Gosford High School

Year 11

2011
Preliminary
Higher School Certificate

Mathematics Extension 1

Assessment Task 3

Time Allowed – 90 minutes

(reading time 5 minutes)

Remember to start each new question **in a new booklet**
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

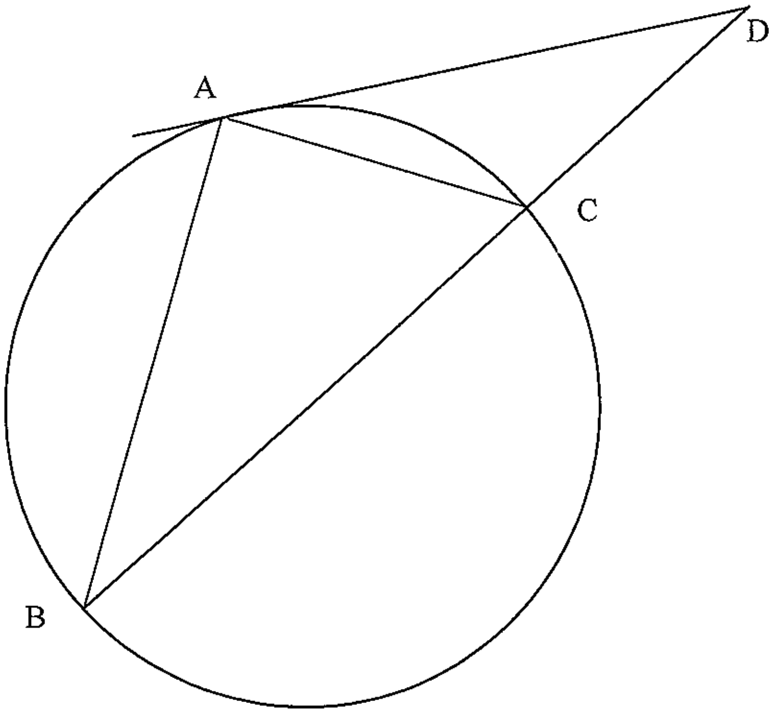
* 'bald' answers may not gain full marks.
* untidy and/or poorly organised solutions may not gain full marks.

QUESTION 1 (10 marks) ***(start a new booklet)***

a) For the points $A(-4, 7)$ and $B(-1, 2)$, find the co-ordinates of the point which divides AB externally in the ratio $3:1$ (3)

b) Solve $x-3<\frac{4}{x}$ (3)

c)



BC is a diameter of a circle. The tangent to the circle at A meets BC produced at D.

$\angle ABC = 40°$.

(i) Find the size of $\angle DAC$, giving reasons (1)

(ii) Find the size of $\angle CDA$, giving reasons. (3)

QUESTION 2 (10 marks) ***(start a new booklet)***

a) Fully factorise $ap^3 - aq^3 - ap^2q + apq^2$ (2)

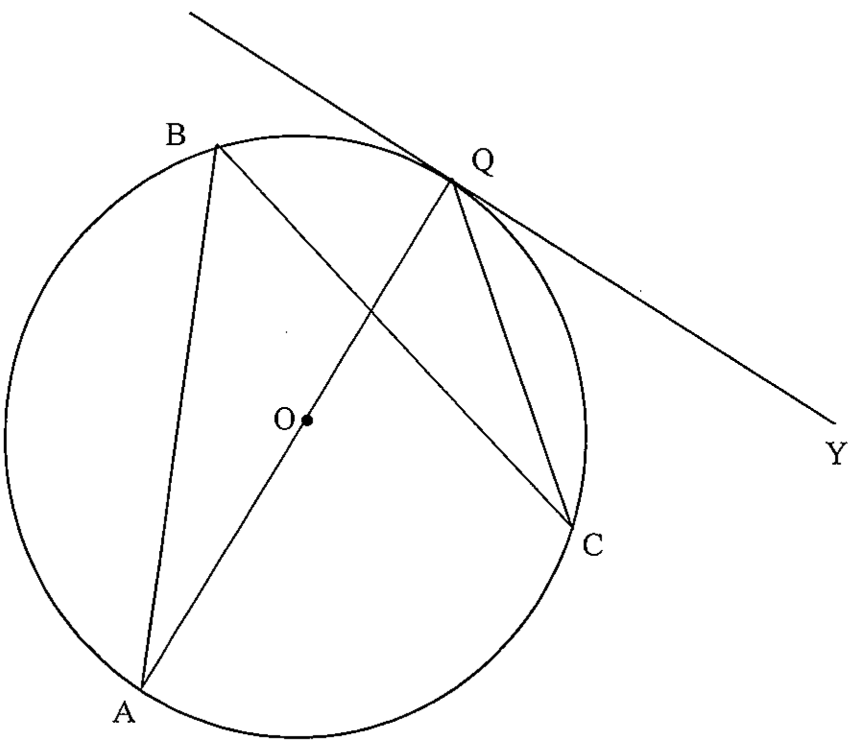
b) Find a linear factor of $4x^3 + 13x^2 + 7x - 6$ (2)

c) Four couples are to be seated at a round table for a game of Pictionary.
How many different seating arrangements are possible if :

(i) there are no restrictions? (1)

(ii) each person sits next to their partner? (2)

d)



In the diagram QY is a tangent to the circle with centre O.

AQ is a diameter and $\angle YQC = 32°$.

Find $\angle ABC$, giving reasons. (3)

QUESTION 3 (10 marks) ***(start a new booklet)***

a) Find, to the nearest degree, the acute angle formed by the lines $3y = x - 1$ and $4x + 2y - 5 = 0$ (3)

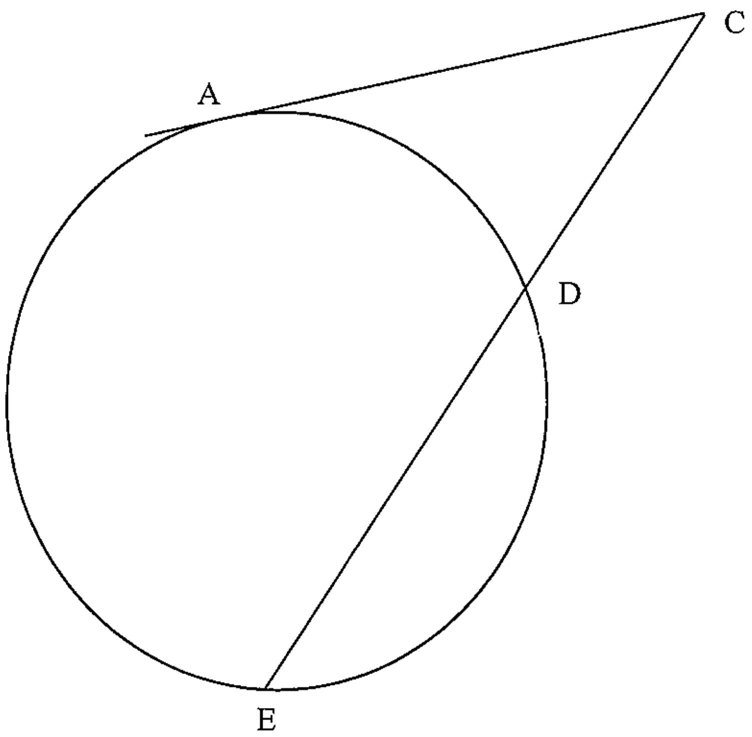
b) Using standard expressions for $\sin 2\theta$ and $\cos 2\theta$, prove that

$$\cos 3\theta = -3\cos\theta + 4\cos^3\theta$$ (4)

c) In the diagram the tangent AC intersects the secant EC at C.

Given $AC = 6cm$ and $ED = 5cm$.

Find the length of CD, giving reasons . (3)



QUESTION 4 (10 marks) ***(start a new booklet)***

a) A committee of three is to be chosen from a group of four males and five females.

The committee must include at least one male and at least one female.

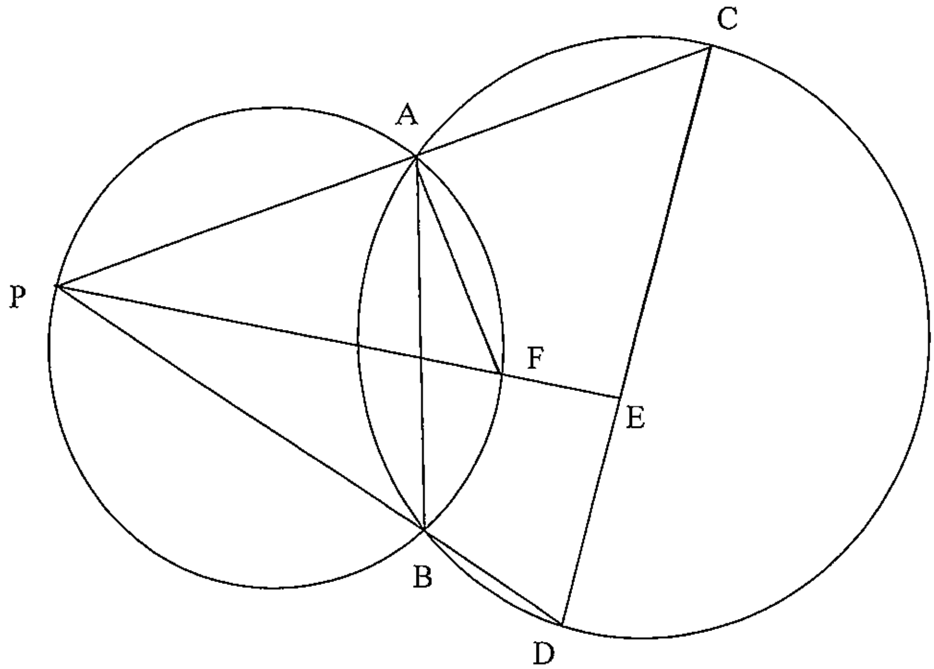
How many different committees can be chosen? (2)

b) (i) State the sum of the roots of the general quadratic equation $ax^2 + bx + c = 0$ (1)

(ii) The line $px + y - p^2 = 0$ intersects the parabola $x^2 = 4y$ at A and B. Show that the x co-ordinates of A and B are the roots of the equation $x^2 + 4px - 4p^2 = 0$. (1)

(iii) Hence, or otherwise, find the co-ordinates of the midpoint of the interval AB. (2)

c)



In this diagram, two circles intersect at A and B.

P is a point on the first circle such that PA produced and PB produced meet the second circle at C and D respectively.
E is a point on DC such that PE cuts the first circle at F.

(i) Explain why $\angle PBA = \angle PFA$ (1)

(ii) Show that $AFEC$ is a cyclic quadrilateral. (3)

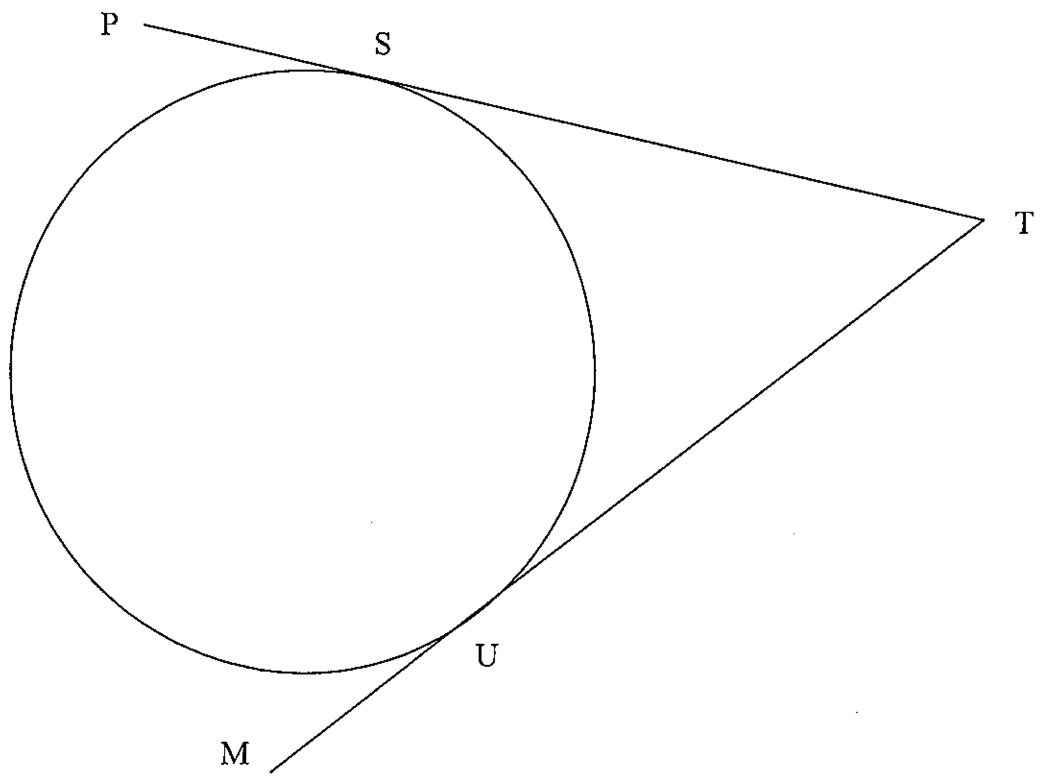
QUESTION 5 (10 marks) ***(start a new booklet)***

a) The polynomial equation $x^3 + bx^2 + cx + d = 0$ has roots α , α^2 and α^3 for some real number $\alpha \neq 0$

(i) Find in terms of c and d the value of $\frac{1}{\alpha}+\frac{1}{\alpha^2}+\frac{1}{\alpha^3}$ (3)

(ii) Show that $b^3 d - c^3 = 0$ (2)

b)



S and U are points on a circle. Tangents to the circle at S and U intersect at T.

P and M are points on the tangents TS and TU respectively.

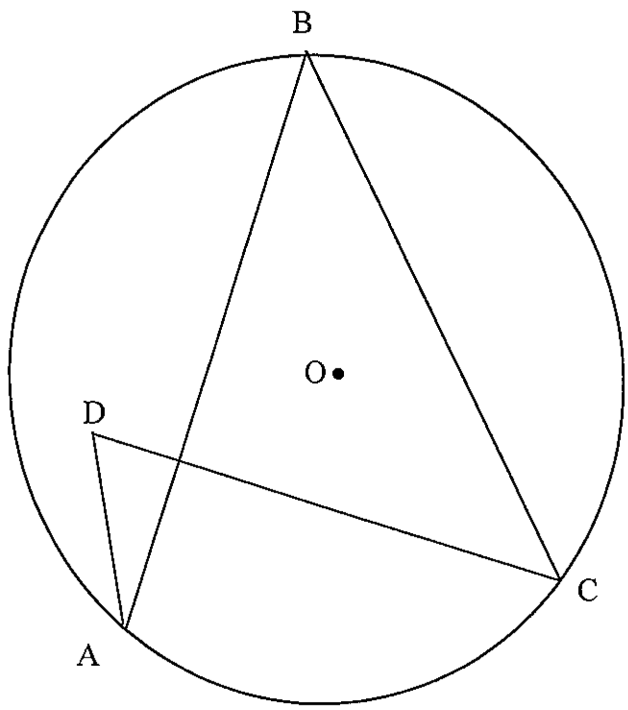
R is a point on the circle so that the chord SR is parallel to UT.

(i) Draw a neat diagram showing the given information. (1)

(ii) Prove that $SU = UR$ (2)

Question 5 is continued on the next page

c)



Points A, B and C lie on the circumference of a circle with centre O.
The point D lies inside the circle. $\angle ABC = 17°$ and $\angle ADC = 34°$

Prove that $ADOC$ is a cyclic quadrilateral. (2)

QUESTION 6 (10 marks) ***(start a new booklet)***

a) Simplify $\sin 4x \cos 3x - \sin 3x \cos 4x$ (1)

b) (i) Express $\tan 2x$ in terms of $\tan x$ (1)

(ii) Hence, or otherwise, show that the exact value of $\tan 15° = 2 - \sqrt{3}$ (3)

c) In ΔABC, $\cos B = \frac{1}{3}$ and $\cos C = \frac{2}{3}$.

(i) Find the exact value of $\sin B$ and $\sin C$ (2)

(ii) Hence, show that the exact value of $\cos A = \frac{2(\sqrt{10} - 1)}{9}$ (3)

GOSFORD HIGH SCHOOL YEAR 11 2011

PRELIMINARY HSC EXTENSION 1

MATHEMATICS EXTENSION 1 TASK 3

Question 1

a) Let (x, y) be the required point

$$x = \frac{3(-1) + (-1)(-4)}{2}, \quad y = \frac{3(2) + (-1)(7)}{2}$$

$\therefore$ Point is $\left(\frac{1}{2}, -\frac{1}{2}\right)$

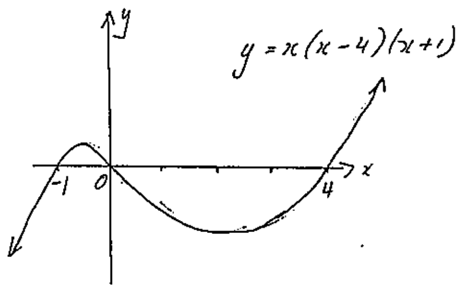
b) $x - 3 < \frac{4}{x}$

$\therefore x^2(x-3) < 4x$

$\therefore x^3 - 3x^2 - 4x < 0$

$x(x^2 - 3x - 4) < 0$

$x(x-4)(x+1) < 0$



Using the graph solution is $-1 < x < 0$ and, or $x > 4$

c) (i) $\angle DAC = \angle ABC$
$= 40°$
(angle between a tangent and a chord drawn at the point of contact is equal to the angle in the alternate segment.)

(ii) $\angle BAC = 90°$ (angle in a semi-circle is a right angle)

$\angle BAD = \angle BAC + \angle DAC$ (adjacent $\angle$'s)
$= 40° + 90°$
$= 130°$

$\therefore \angle CDA = 180° - \angle BAD - \angle ABC$ ($\angle$. sum of a Δ is $180°$)
$= 180° - 130° - 40°$
$= 10°$

Question 2

a) $ap^3 - aq^3 - ap^2q + apq^2 = a(p^3 - q^3) - apq(p-q)$
$= a(p-q)(p^2 + pq + q^2) - apq(p-q)$
$= a(p-q)(p^2 + pq + q^2 - pq)$
$= a(p-q)(p^2 + q^2)$

b) $P(-1) \neq 0$, $P(-2) = 4(-2)^3 + 13(-2)^2 + 7(-2) - 6$
$= -32 + 52 - 14 - 6$
$= 0$

$\therefore (x+2)$ is a factor

c) (i) No. of arrangements $= 7!$
$= 5040$

(ii) Four couples can arrange themselves in $3!$ w
Each couple can arrange themselves in 2 wa
$\therefore$ No. of arrangements $= 3!(2)^4$
$= 96$

d) $\angle AQY = 90°$ ($\angle$. between a tangent and a chord drawn at the point of contact is a right angle)

$\angle AQC = 90° - \angle YQC$ (adjacent complementary $\angle$'s.
$= 90° - 32°$
$= 58°$

$\angle ABC = \angle AQC$ ($\angle$'s at the circumference standing on the same arc are equal)
$= 58°$

Question 3.

a) $m_1 = \frac{1}{3}$, $m_2 = -2$

Let θ be the required angle

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{3} + 2}{1 - \frac{2}{3}}\right|$$

$$= \left|\frac{1 + 6}{3 - 2}\right|$$

$\tan\theta = 7$

$\theta = 82^\circ$ (to the nearest degree)

b) $\cos 3\theta = \cos(2\theta + \theta)$

$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$.

$= [2\cos^2\theta - 1]\cos\theta - 2\sin\theta\cos\theta \cdot \sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1 - \cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$.

$= 4\cos^3\theta - 3\cos\theta$

c) $AC^2 = EC \times CD$ (the square of the tangent is equal to the product of the intercepts of the intersecting secant)

Let $x = CD$

$\therefore 6^2 = (5 + x)x$

$\therefore 36 = 5x + x^2$

$\therefore x^2 + 5x - 36 = 0$

$(x + 9)(x - 4) = 0$

$\therefore x = 4$ since $x > 0$

Question 4

a) Possible committees have 1 male, 2 females or 2 males, 1 females

$\therefore$ No. of different committees $= {}^4C_1 \times {}^5C_2 + {}^4C_2 \times {}^5C_1$

$= 4 \times 10 + 6 \times 5$

$= 70$

b (i) Sum of roots $= -\frac{b}{a}$

(ii) Solve $y = p^2 - px$ and $x^2 = 4y$ simultaneously

$\therefore x^2 = 4(p^2 - px)$

$\therefore x^2 = 4p^2 - 4px$

$\therefore x^2 + 4px - 4p^2 = 0$ is the required equation

(iii) x-coordinate of midpoint $= \dfrac{\text{Sum of Roots}}{2}$

$= \dfrac{-4p}{2}$

$= -2p$.

when $x = -2p$, $y = p^2 - p(-2p)$

$= 3p^2$

$\therefore$ Midpoint is $(-2p, 3p^2)$

Question 4 (continued)

(i) $\angle PBA = \angle PFA$ ($\angle$'s. at the circumference standing on the same arc (or chord) are equal)

(ii) Let $\angle PBA = \angle PFA = x$

$\therefore \angle ABD = 180° - x$ (adjacent supplementary $\angle$'s)

$\therefore \angle ACE = 180° - (180 - x)°$ (the opposite $\angle$'s. of cyclic quadrilateral ABDC are supplementary)

$= x$

$= \angle AFP$

$\therefore$ AFEC is a cyclic quadrilateral as one angle of the quadrilateral AFEC is equal to the opposite exterior angle.

Question 5

a) (i) Sum of roots $= \frac{-b}{a}$.

$\therefore \alpha + \alpha^2 + \alpha^3 = -b$

Sum of roots taken two at a time $= \frac{c}{a}$.

$\therefore \alpha^3 + \alpha^4 + \alpha^5 = c$

and Product of Roots $= \frac{-d}{a}$.

$\therefore \alpha^6 = -d$

Now $\frac{1}{\alpha} + \frac{1}{\alpha^2} + \frac{1}{\alpha^3} = \frac{\alpha^3 + \alpha^4 + \alpha^5}{\alpha^6}$

$= \frac{c}{-d}$.

(ii) $c = \alpha^3 + \alpha^4 + \alpha^5$

$= \alpha^2(\alpha + \alpha^2 + \alpha^3)$

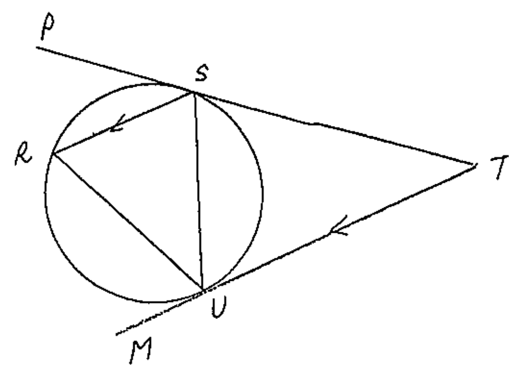
$= \alpha^2 \cdot (-b)$

$c^3 = \alpha^6 \cdot -b^3$

$\therefore c^3 = -d \cdot (-b^3)$

$b^3 d - c^3 = 0$

b)



Question 5 (continued)

$\therefore \angle MUR = \angle SRU$ (alternate $\angle$'s, $MU \parallel RS$)

$\angle MUR = \angle RSU$ ($\angle$. between a tangent and a chord is equal to the angle in the alternate segment)

$\therefore \angle SRU = \angle RSU$ $(= \angle MUR)$

$\therefore SU = UR$ (equal sides opposite equal $\angle$'s. of isosceles Δ

c) $\angle AOC = 2 \times \angle ABC$ (angle at the centre is twice the angle at the circumference, standing on the same arc)

$= 2 \times 17^\circ$

$= 34^\circ$

$\therefore \angle AOC = \angle ADC$ $(= 34^\circ)$

$\therefore ADOC$ is a cyclic quadrilateral

(interval AC subtends two equal $\angle$'s at $D \neq O$ on the same side of it.)

Question 6

a) $\sin 4x \cos 3x - \sin 3x \cos 4x = \sin(4x - 3x)$

$= \sin x$

b) (i) $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

(ii) $\tan 30^\circ = \dfrac{2\tan 15^\circ}{1 - \tan^2 15}$

$\dfrac{1}{\sqrt{3}} = \dfrac{2\tan 15^\circ}{1 - \tan^2 15}$

$1 - \tan^2 15 = 2\sqrt{3}\tan 15^\circ$

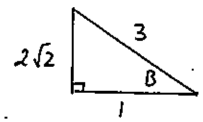
$\therefore \tan^2 15 + 2\sqrt{3}\tan 15 - 1 = 0$

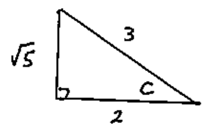
$\therefore \tan 15^\circ = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$

$\tan 15^\circ = \dfrac{-2\sqrt{3} \pm 4}{2}$

$\therefore \tan 15^\circ = -2\sqrt{3} + 2$ since $\tan 15 > 0$

c)





(i) $\sin B = \dfrac{2\sqrt{2}}{3}$

$\sin C = \dfrac{\sqrt{5}}{3}$

(ii) $\cos A = \cos(180 - (B + C))$ ABC is a Δ

$= -\cos(B + C)$

$= -[\cos B \cos C - \sin B \sin C]$

$= \dfrac{2\sqrt{2}}{3} \cdot \dfrac{\sqrt{5}}{3} - \dfrac{1}{3} \cdot \dfrac{2}{3}$

$\therefore \cos A = \dfrac{2\sqrt{10} - 2}{9} = \dfrac{2(\sqrt{10} - 1)}{9}$

Gosford High School

Year 11

2011
Preliminary
Higher School Certificate

Mathematics Extension 1

Assessment Task 3
Paper 1

Time Allowed – 50 minutes

Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

- * 'bald' answers may not gain full marks.
- * untidy and/or poorly organised solutions may not gain full marks.

POLYNOMIALS Part A *(start a new page)*

1) Sketch the graph of $y = 2x^2(1-x)^3$ *(no calculus required)* (2)

2) Find the remainder when $x^3 - x^2 + 11x - 17$ is divided by $x+1$ (2)

3) Given $P(x) = x^4 - 3x^3 - 2x + 7$ and $A(x) = x^2 + 3$,

(i) find the quotient *Q(x)* and the remainder *R(x)* when *P(x)* is divided by *A(x)*. (2)

(ii) Express your result in the form $P(x) = A(x) \cdot Q(x) + R(x)$. (1)

4) When *P(x)* is divided by $x^2 - 3x - 4$ the remainder is $2x + 3$.
Find the remainder when *P(x)* is divided by $x - 4$. (2)

5) If α, β and γ are the roots of the equation $2x^3 - 4x^2 + 5x - 3 = 0$, find the values of
(i) $\alpha + \beta + \gamma$ (ii) $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ (iii) $\alpha^2 + \beta^2 + \gamma^2$ (5)

POLYNOMIALS Part B *(start a new page)*

6) (i) If $P(x) = 2x^3 + x^2 - 13x + 6$, find the value of $P(2)$ (1)
(ii) Hence, solve $2x^3 + x^2 - 13x + 6 = 0$ (3)

7) The polynomials x^3 and $A(x-1)(x-2)(x-3) + B(x-1)(x-2) + C(x-1) + D$ are equal for more than 3 values of *x*. Find the values of the constants A, B, C and D. (4)

8) Two of the roots of $2x^4 - 5x^3 + cx^2 + 45x - 18 = 0$ are reciprocals, while the remaining two roots are opposites of each other. Find the four roots and hence evaluate *c*. (4)

INEQUALITIES *(start a new page)*

1) Solve $2x(x-4)(3-x) > 0$ (2)

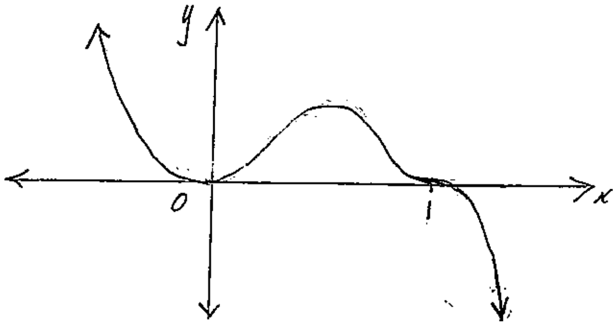
2) Solve $\dfrac{x^2 - 5x}{x - 4} \le 3$ (3)

3) Find the set of values of *x* for which $x^2 < 25$ and $\dfrac{5}{|x-4|} \le 1$ hold simultaneously. (3)

2011 Preliminary HSC
X1 Assessment Task 3 Paper 1

Polynomials Part A.

1)



2) Let $P(x) = x^3 - x^2 + 11x - 17$

Remainder $= P(-1)$

$= (-1)^3 - (-1)^2 + 11(-1) - 17$

$= -1 - 1 - 11 - 17$

$= -30$

3) (i)

$$\begin{array}{r} x^2 - 3x - 3 \\ x^2+3 \,\big)\, x^4 - 3x^3 \qquad - 2x + 7 \\ x^4 \qquad + 3x^2 \qquad\qquad \\ \hline -3x^3 - 3x^2 - 2x + 7 \\ -3x^3 \qquad - 9x \qquad \\ \hline -3x^2 + 7x + 7 \\ -3x^2 \qquad - 9 \\ \hline 7x + 16 \end{array}$$

(ii)

$\therefore P(x) = (x^2+3)(x^2-3x-3) + 7x + 16$

4)
$P(x) = (x^2 - 3x - 4) \cdot A(x) + (2x+3)$

$= (x-4)(x+1) \cdot A(x) + (2x+3)$

Remainder when divided by $(x-4)$ is $P(4)$

$P(4) = 0 + 2(4) + 3$

$= 11$

Remainder is 11

5) (i) $\alpha + \beta + \gamma = \frac{-b}{a}$

$= 2$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{5/2}{3/2}$

$= \frac{5}{3}$

(iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - \sum 2\alpha\beta$

$= 2^2 - 2 \times \frac{3}{2}$

$= 1$

Polynomials Part B.

6) (i) $P(2) = 2(2)^3 + (2)^2 - 13(2) + 6$

$= 16 + 4 - 26 + 6$

$= 0$

$\therefore x - 2$ is a factor

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x-2 \,\big)\, 2x^3 + x^2 - 13x + 6 \\ 2x^3 - 4x^2 \qquad\qquad \\ \hline 5x^2 - 13x + 6 \\ 5x^2 - 10x \qquad \\ \hline -3x + 6 \\ -3x + 6 \\ \hline 0 \end{array}$$

$\therefore (x-2)(2x^2 + 5x - 3) = 0$

$(x-2)(2x-1)(x+3) = 0$

$\therefore x = 2, \frac{1}{2}, -3$

7) $x^3 \equiv A(x-1)(x-2)(x-3) + B(x-1)(x-2) + C(x-1) + D$

True for $x = 1$ $\therefore 1 = D.$

True for $x = 2$ $\therefore 8 = C + 1$

$7 = C$

True for $x = 3$ $\therefore 27 = B(2)(1) + 7(2) + 1$

$12 = 2B$

$B = 6$

and $A = 1$ by inspection of coefficient of x^3.

$\therefore A = 1,\ B = 6,\ C = 7,\ D = 1$

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(8) Let the roots be

$\alpha, \frac{1}{\alpha}, \beta, -\beta$

$\sum \alpha = -\frac{b}{a}$

$\therefore \alpha + \frac{1}{\alpha} = \frac{5}{2}$

$2\alpha^2 + 2 = 5\alpha$

$\therefore 2\alpha^2 - 5\alpha + 2 = 0$

$(2\alpha - 1)(\alpha - 2) = 0$

$\therefore \alpha = \frac{1}{2}, 2$

Product of Roots $= \frac{e}{a}$

$-\beta^2 = -9$

$\beta^2 = 9$

$\beta = \pm 3$

$\therefore$ Roots are $2, \frac{1}{2}, 3, -3$

$x = 2$ satisfies

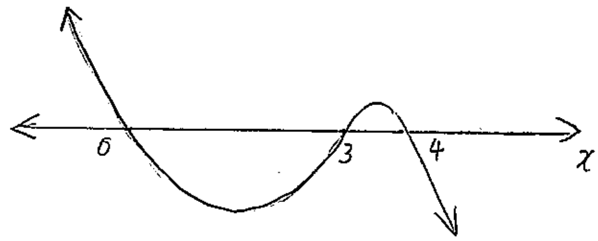
$\therefore 32 - 40 + 4c + 90 - 18 = 0$

$4c = -64$

$c = -16$

Inequalities

(1)



$\{x : x < 0\} \cup \{x : 3 < x < 4\}$

(2) Critical values $x \neq 4$

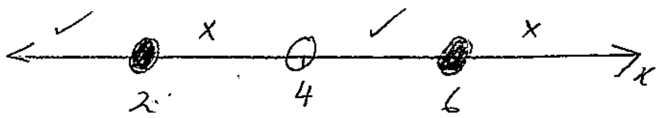
and when $\frac{x^2 - 5x}{x - 4} = 3$

$x^2 - 5x = 3x - 12$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2)=0$

$x = 2, 6$



Test $x = 0$, True

Test $x = 3$, False

Test $x = 5$, True

Test $x = 7$, False

$\therefore \{x : x \leq 2\} \cup \{x : 4 < x \leq 6\}$.

(3) $x^2 < 25$

$\therefore -5 < x < 5$

$5 \leq |x-4|$

$\therefore |x-4| \geq 5$

$\therefore x-4 \leq -5$ or $x-4 \geq 5$

$x \leq -1$ $\quad$ $x \geq 9$

$\therefore$ Require set of points satisfying

$-5 < x < 5$ and

$x \leq -1 \cup x \geq 9$

$\therefore \underline{\underline{-5 < x \leq -1}}$

GHS

SPECTEMUR AGENDO

Gosford High School

Year 11

2011
Preliminary
Higher School Certificate

Mathematics Extension 1

Assessment Task 3
Paper 2

Time Allowed – 60 minutes

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COUNTING TECHNIQUES *(start a new page)*

1) In how many ways can the letters of the word CYCLIC be arranged

(i) without any restrictions? (1)

(ii) if all the C's are to be together? (2)

(iii) if there is to be a vowel on an end and *no* C's are to be together? (2)

2) In how many ways can 4 boys and 4 girls be seated around a table if

(i) there are no restrictions (1)

(ii) the boys and girls alternate (1)

(iii) two particular boys wish to sit next to each other (1)

(iv) two particular girls wish to sit opposite each other. (1)

3) A committee of 5 is to be chosen from 6 boys and 4 girls.

(a) Find the total number of different committees possible (1)

(b) Find the total number of different committees possible if the committee contains

(i) 3 boys and 2girls. (1)

(ii) a particular boy X but excludes a particular girl Y. (1)

(iii) a majority of girls (2)

DIVISION OF A LINE IN A GIVEN RATIO *(start a new page)*

4) $A(2,-1)$ and $B(-5,-3)$ are two points.
Find the coordinates of the point S which divides AB externally in the ratio 4 : 7. (3)

5) The points $A(-1, 5)$, $B(6,-4)$ and $P(20,-22)$ are collinear.
Find the ratio in which P divides the interval AB. (3)

6) $P(x, y)$ divides the join of $(-1, 0)$ and $(3,5)$ in the ratio $k:1$.
Find the values of k for which $x+y>4$. (4)

HARDER 2 UNIT *(start a new page)*

7) The length of a rectangle is increased by 12% and the width is decreased by 10%.
What is the percentage change in the area. (1)

8) Find x and y given that $25^x = 125^y$ and $\dfrac{16^x}{8} = 2\times 4^{2y}$ (3)

9) Make x the subject of the equation $t = \dfrac{1}{2(4-x)} - \dfrac{1}{8}$, expressing your answer as a single fraction. (3)

10) Factorise $m^3 - a^3 - m^2 + 2am - a^2$ (3)

11) Solve $x^2 - 4x + \dfrac{5}{x^2 - 4x} = 6$ (3)

12) Find the average of the roots of $2x^2 + 14x + 17 = 0$ (2)

Task 3 (Paper 2)

Counting Techniques

① (i) Nº. of arrangements $= \frac{6!}{3!} = 120$ ways

(ii) c's can only arrange themselves as a group in 1 way then the c's (as one) and the other 3 letters can arrange themselves in $4!$ ways

$\therefore$ Nº. of arrangements $= 1 \times 4! = 24$ ways

(iii) | i | c | 2 | c | 1 | c |

With vowel (i) in first position c's must occupy alternate positions resulting in $2!$ ways.

But vowel (i) can be on the end with c's again in alternate positions resulting again in $2!$ ways

$\therefore$ Nº. of arrangements $= 2 \times 2!$ ways

$= 4$ ways

② i) Nº. of arrangements $= (8-1)! = 7! = 5040$ ways

ii) Nº. of arrangements $= 3!\,4! = 144$ ways

iii) Nº. of arrangements $= 2 \times 6! = 1440$ ways

iv) Nº of arrangements $= 1 \times 6! = 720$ ways.

(3) a) N°. of Committees $= {}^{10}C_5 = 252$

b) i) N°. of Committees $= {}^{6}C_3 \times {}^{4}C_2 = 120$

ii) N° of Committees $= {}^{8}C_4 = 70$

iii) N°. of Committees $= {}^{4}C_4 \times {}^{6}C_1 + {}^{4}C_3 \times {}^{6}C_2$

$= 6 + 60$

$= 66$

Division of a Line in a Given Ratio

(4) $A(2,-1)$ $B(-5,-3)$ $S(x,y)$

$-4 : 7$

$\therefore x = \dfrac{(-4)(-5) + 7(2)}{7-4}$

$x = \dfrac{34}{3}$

and $y = \dfrac{(-4)(-3) + 7(-1)}{3}$

$= \dfrac{5}{3}$

$\therefore S(11\tfrac{1}{3}, 1\tfrac{2}{3})$

(5) Let the ratio be $k:1$

$A(-1,5)$ $B(6,-4)$

$k : 1$

$\therefore \dfrac{6k-1}{k+1} = 20$

$6k-1 = 20k+20$

$-21 = 14k$

$-\dfrac{3}{2} = k$.

$\therefore$ Ratio is $-\frac{3}{2} : 1$

ie $-3 : 2$ or $3 : -2$ or $3 : 2$ Externally

(6) $(-1, 0)$ $(3, 5)$, $k : 1$

$$x = \frac{3k-1}{k+1} \quad \text{and} \quad y = \frac{5k}{k+1}$$

$$\therefore \frac{3k-1}{k+1} + \frac{5k}{k+1} > 4$$

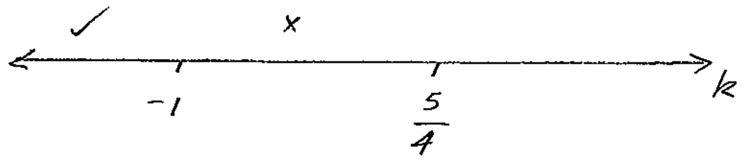
$$\therefore \frac{8k-1}{k+1} > 4$$

Critical values occur when $k = 1$ & when $\frac{8k-1}{k+1} = 4$

ie $8k - 1 = 4k + 4$

$4k = 5$

$k = \frac{5}{4}$.



<u>Test</u> $k < -1$ (say $k = -2$) True.

$-1 < k < \frac{5}{4}$ (say $k = 0$) False

$k > \frac{5}{4}$ (say $k = 2$) True

$\therefore$ Solution is $k < -1$ or $k > \frac{5}{4}$.

Harder 2 Unit

(7) Let original dimensions be x, y $\therefore$ Area $= xy$.

New dimensions are $1.12x, 0.9y$ $\therefore$ Area $= 1.008xy$.

$\therefore$ % change is 0.8%

⑧ $5^{2x} = 5^{3y}$

$\therefore 2x = 3y \quad \ldots (1)$

$\frac{2^{4x}}{2^3} = 2 \times 2^{4y}$

$2^{4x-3} = 2^{4y+1}$

$\therefore 4x - 3 = 4y + 1$

But $4x = 6y$ from (1)

$6y - 3 = 4y + 1$

$2y = 4$

$y = 2$ and $\therefore x = 3$

⑨ $t + \frac{1}{8} = \frac{1}{2(4-x)}$

$8t + 1 = \frac{4}{4-x}$

$4 - x = \frac{4}{8t+1}$

$x = 4 - \frac{4}{8t+1}$

$x = \frac{4(8t+1) - 4}{8t+1}$

$x = \frac{32t}{8t+1}$

⑩ $(m-a)(m^2+am+a^2) - (m^2-2am+a$

$= (m-a)(m^2+am+a^2) - (m-a)^2$

$= (m-a)(m^2+am+a^2-m+a)$

(11) Let $u = x^2 - 4x$

$\therefore \quad u + \frac{5}{u} = 6$

$u^2 + 5 = 6u$

$u^2 - 6u + 5 = 0$

$(u-5)(u-1) = 0$

$u = 5, 1$

If $u = 5$ then $x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5, -1$

If $u = 1$ then $x^2 - 4x = 1$

$x^2 - 4x - 1 = 0$

$x = \frac{4 \pm \sqrt{16+4}}{2}$

$x = \frac{4 \pm \sqrt{20}}{2}$

$x = 2 \pm \sqrt{5}$

$\therefore x = -1, 5, 2 \pm \sqrt{5}$

(12) Sum of Roots $= \frac{-b}{a}$

$= -7$

Average of Roots $= \frac{\text{Sum of Roots}}{2}$

$= \frac{-7}{2}$