



**BAULKHAM HILLS HIGH SCHOOL**

**Assessment Task 2 2015**  
**YEAR 11**

# **Mathematics Extension 1**

## **General Instructions**

- Reading time – 5 minutes
- Working time – 50 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working
- Marks may be deducted for careless or badly arranged work

**Total marks – 29**

**Exam consists of 3 pages.**

This paper consists of TWO sections.

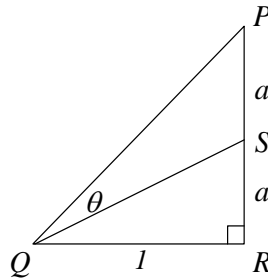
Question 1 (7 marks)		Marks
a)	Find the acute angle between the lines $2x + y = 4$ and $x - y = 2$ , to the nearest minute.	2
b)	$A$ is the point $(-4, 1)$ and $B$ is the point $(2, 4)$ . $R$ is the point which divides $AB$ externally in the ratio $2 : 1$ . Find the co-ordinates of $R$	2
c)	If $\sin \alpha = \frac{3}{4}$ , $0^\circ < \alpha < 90^\circ$ and $\sin \beta = \frac{2}{3}$ , $90^\circ < \beta < 180^\circ$ find the exact values of $\sin(\alpha - \beta)$	3

Question 2 (8 marks)		
a)	Given $\cos \theta = x$ . Express $\sin(270^\circ + \theta)$ in terms of $x$	2
b)	i) Express $\sqrt{3} \cos x - \sin x$ in the form $R \cos(x + \alpha)$ , where $\alpha$ is in degrees.	2
	ii) Hence, or otherwise, solve the equation $\sqrt{3} \cos x - \sin x = 2$ for $0^\circ \leq x \leq 360^\circ$	2
c)	Prove $\tan(45^\circ + x) = \frac{\cos x + \sin x}{\cos x - \sin x}$	2

**Question 3 (8 marks)**

**Marks**

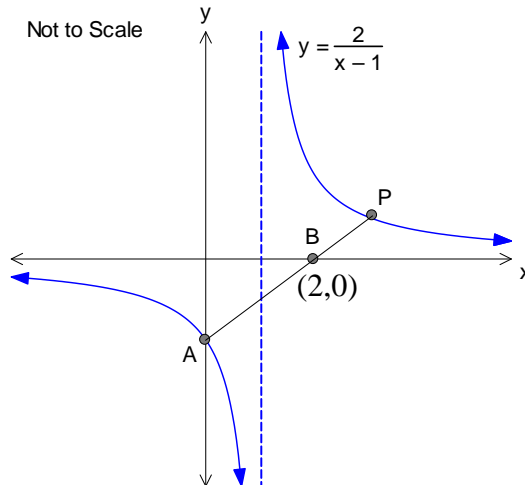
a) **2**



In the diagram, angle  $PRQ$  is a right angle, and  $PS$  and  $SR$  are both of length  $a$  cm.  $QR$  is of length 1 cm. Express  $\tan \theta$  in terms of  $a$

b) Solve for  $7 \sin x - 4 \cos x = 4$  for  $0^\circ \leq x \leq 360^\circ$  by using the substitution  $t = \tan \frac{x}{2}$  **3**

c) It is given that  $A$  is the  $y$ -intercept of the graph  $y = \frac{2}{x-1}$ .  $B$  is  $(2, 0)$  as shown in the diagram below. **3**



The line  $AB$  is extended to intersect the curve at point  $P$   
Find the ratio in which  $P$  divides the interval  $AB$ .

**Question 4 (6 marks)**

a) i) Evaluate  $\cos 105^\circ$  in exact form. **2**

ii) Hence or otherwise, find the exact value of  $\cos 1005^\circ$  **1**

b) Given that that  $\cos 3x = 4 \cos^3 x - 3 \cos x$ ,  
find the exact value of  $\sin 18^\circ$  **3**

**End of the Exam**



Q. a)  $2x + y = 4$        $x - y = 2$

$$m_1 = -2$$

$$m_2 = 1$$

$$\tan \theta = \left| \frac{1 - (-2)}{1 + (-2)(1)} \right|$$

Imk

$$\tan \theta = 3.$$

$$\theta = 71^\circ 34'$$

b)  $(-4, 1)$        $(2, 4)$

$$-2 : 1$$

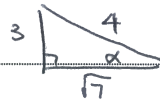
$$R = \left( \frac{-2 \times 2 - 4 \times 1}{-2 + 1}, \frac{1 \times 1 - 2 \times 4}{-2 + 1} \right)$$

Imk = -ve for external ratio  
 • correct internal division  
 ie  $R(0, 3)$ .

$$R(8, 7)$$

c)  $\sin \alpha = \frac{3}{4}$

$\alpha$  is acute

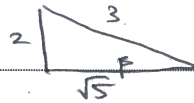


Imk • correct expansion  
 of  $\sin(\alpha - \beta)$

$$\cos \alpha = \frac{\sqrt{7}}{4}$$

$$\sin \beta = \frac{2}{3}$$

$\beta$  is obtuse



Imk • correct  $\cos \alpha \cos \beta$

$$\cos \beta = \frac{-\sqrt{5}}{3}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta.$$

$$= \frac{3}{4} \times \frac{-\sqrt{5}}{3} - \frac{\sqrt{7}}{4} \times \frac{2}{3}$$

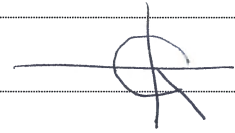
$$= \frac{-3\sqrt{5} - 2\sqrt{7}}{12}$$

Q2. a)  $\cos \theta = x$

$$\sin(270^\circ + \theta) = -\sin(90 - \theta)$$

$$= -\cos \theta$$

$$= -x$$



Imk acute  $\angle = 90 - \theta$ .

Imk  $-\sin \theta$ .

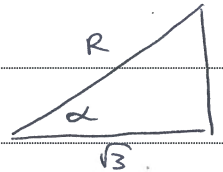
b) (i)  $\sqrt{3} \cos x - \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$

$$R \cos \alpha = \sqrt{3}$$

$$R \sin \alpha = 1$$

$$\tan \alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^\circ$$



$$R = 2$$

Imk. correct  $\alpha$  or  $R$ .

$$\sqrt{3} \cos x - \sin x = 2 \cos(x + 30^\circ)$$

Imk correct expression with correct  $\alpha$  and  $R$ .

(ii)

$$2 \cos(x + 30^\circ) = 2$$

$$+30 < x + 30 < 390$$

$$\cos(x + 30^\circ) = 1$$

Imk changing domain.



Imk exact value of  $\cos \theta = 1$   
 $\theta = 360^\circ$

\* excluding  $\theta = 0^\circ$ .

$$x + 30^\circ = 360^\circ$$

$$x = 330^\circ$$

c)  $\tan(45^\circ + x) = \frac{\tan 45 + \tan x}{1 - \tan 45 \tan x}$

$$= \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} \times \cos x$$

Imk expanding  $\tan x$ .

$$= \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} \times \cos x$$

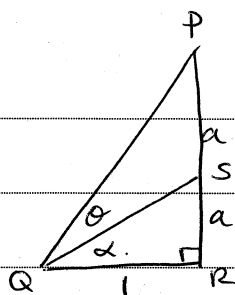
$$= \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$= \text{RHS.}$$

Q3. a)



$$\tan \alpha = \frac{a}{1}$$

$$\tan(\theta + \alpha) = \frac{\tan \theta + \tan \alpha}{1 - \tan \theta \tan \alpha}$$

Imp. expanding  $\tan(\theta + \alpha)$   
and using  $\tan \alpha = 2a$

$$2a = \frac{\tan \theta + a}{1 - \tan \theta \times a}$$

$$2a - 2a \times a \tan \theta = \tan \theta + a$$

$$a = \tan \theta (1 + 2a^2)$$

$$\tan \theta = \frac{a}{1 + 2a^2}$$

Imp correct soln  
for  $\tan \theta$

b)  $7 \sin x - 4 \cos x = 4$  for  $0^\circ < x < 360^\circ$

$$\frac{7 \times 2t}{1+t^2} - \frac{4(1-t^2)}{1+t^2} = 4$$

Imp correct t formulae  
in equation.

$$4t^2 + 14t - 4 = 4 + 4t^2$$

$$7t = 4$$

$$\tan \frac{x}{2} = \frac{4}{7} \quad \text{for } 0 < \frac{x}{2} < 180^\circ$$

$$\frac{x}{2} = 29^\circ 45'$$

$$x = 59^\circ 29'$$

Imp = Finding  $x$

When  $x = 180^\circ$

$$7 \sin 180 - 4 \cos 180 = 4$$

$$\therefore x = 59^\circ 29', 180^\circ$$

Imp. including  
 $x = 180^\circ$

Q3 c) A(0,-2) B(2,0)

∴ eqn of line AB is  $y = x - 2$ .

$$x - 2 = \frac{2}{x - 1}$$

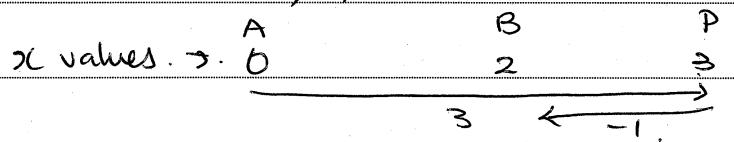
$$x^2 - 3x + 2 = 2$$

$$x(x - 3) = 0$$

$$x = 3$$

∴ P(3, 1)

Imp Finding P.



∴ ratio is 3:-1 or -3:1.

Imp working towards finding Ratio.

Imp correct ratio with negative

or correct external ratio.



$$Q4. a) (i) \cos 105 = \cos (60 + 45)$$

Imk expanding  $\cos(60+45)$

$$= \cos 60 \cos 45 - \sin 60 \sin 45$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

Imk correct exact values

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{2}$$

$$(ii) \cos 1005^\circ = \cos(105 + 180 \times 5)$$



$$= -\cos 105^\circ$$

$$= \frac{\sqrt{6} - \sqrt{2}}{2}$$

Imk correct soln.

$$b) \cos(3 \times 18) = 4\cos^3 18 - 3\cos 18$$

$$\cos 54 = 4\cos^3 18 - 3\cos 18$$

Imk correct eqn to find  $\sin 18^\circ$

$$\sin 36 = 4\cos^3 18 - 3\cos 18$$

$$2\cos 18 \sin 18 = 4\cos^3 18 - 3\cos 18$$

$$0 = 4\cos^3 18 - 3\cos 18 - 2\cos 18 \sin 18$$

$$0 = \cos 18 (4\cos^2 18 - 2\sin 18 - 3)$$

$$0 = \cos 18 (4(1 - \sin^2 18) - 2\sin 18 - 3)$$

$$\therefore 4 - 4\sin^2 18 - 2\sin 18 - 3 = 0$$

$$4\sin^2 18 + 2\sin 18 - 1 = 0$$

Imk manipulative equation to something useful

$$\therefore \sin 18 = \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times -1}}{2 \times 4}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$

Since  $\sin 18 > 0$

Imk. correct value of  $\sin 18^\circ$  and excludes negative value

$$\therefore \sin 18 = \frac{-1 + \sqrt{5}}{4}$$