coordinates of P which divides AB externally in the ratio $5:2$.	2
2.	Evaluate $\sin 75°$ in exact form.	2
3.	Find the acute angle between the lines $y = 4x + 5$ and $6x + 3y = 7$.	2
4.	Solve $3\cos^2 2\theta = \sin^2 2\theta$, for $0^\circ \leq \theta \leq 180^\circ$.	3
5.	Express $1 - 2\sin^2 2x$ in terms of $4x$.	1
6.	Show that $\frac{1+\cos x}{\sin x} = \frac{1}{t}$ where $t = \tan\frac{x}{2}$.	2
7.	(i) Show that the points $A\left(\frac{1}{2}, 4\right)$, $P(-3, -3)$ and $Q\ (1, 5)$ are collinear. (ii) Find the ratio in which A divides the line segment PQ.	2 2
8.	(i) Express $3\cos x + 4\sin x$ in the form $A\cos(x - \alpha)$ where $A > 0$ and α is acute. (ii) Hence solve $3\cos x + 4\sin x = -3$ for $0^\circ \leq x \leq 360^\circ$	2 2
9.	Prove the identity $\frac{\cot A - \tan A}{\cot 2A} \equiv 2$	3
10.	Solve $8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$ for $0° \leq \theta \leq 360°$	3

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11.	Use the substitution $t = \tan\theta$ to solve the equation $3\sin 2\theta - 4\cos 2\theta = 4$ for $0^\circ \leq \theta \leq 180^\circ$	4
12.	If $\sin x = \frac{\sqrt{32}}{9}$ and $\sin y = \frac{\sqrt{8}}{3}$ where x and y are acute. Without finding the value of x or y (i) Show that $x = 2y$ (ii) Hence show that the exact value of $\sin 3y = -\frac{10\sqrt{2}}{27}$	 2 2
13.	The angle between the line $4x + 3y = 8$ and the line $ax + by + c = 0$ is 45°. Find the possible values of $a : b$.	3
14.	(a) Prove that $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$ (b) Hence find the exact value of $\sin 157\frac{1}{2}^\circ \cos 67\frac{1}{2}^\circ$	1 2
15.	Given that $\sin x + \cos x = a$ and $\cos 2x = b$. Prove that $a^4 - 2a^2 + b^2 = 0$	3
16.	(a) Prove that $\frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x$ (b) Hence prove that if $\cos 2A = \tan^2 B$ then $\cos 2B = \tan^2 A$	2 3

END OF EXAMINATION

Yr 11 EXTENSION 1 TERM 2 2014 SOLUTIONS

1. $A(-5,12)\ B(4,9)$

$-5 : 2$ ✓ 1 for correct internal division

$x = \dfrac{-10-20}{-5+2}$ $y = \dfrac{24-45}{-3}$

$x = \dfrac{-30}{-3}$ $y = \dfrac{-21}{-3}$

$x = 10$ $y = 7$ ✓

$\therefore P$ is $(10,7)$

2. $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$ ✓

$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2}$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ ✓

$= \dfrac{\sqrt{6}+\sqrt{2}}{4}$

3. $m_1 = 4$ $m_2 = \dfrac{-6}{3} = -2$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\dfrac{4 - -2}{1 - 8}\right|$ ✓

$= \left|-\dfrac{6}{7}\right|$

$\tan\theta = \dfrac{6}{7}$

$\theta = 40^\circ 36'$ ✓

4. $3\cos^2 2\theta = \sin^2 2\theta$ $\quad 0 \leq \theta \leq 180°$

$3 = \tan^2 2\theta$ ✓ $\quad 0 \leq 2\theta \leq 360°$

$\tan 2\theta = \pm\sqrt{3}$

$2\theta = 60°, 120°, 240°, 300°$

$\theta = \underbrace{30°, 60°}_{✓}, \underbrace{120°, 150°}_{✓}$

5. $1 - 2\sin^2 2x = \cos 4x$ ✓

6. $LHS = \dfrac{1+\cos x}{\sin x}$

$= \dfrac{1 + \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$ ✓

$= \dfrac{1+\cancel{t^2}+1-\cancel{t^2}}{2t}$

$= \dfrac{2}{2t}$

$= \dfrac{1}{t}$ ✓

$= RHS$ as req'd.

7. i) $m_{AP} = \dfrac{4 - -3}{\frac{1}{2} - -3}$ $\quad m_{PQ} = \dfrac{5 - -3}{1 - -3}$

$= \dfrac{7}{3\frac{1}{2}}$ $\quad = \dfrac{8}{4}$

$= 2$ ✓ $\quad = 2$

$= m_{AP}$ ✓

$\therefore$ A, P, Q are collinear

ii) $P(-3,-3) \quad Q(1,5)$

$k : 1$

$\therefore 4 = \dfrac{-3+5k}{k+1}$

$4k+4 = -3+5k$

$7 = k$

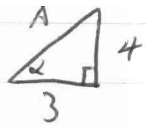
$\therefore$ Ratio is $7:1$

8 (i) $3\cos x + 4\sin x = A\cos(x-\alpha)$

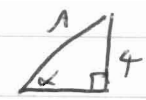
$3\cos x + 4\sin x = A\cos x\cos\alpha + A\sin x\sin\alpha$

Equating:

$\cos x$: $3 = A\cos\alpha$ $\qquad$ $\sin x$: $4 = A\sin\alpha$



$\sin\alpha = \frac{4}{A}$



$\therefore A = 5$ ✓

$\cos\alpha = \frac{3}{5}$

$\alpha = 53^\circ 8'$ nearest minute ✓

$\therefore 3\cos x + 4\sin x = 5\cos(x - 53^\circ 8')$

(ii) $3\cos x + 4\sin x = -3$ $\qquad$ $0^\circ \le x \le 360^\circ$

$5\cos(x-53^\circ 8') = -3$ $\qquad$ $-53^\circ 8' \le x - 53^\circ 8' \le 306^\circ 52'$

$\cos(x - 53^\circ 8') = -\frac{3}{5}$

$x - 53^\circ 8' = 126^\circ 52', 233^\circ 8'$ ✓

$x = 180^\circ, 286^\circ 16'$ ✓

9. $LHS = \dfrac{\cot A - \tan A}{\cot 2A}$

$= \dfrac{\frac{1}{\tan A} - \tan A}{\frac{1-\tan^2 A}{2\tan A}}$

$= \dfrac{1-\tan^2 A}{\tan A} \times \dfrac{2\tan A}{1-\tan^2 A}$

$= 2$

$= RHS$.

10. $8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$

$8\sin^2\theta - 2\cos^2\theta = 2\cos^2\theta - 1$

$8\sin^2\theta - 4\cos^2\theta + 1 = 0$

$8\sin^2\theta - 4(1-\sin^2\theta) + 1 = 0$

$8\sin^2\theta - 4 + 4\sin^2\theta + 1 = 0$

$12\sin^2\theta - 3 = 0$

$\sin^2\theta = \frac{3}{12}$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$\theta = 30°, 150°, 210°, 330°$

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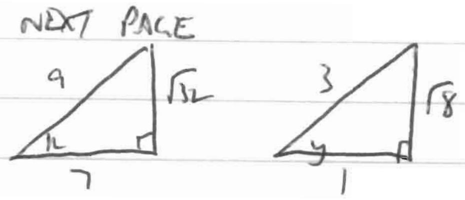
12 

i) $\sin x = \frac{\sqrt{32}}{9}$

$\sin 2y = 2\sin y\cos y$

$= 2\,\frac{\sqrt{8}}{3}\cdot\frac{1}{3}$

$= \frac{2\times 2\sqrt{2}}{9}$

$= \frac{4\sqrt{2}}{9}$

$= \frac{\sqrt{32}}{9}$

since $\sin x = \sin 2y$ (x, y acute)

$\therefore x = 2y$

ii) $\sin 3y = \sin(2y+y)$

$= \sin 2y\cos y + \cos 2y\sin y$

$= \frac{4\sqrt{2}}{9}\cdot\frac{1}{3} + (1-2\sin^2 y)\frac{\sqrt{8}}{3}$

$= \frac{4\sqrt{2}}{27} + \frac{\sqrt{8}}{3} - 2\frac{\sqrt{8}}{3}\times\frac{8}{9}$

$= \frac{4\sqrt{2}}{27} + \frac{18\sqrt{2}}{27} - \frac{32\sqrt{2}}{27}$

$= -\frac{10\sqrt{2}}{27}$ as req'd.

ii. $3\sin 2\theta - 4\cos 2\theta = 4$ $\quad 0^\circ \leq \theta \leq 180^\circ$

$\frac{3\times 2t}{1+t^2} - 4\frac{(1-t^2)}{1+t^2} = 4$ ✓

$6t - 4 + 4t^2 = 4 + 4t^2$

$6t = 8$ ✓

$t = \frac{4}{3}$

$\tan\theta = \frac{4}{3}$

$\theta = 53^\circ 8'$ ✓

Checking $\theta = 90^\circ$

$LHS = 3\sin 180^\circ - 4\cos 180^\circ$

$= 0 - 4\times -1$

$= 4$

$= RHS$

$\therefore \theta = 90^\circ$ is also a solution ✓

$\therefore \theta = 53^\circ 8',\ 90^\circ.$

13.

$4x+3y=8$

$y=-\frac{4x}{3}+\frac{8}{3}$

$m_1=-\frac{4}{3}$

$ax+by+c=0$

$y=-\frac{ax}{b}-\frac{c}{a}$

$m_2=-\frac{a}{b}$

$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan 45^\circ=\left|\frac{-\frac{4}{3}+\frac{a}{b}}{1+\frac{4a}{3b}}\right|$ ✓

$1=\left|\frac{-4b+3a}{3b+4a}\right|$

$|4a+3b|=|3a-4b|$

$4a+3b=3a-4b$ or $4a+3b=-(3a-4b)$ ✓

$a=-7b$

$\therefore \frac{a}{b}=-7$

$\therefore a:b=-7:1$

$4a+3b=-3a+4b$

$7a=b$

$\frac{a}{b}=\frac{1}{7}$ ✓

$\therefore a:b=1:7$

14 a) $\sin(A+B)+\sin(A-B)$

$=\sin A\cos B+\cancel{\cos A\sin B}+\sin A\cos B-\cancel{\sin B\cos A}$ ✓

$=2\sin A\cos B$

b) $\sin 157.5^\circ\cos 67.5^\circ=\frac{1}{2}\left[\sin(157.5^\circ+67.5^\circ)+\sin(157.5^\circ-67.5^\circ)\right]$

$=\frac{1}{2}(\sin 225^\circ+\sin 90^\circ)$ ✓

$=\frac{1}{2}(-\sin 45^\circ+1)$

$=\frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)$ ✓

$=\frac{\sqrt{2}-1}{2\sqrt{2}}$

$=\frac{2-\sqrt{2}}{4}$

15.

$\sin x + \cos x = a \qquad \cos 2x = b$

$\sin^2 x + 2\sin x\cos x + \cos^2 x = a^2 \qquad \cos^2 2x = b^2$

$\sin^2 x + \cos^2 x + \sin 2x = a^2$

$1 + \sin 2x = a^2$ ✓

$(1+\sin 2x)^2 = a^4$

$1 + 2\sin 2x + \sin^2 2x = a^4$

Now $a^4 - 2a^2 + b^2$

$= 1 + 2\sin 2x + \sin^2 2x - 2(1+\sin 2x) + \cos^2 2x$ ✓

$= 1 + \sin^2 2x + \cos^2 2x - 2 + 2\sin 2x - 2\sin 2x$ ✓

$= 0$ as req'd.

16.

a) LHS $= \dfrac{1-\tan^2 x}{1+\tan^2 x}$

$= \dfrac{1-\tan^2 x}{\sec^2 x}$

$= \dfrac{1 - \frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} \times \dfrac{\cos^2 x}{\cos^2 x}$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

$=$ RHS as req'd.

b) if $\cos 2A = \tan^2 B$

$\dfrac{1-\tan^2 A}{1+\tan^2 A} = \tan^2 B$

$1 - \tan^2 A = \tan^2 B(1+\tan^2 A)$

$1 - \tan^2 A = \tan^2 B + \tan^2 A\tan^2 B$

$1 - \tan^2 B = \tan^2 A + \tan^2 A\tan^2 B$

$1 - \tan^2 B = \tan^2 A(1+\tan^2 B)$

$\dfrac{1-\tan^2 B}{1+\tan^2 B} = \tan^2 A$

But from (a) $\dfrac{1-\tan^2 B}{1+\tan^2 B} = \cos 2B$

$\therefore \tan^2 A = \cos 2B$