BAULKHAM HILLS HIGH SCHOOL

Assessment Task 2, 2016
YEAR 11

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 1-4
- Marks may be deducted for careless or badly arranged work

Total marks – 41
Exam consists of 3 pages.

This paper consists of 4 Questions.

QUESTION 1 (9 marks)

a) Given the points $A(-5,6)$ and $B(-2,3)$, find the point P which divides the interval AB externally in the ratio 3:2. **2**

b) Find the exact value of $\sin 255°$. **3**

c) Solve $\cos 2\theta + \sin 2\theta = 2\cos^2\theta$ for $0° \leq \theta \leq 360°$. **2**

d) If $\sin x = \frac{1}{3}$ and $\cos y = \frac{2}{3}$, and x and y are acute angles, evaluate $\cos(x - y)$ **2**

QUESTION 2 (9 marks)

a) The point $P(1, y)$ divides the interval AB internally in the ratio m:1, where $A = (-5,2)$ and $B = (3, -1)$.

(i) Find the value of m. **2**

(ii) What is the y-coordinate of P? **1**

b) Given $\sin 18° = \frac{1}{4}(\sqrt{5} - 1)$, find the exact value of $\cos 36°$ in simplest form. **2**

c) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x + \alpha)$

where $R > 0$ and $0° < \alpha < 90°$. **2**

(ii) Hence or otherwise, solve $\sqrt{3}\cos x - \sin x = \sqrt{3}$ for $0° \leq x \leq 360°$. **2**

QUESTION 3 (11 marks)

a) The line $y = mx$ makes an angle of 30° with the line $6x - 2y + 7 = 0$. Find the possible values of m in exact form. **3**

b) If $\tan\frac{\theta}{2} = \frac{a}{b}$, express $b\cos\theta + a\sin\theta$ in its simplest form. **3**

c) (i) Prove

$$\frac{\sin 2A}{1-\cos 2A} = \cot A$$ **2**

(ii) Hence find integers a and b such that $\cot 67.5° = a + \sqrt{b}$ **3**

QUESTION 4 (12 marks)

a) A, B and C are three acute angles whose sum is 45°. **3**

If $\tan A = 0.5$ and $\tan B = 0.25$, find the exact value of $\tan C$.

b) Prove that

$$\frac{\sin x+\sin(x+2y)}{2\cos y} = \sin(x+y)$$ **3**

c) (i) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ **2**

(ii) Hence or otherwise solve $\sin 2\theta = \cos 3\theta$ for $-180° \leq \theta \leq 180°$. **4**

End of Exam

Year 11 Assessment Task 2, 2016 - SOLUTIONS

Question 1.

TOTAL /41

a) $A(-5,6)$ $B(-2,3)$

$3 : -2$

$x = \dfrac{3(-2) - 2(-5)}{3 - 2}$ $y = \dfrac{3(3) - 2(6)}{3 - 2}$

$= 4$ $= -3$

$\therefore P = (4, -3)$

(1) x-value

(1) y-value

[For internal, max 1 mark]

b) $\sin 255^\circ = -\sin 75^\circ$ $\leftarrow$ (1)

$= -\sin(45^\circ + 30^\circ)$

$= -(\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ)$ $\leftarrow$ (1)

$= -\left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)$

$= -\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)$ or $\dfrac{-\sqrt{3}-1}{2\sqrt{2}}$ $\leftarrow$ (1)

c) $\cancel{2\cos^2\theta} - 1 + \sin 2\theta = \cancel{2\cos^2\theta}$

$\left[\begin{array}{l} 0 \le \theta \le 360^\circ \\ 0 \le 2\theta \le 720^\circ \end{array}\right]$

$\sin 2\theta = 1$ $\leftarrow$ (1)

$2\theta = 90^\circ, 450^\circ$

$\theta = 45^\circ, 225^\circ$ $\leftarrow$ (1)

d) $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \frac{\sqrt{8}}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{\sqrt{5}}{3}$ $\leftarrow$ (1)

$\left.\begin{array}{l} = \dfrac{2\sqrt{8} + \sqrt{5}}{9} \\ = \dfrac{4\sqrt{2} + \sqrt{5}}{9} \end{array}\right\}$ $\leftarrow$ (1)

Question 2

a) i) $A(-5,2)$ $\quad B(3,-1)$ $\quad P(1,y)$

$m:1$

~~$P(1,y)$~~

$$1 = \frac{m(3) + 1(-5)}{m+1} \quad \longleftarrow (1)$$

$$m + 1 = 3m - 5$$

$$6 = 2m$$

$$m = 3 \quad \longleftarrow (1)$$

ii) $$y = \frac{3(-1) + 1(2)}{3+1} = -\frac{1}{4} \quad \longleftarrow (1)$$

b) $$\cos 36^\circ = 1 - 2\sin^2 18^\circ \quad \longleftarrow (1)$$

$$= 1 - 2 \cdot \frac{1}{16}(\sqrt{5}-1)^2$$

$$= 1 - \frac{1}{8}(6 - 2\sqrt{5})$$

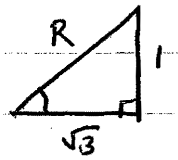
$$= \frac{1}{4} + \frac{\sqrt{5}}{4} \quad \text{or} \quad \frac{1+\sqrt{5}}{4} \quad \longleftarrow (1)$$

c) (i) $$\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$$

$$= R\cos x\cos\alpha - R\sin x\sin\alpha$$

$\cancel{R}\cos\alpha = \sqrt{3}/R$

$\cancel{R}\sin\alpha = 1/R$



$\therefore R = 2 \quad \longleftarrow (1)$

$\alpha = 30^\circ \quad \longleftarrow (1)$

$$\therefore \sqrt{3}\cos x - \sin x = 2\cos(x + 30^\circ)$$

(ii) $$\cancel{2}\cos(x+30^\circ) = \frac{\sqrt{3}}{2}$$

$$\begin{bmatrix} 0 \le x \le 360^\circ \\ 30^\circ \le x \le 390^\circ \end{bmatrix}$$

$$x + 30^\circ = 30^\circ, 330^\circ, 390^\circ$$

$$x = 0^\circ, 300^\circ, 360^\circ \quad \longleftarrow (2)$$

$$\begin{pmatrix} \text{2 correct} = \text{1 mark} \\ \text{1 correct} = \text{0 mark} \end{pmatrix}$$

Question 3

a) $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan 30^\circ = \left|\dfrac{m-3}{1+3m}\right|$

$\dfrac{1}{\sqrt{3}} = \dfrac{|m-3|}{|1+3m|}$ ← (1)

$\sqrt{3}(m-3) = 1 + 3m$ or $\sqrt{3}(m-3) = -1-3m$

$(\sqrt{3}-3)m = 1 + 3\sqrt{3}$ $\quad$ $(\sqrt{3}+3)m = -1+3\sqrt{3}$

$m = \dfrac{1+3\sqrt{3}}{\sqrt{3}-3}$ ← (1) $\quad$ $m = \dfrac{-1+3\sqrt{3}}{\sqrt{3}+3}$ ← (1)

b) Using $t = \tan\frac{\theta}{2}$,

$b\cos\theta + a\sin\theta = b\left(\dfrac{1-t^2}{1+t^2}\right) + a\left(\dfrac{2t}{1+t^2}\right)$ ← (1)

$= \dfrac{b(1-t^2) + 2at}{1+t^2}$

$= \dfrac{b - bt^2 + 2at}{1+t^2}$

$= \dfrac{b - b.\frac{a^2}{b^2} + 2a.\frac{a}{b}}{1 + \frac{a^2}{b^2}}$ ← (1)

$= \dfrac{b - \frac{a^2}{b} + \frac{2a^2}{b}}{1 + \frac{a^2}{b^2}}$

$= \dfrac{b + \frac{a^2}{b}}{1 + \frac{a^2}{b^2}} \times \dfrac{b^2}{b^2}$

$$= \frac{b^3 + a^2 b}{b^2 + a^2}$$

$$= \frac{b(b^2 + a^2)}{b^2 + a^2}$$

$$= b. \quad \longleftarrow (1)$$

c) (i) $\text{LHS} = \dfrac{\sin 2A}{1 - \cos 2A}$

$$= \frac{2 \sin A \cos A}{1 - (1 - 2\sin^2 A)}$$

$$= \frac{\cancel{2} \cancel{\sin A} \cos A}{\cancel{2} \sin^{\cancel{2}} A}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A \quad = \text{RHS}.$$

(ii) $\cot 67.5^\circ = \dfrac{\sin 135^\circ}{1 - \cos 135^\circ}$ (1)

$$= \frac{1/\sqrt{2}}{1 + 1/\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \sqrt{2} - 1$$

$\therefore \quad a = -1 \leftarrow (1), \quad b = 2 \leftarrow (1)$

Question 4.

a) $\tan((A+B)+C) = \tan 45°$

$$\frac{\tan(A+B) + \tan C}{1 - \tan(A+B)\tan C} = 1 \quad \longleftarrow (1)$$

Now $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{0.5 + 0.25}{1 - 0.5 \times 0.25}$$

$$= \frac{.75}{.875}$$

$$= \frac{6}{7} \quad \longleftarrow (1)$$

$$\frac{\frac{6}{7} + \tan C}{1 - \frac{6}{7}\cdot\tan C} = 1$$

$$\frac{6}{7} + \tan C = 1 - \frac{6}{7}\tan C$$

$$\frac{13}{7}\tan C = \frac{1}{7}$$

$$\tan C = \frac{1}{13} \quad \longleftarrow (1)$$

b) $\text{LHS} = \dfrac{\sin x + \sin(x+2y)}{2\cos y}$

$$= \frac{\sin x + \sin x \cos 2y + \cos x \sin 2y}{2\cos y}$$

$$= \frac{\sin x(1 + \cos 2y) + \cos x . 2\sin y \cos y}{2\cos y} \quad \longleftarrow (1)$$

$$\left\{ \begin{array}{l} \cos 2y = 2\cos^2 y - 1 \\ \therefore 1 + \cos 2y = 2\cos^2 y \end{array} \right.$$

$$= \frac{\sin x . 2\cos^2 y + 2\cos x \sin y \cos y}{2\cos y}$$

$$= \frac{\cancel{2\cos y}\,(\sin x \cos y + \cos x \sin y)}{\cancel{2\cos y}}$$

$$= \sin x \cos y + \cos x \sin y \qquad \longleftarrow (1)$$

$$= \sin(x+y)$$

$$= RHS$$

c) i) $\cos 3\theta = \cos(2\theta + \theta)$

$$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$

$$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta \cdot \sin\theta \qquad \longleftarrow (1)$$

$$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cdot\cos\theta$$

$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \qquad \longleftarrow (1)$$

$$= 4\cos^3\theta - 3\cos\theta$$

ii) $\sin 2\theta = \cos 3\theta$

$$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$$

$$4\cos^3\theta - 3\cos\theta - 2\sin\theta\cos\theta = 0$$

$$\cos\theta\,(\underbrace{4\cos^2\theta}_{1-\sin^2\theta} - 2\sin\theta - 3) = 0$$

$$\cos\theta\,(4 - 4\sin^2\theta - 2\sin\theta - 3) = 0 \qquad \longleftarrow (1)$$

$$\cos\theta\,(-4\sin^2\theta - 2\sin\theta + 1) = 0$$

$\therefore \cos\theta = 0$ or $-4\sin^2\theta - 2\sin\theta + 1 = 0$

$$4\sin^2\theta + 2\sin\theta - 1 = 0$$

$$\sin\theta = \frac{-2 \pm \sqrt{4+16}}{8}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$\doteqdot \cancel{0.0}\ 0.3090, -0.8090$$

$$\therefore \theta = \underbrace{90^\circ, -90^\circ}, \underbrace{18^\circ, 162^\circ}, -54^\circ, \underline{-126^\circ}$$