BAULKHAM HILLS HIGH SCHOOL

2017

YEAR 11 TASK 2

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 1 hour.
- Write using non erasable black or blue pen.
- Board-approved calculators may be used
- All relevant mathematical reasoning and/or calculations must be shown

Total marks – 36

This paper consists of 3 Questions.

Start each question on the appropriate page of your answer booklet
In Questions 1-3, your responses should include relevant mathematical reasoning and/ or calculations.

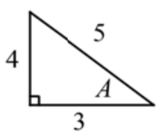
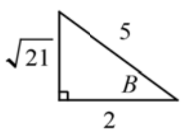
Question 1 (12 marks)		**Marks**
(a)	Find the exact value of $\tan 105°$.	**2**
(b)	Solve for x if $$\sin 2x = \cos x \quad \text{for} \quad 0° \leq x \leq 360°$$	**3**
(c)	Find the acute angle between the lines $x - y + 2 = 0$ and $2x - y - 4 = 0$. Give your answer to the nearest degree.	**2**
(d)	Find the point P which divides the interval AB internally in the ratio of $2:3$, where A is $(-4,-9)$ and B is $(5,9)$.	**2**
(e)	Simplify the following function in terms of t, if $t = \tan\frac{x}{2}$: $$\frac{\sin x - \cos x + 1}{\sin x + \cos x - 1}$$	**3**
	End of Question 1	

Question 2 (12 marks)		**Marks**
(a)	By using $t = \tan\frac{x}{2}$, solve for x $$3\sin x - 2\cos x = 2 \quad \text{for } 0 \le x \le 360°.$$ Give your answer to the nearest degree.	**3**
(b)	Given that the point $P(13,3)$ lies on the line joining $A(7,1)$ and $B(-5,-3)$, find the ratio in which P divides the interval AB.	**2**
(c)	(i) Express $\sqrt{3}\cos x + \sin x$ in the form $R\cos(x - \alpha)$ where α is in degrees.	**2**
	(ii) Hence, find the solution of the equation. $$\sqrt{3}\cos x + \sin x = \frac{11}{6} \quad \text{for } 0° \le x \le 360°.$$ Give your answer to the nearest degree.	**3**
(d)	If $\sin A = \frac{4}{5}$ where A is obtuse and $\cos B = \frac{2}{5}$ where B is acute, find the exact value of $\sin(A + B)$	**3**
	End of Question 2	

Question 3 (11 marks)		
(a)	(i) Prove that $\sqrt{\frac{1-\cos 2x}{1+\cos 2x}} = \tan x$, where x is acute.	**2**
	(ii) Hence find a and b if $\tan 15° = a - \sqrt{b}$.	**3**
(b)	(i) Prove that $\dfrac{\sin 3x}{\sin x} + \dfrac{\cos 3x}{\cos x} = 4\cos 2x$	**3**
	(ii) Hence solve for $\dfrac{\sin 3x}{\sin x} + \dfrac{\cos 3x}{\cos x} = 4\cos^2 2x \quad \text{for } 0° \le x \le 180°$	**3**
	End of Exam	

Year 11 Task 2 X1 2017

	Solutions	Mks	Comments
1a	$\tan 105° = \tan(60° + 45°)$ $= \dfrac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$ $= \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}}$ $= -2 - \sqrt{3}$	2	**2 marks** • correct solution. **1 mark** • Correct expansion • Finds exact values
1b	$\sin 2x = \cos x$ $2 \sin x \cos x - \cos x = 0$ $\cos x\,(2 \sin x - 1) = 0$ $\cos x = 0$ $x = 90°$ or $270°$ $\sin x = \dfrac{1}{2}$ $x = 30°$ or $150°$ $\therefore x = 30°, 90°, 150°, 270°$	3	**3 marks** • correct solution. **2 marks** • Solving $\cos x = 0$ or $\sin x = \frac{1}{2}$ **1 mark** • expanding $\sin 2x$.
1c	$m_1 = 1, m_2 = 2$ $\tan \theta = \left\|\dfrac{2 - 1}{1 + 2 \times 1}\right\|$ $\tan \theta = \dfrac{1}{3}$ $\theta = 18°26'$ $\theta = 18°$ (nearest degree)	2	**2 marks** • correct solution. **1 mark** • correct gradients • correct use of formula
1d	$\left(\dfrac{-4 \times 3 + 2 \times 5}{2 + 3}, \dfrac{-9 \times 3 + 2 \times 9}{2 + 3}\right)$ $= \left(\dfrac{-2}{5}, \dfrac{-9}{5}\right)$	2	**2 marks** • correct solution. **1 mark** • correct use of formula • One ordinate correct
1e	$\dfrac{\sin x - \cos x + 1}{\sin x + \cos x - 1} = \dfrac{\left(\frac{2t}{1+t^2}\right) - \left(\frac{1-t^2}{1+t^2}\right) + 1}{\left(\frac{2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) - 1}$ $= \dfrac{2t - 1 + t^2 + 1 + t^2}{2t + 1 - t^2 - 1 - t^2}$ $= \dfrac{2t^2 + 2t}{2t - 2t^2}$ $= \dfrac{2t(t + 1)}{2t(1 - t)}$ $= \dfrac{t + 1}{1 - t}$	3	**3 marks** • correct solution. **2 marks** • significant progress towards solution **1 mark** • correct substitution of t

2a	$3\left(\frac{2t}{1+t^2}\right)-2\left(\frac{1-t^2}{1+t^2}\right)=2$ $6t-2+2t^2=2+2t^2$ $\tan\frac{x}{2}=\frac{2}{3}$ $\frac{x}{2}=33.69°$ $x=67°$ Test $x=180°$ $LHS=3\sin 180°-2\cos 180°$ $=2$ $=RHS$ $\therefore x=67° \text{ or } 180°$	**3**	**3 marks** • correct solution. **2 marks** • Finds $x=67°$ using t-method **1 mark** • expanding $\sin 2x$. **0 mark** • solving without using t method.
2b	$x_{AP}=13-7=6$ $x_{PB}=13--5=18$ $x_{AP}:x_{PB}=6:-18$ $\therefore ratio\ AP:PB=1:-3$	**2**	**2 marks** • correct solution. **1 mark** • correct lengths
2c(i)	$\sqrt{3}\cos x+\sin x=R\cos x\cos\alpha+R\sin x\sin\alpha$ $R\cos\alpha=\sqrt{3}$ $R\sin\alpha=1$ $\tan\alpha=\frac{1}{\sqrt{3}}$ $\alpha=30°$ $R=2$ $\sqrt{3}\cos x+\sin x=2\cos(x-30°)$	**2**	**2 marks** • correct solution. **1 mark** • correct R • correct α
2c(ii)	$2\cos(x-30°)=\frac{11}{6}$ $\cos(x-30°)=\frac{11}{12}$ $x-30°=\pm 23.55° \quad \text{for } -30°\le x\le 330°$ $x=6° \text{ or } 54°$	**3**	**3 marks** • correct solution. **2 marks** • Find the 2 solutions for $(x-30°)$ **1 mark** • Finds acute angle of 23.55°
2d	 $\sin A=\frac{4}{5}$ $\cos A=\frac{-3}{5}$ |  $\cos B=\frac{2}{5}$ $\sin B=\frac{\sqrt{21}}{5}$ $\sin(A+B)=\sin A\cos B+\sin B\cos A$ $\sin(A+B)=\frac{4}{5}\times\frac{2}{5}+\frac{\sqrt{21}}{5}\times-\frac{3}{5}$ $=\frac{8-3\sqrt{21}}{25}$	**3**	**3 marks** • correct solution. **2 marks** • correct expansion and use of triangles to find trig ratios **1 mark** • correct expansion of $\sin(A+B)$ • Uses triangles to find exact trig ratios.

3a(i)	$LHS = \sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}$ $= \sqrt{\dfrac{1-(2\cos^2 x - 1)}{1+(2\cos^2 x - 1)}}$ $= \sqrt{\dfrac{2(1-\cos^2 x)}{2\cos^2 x}}$ $= \sqrt{\dfrac{\sin^2 x}{\cos^2 x}}$ $= \tan x$	**2**	**2 marks** • correct solution. **1 mark** • correct use of $\cos 2x$ identity
3a(ii)	$\tan 15° = \sqrt{\dfrac{1-\cos 2\times 15°}{1+\cos 2\times 15°}}$ $\tan 15° = \sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}$ $= \sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}}$ $= \sqrt{\dfrac{\left(2-\sqrt{3}\right)^2}{4-3}}$ $= 2-\sqrt{3}$ $\therefore a = 2, b = 3$	**3**	**3 marks** • correct solution. **2 marks** • $\sqrt{7-4\sqrt{3}}$ • substitutes values and rationalises **1 mark** • correct substitution for $x = 15°$ (Maximum of 1 mark if not using hence)
3b(i)	$LHS = \dfrac{\sin 3x}{\sin x} + \dfrac{\cos 3x}{\cos x}$ $= \dfrac{\sin 3x \cos x + \cos 3x \sin x}{\sin x \cos x}$ $= \dfrac{\sin(3x + x)}{\sin x \cos x}$ $= \dfrac{\sin 4x}{\sin x \cos x}$ $= \dfrac{2\sin 2x \cos 2x}{\sin x \cos x}$ $= \dfrac{2 \times 2\sin x \cos x \cos 2x}{\sin x \cos x}$ $= 4\cos 2x$	**3**	**3 marks** • correct solution. **2 marks** • significant progress towards solution **1 mark** • identifies $\sin(5x - x)$

3b(ii)	$4\cos 2x = 4\cos^2 2x$ $\cos 2x - \cos^2 2x = 0$ $\cos 2x\,(1 - \cos 2x) = 0$ $\cos 2x = 0$	$\cos 2x = 1$ $2x = 90°, 270°$	$2x = 0°, 360°$ $x = 45°, 135°$	$x = 0°, 180°$ But $\sin x \neq 0$ and $\cos x \neq 0$ $\therefore x = 45°, 135°.$	**3**	**3 marks** • correct solution. **2 marks** • Solves for $\cos 2x = 0$ and $\cos 2x = 1$ **1 mark** • uses equation in (i) to from a simplified equivalent expression.