# SYDNEY TECHNICAL HIGH SCHOOL

*(Est 1911)*

# MATHEMATICS EXTENSION I

## YEAR 11 COMMON TEST

## JULY 2002

**Time allowed :** 70 minutes

**Instructions :**

- **Show all necessary working in every question.**
- **Start each question on a new page.**
- **Attempt all questions.**
- **All questions are not of equal value.**
- **Full marks may not be awarded for careless or badly arranged work.**
- **Approved calculators may be used.**
- **These questions are to be handed in with your answers.**

**Name :** ____________________________

**Class :** ____________________________

| Question 1 | Question 2 | Question 3 | Question 4 | Question 5 | Question 6 | Total |
|---|---|---|---|---|---|---|
| | | | | | | |

## Question 1 Marks

a) Write the expansion of $\tan(A+B)$ 1

b) Let A be the point $(6,-12)$ and let B be the point $(-4, 8)$. 2

Find the coordinates of the point P which divides the interval AB internally in the ratio $5:3$.

c) Evaluate $\lim_{x\to\infty} \frac{2x^2 - x - 3}{x^2 + 4x + 8}$ 1

d) Express $3\sin\theta$ in terms of $t$ where $t = \tan\frac{\theta}{2}$. 1

e) Express as a single trigonometric ratio 1

$\sin\alpha\cos(\alpha-\beta) - \cos\alpha\sin(\alpha-\beta)$

f) Find the point on the curve $y = x^2 - 8x + 4$ where its tangent 2

is parallel to the line $2x + y + 6 = 0$.

## Question 2 (Start a new page)

a) Evaluate $\lim_{x\to 3} \frac{x^2 + 2x - 15}{x^2 - 9}$ 2

b) Show that $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\operatorname{cosec} x$ 3

c) Use the formula $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ 3

to find the derivative of $f(x) = 4x^2 - x$.

## Question 3 (Start a new page)

a) Find to the nearest degree the acute angle between the lines 3

$4x - y - 3 = 0$ and $x - 3y - 6 = 0$.

b) i) Show that $\cos 3A = 4\cos^3 A - 3\cos A$. 3

ii) Hence solve the equation

$$4\cos^3 A - 3\cos A = \frac{1}{2} \qquad \text{for } 0° \le A \le 180°$$ 3

## Question 4 (Start a new page)

a) Find the equation of the normal to the curve $y = x^4 - \frac{3}{x}$ 3

at the point $(1,-2)$.

b) Solve $2\sin^2\theta\sec\theta - \tan\theta = 0$ for $0 \le \theta \le 2\pi$ (answer in radians) 3

c) Find the value, or values of $k$ given that the perpendicular distance 3

of the point $(3, k)$ from the line $2x - y + 4 = 0$ is equal to $2\sqrt{5}$ units.

## Question 5 (Start a new page)

a) Differentiate $y = \dfrac{3x^2}{x-1}$ 2

b) Find the equation of the line which passes through the point 3

of intersection of the lines $x + 4y - 8 = 0$ and $5x + 6y - 6 = 0$

and the point $(4,-1)$. Give your answer in general form.

c) Solve to the nearest degree the equation 3

$2\cos 2x - 7\cos x = 0$ for $0° \le x \le 360°$.

## Question 6 (Start a new page)

a) Find the acute angle between the line $AB$ and the line $BC$ given 3

given that $A$, $B$ and $C$ have the coordinates

$(-3,4)$, $(2,1)$ and $(2,6)$ respectively.

b) If $y = (2x-1)\sqrt{4x-1}$ 3

show that $\dfrac{dy}{dx} = \dfrac{12x-4}{\sqrt{4x-1}}$

c) Solve the equation $3\cos\theta - \sin\theta = 2$ for $0° \le \theta \le 360°$ 3

giving your answer correct to the nearest degree.

## SOLUTIONS

### QUESTION 1

a. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

b. $A(6,-12)$ $B(-4,8)$ $5:3$

$\left(\dfrac{3\times 6 + 5\times -4}{8}, \dfrac{3\times -12 + 5\times 8}{8}\right)$

$= \left(-\dfrac{1}{4}, \dfrac{1}{2}\right)$

c. $\lim_{x\to\infty} \dfrac{2x^2 - x - 3}{x^2 + 4x + 5} = 2$

d. $3\sin\theta = 3 \times \dfrac{2t}{1+t^2}$

$= \dfrac{6t}{1+t^2}$

e. $\sin\alpha \cos(\alpha-\beta) - \cos\alpha \sin(\alpha-\beta)$

$= \sin(\alpha - (\alpha-\beta))$

$= \sin\beta$

f. $2x + y + 6 = 0$ $m = -2$

$\dfrac{dy}{dx} = 2x - 8$

$\therefore 2x - 8 = -2$

$2x = 6$

$x = 3$

$\therefore y = 3^2 - 8\times 3 + 4$

$= -11$

$\therefore$ at point $(3, -11)$

### QUESTION 2

a. $\lim_{x\to 3} \dfrac{(x+5)(x-3)}{(x+3)(x-3)}$

$= \lim_{x\to 3} \dfrac{x+5}{x+3}$

$= \dfrac{8}{6}$

$= \dfrac{4}{3}$

b. $LHS = \dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$

$= \dfrac{\sin^2 x + (1+\cos x)^2}{(1+\cos x)\sin x}$

$= \dfrac{\sin^2 x + \cos^2 x + 2\cos x + 1}{(1+\cos x)\sin x}$

$= \dfrac{2 + 2\cos x}{(1+\cos x)\sin x}$

$= \dfrac{2(1+\cos x)}{(1+\cos x)\sin x}$

$= \dfrac{2}{\sin x}$

$= 2\operatorname{cosec} x$

$= RHS$

c. $\dfrac{dy}{dx} = \lim_{h\to 0} \dfrac{[4(x+h)^2 - (x+h)] - [4x^2 - x]}{h}$

$= \lim_{h\to 0} \dfrac{4x^2 + 8xh + 4h^2 - x - h - 4x^2 + x}{h}$

$= \lim_{h\to 0} \dfrac{8xh + 4h^2 - h}{h}$

$= \lim_{h\to 0} 8x + 4h - 1$

$= 8x - 1$

QUESTION 3

a. $m_1 = 4$ , $m_2 = \frac{1}{3}$

$\therefore \tan\theta = \left| \frac{\frac{1}{3} - 4}{1 + 4 \times \frac{1}{3}} \right|$

$= \frac{11}{7}$

$\therefore \theta = 58^\circ$

b. i) LHS $= \cos 3A$

$= \cos(A + 2A)$

$= \cos A \cos 2A - \sin A \sin 2A$

$= \cos A(2\cos^2 A - 1) - \sin A \, 2\sin A \cos A$

$= 2\cos^3 A - \cos A - (1 - \cos^2 A)2\cos A$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$

$=$ RHS

ii) $4\cos^3 A - 3\cos A = \frac{1}{2}$

$\cos 3A = \frac{1}{2}$

$3A = 60^\circ, 300^\circ, 420^\circ$

$A = 20^\circ, 100^\circ, 140^\circ$

QUESTION 4

a. $y = x^4 - 3x^{-1}$

$\frac{dy}{dx} = 4x^3 + 3x^{-2}$

$= 4x^3 + \frac{3}{x^2}$

when $x = 1$

$m_T = 4 + 3$

$= 7$

$\therefore m_N = -\frac{1}{7}$

$\therefore$ equation $y - y_1 = m(x - x_1)$

$y + 2 = -\frac{1}{7}(x - 1)$

$7y + 14 = -x + 1$

$x + 7y + 13 = 0$

b. $2\sin^2\theta \sec\theta - \tan\theta = 0$

$\frac{2\sin^2\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = 0$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ $\sin\theta = \frac{1}{2}$

$\theta = 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}$

c. $\frac{|2 \times 3 + -1 \times k + 4|}{\sqrt{2^2 + 1^2}} = 2\sqrt{5}$

$\frac{|10 - k|}{\sqrt{5}} = 2\sqrt{5}$

$|10 - k| = 10$

$10 - k = 10$ or $10 - k = -10$

$k = 0$ or $k = 20$

QUESTION 5

a. $\frac{dy}{dx} = \frac{(x-1)6x - 3x^2(1)}{(x-1)^2}$

$= \frac{6x^2 - 6x - 3x^2}{(x-1)^2}$

$= \frac{3x^2 - 6x}{(x-1)^2}$

b. $x + 4y - 8 + k(5x + 6y - 6) = 0$

sub $(4, -1)$

$4 - 4 - 8 + k(20 - 6 - 6) = 0$

$-8 + 8k = 0$

$k = 1$

$6x + 10y - 14 = 0$

$3x + 5y - 7 = 0$

c. $2\cos 2x - 7\cos x = 0$

$2(2\cos^2 x - 1) - 7\cos x = 0$

$4\cos^2 x - 7\cos x - 2 = 0$

$(4\cos x + 1)(\cos x - 2) = 0$

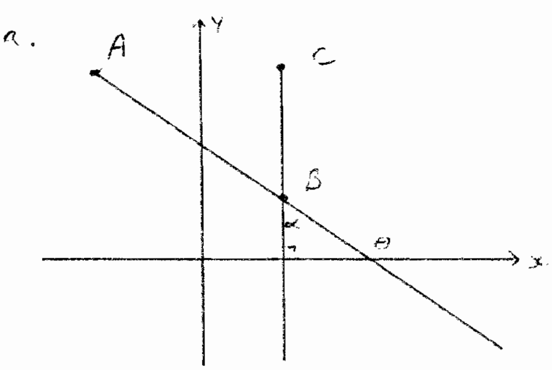
$\cos x = -\frac{1}{4}$ $\quad$ $\cos x = 2$

no solution

$x = 104°, 256°$

QUESTION 6

a.




$\tan\theta = m_{AB}$

$= \frac{4 - 1}{-3 - 2}$

$= -\frac{3}{5}$

$\therefore \theta = 149°$

$\therefore \alpha = 59°$

$\therefore$ angle is $59°$

b $\quad y = (2x - 1)\sqrt{4x - 1}$

$y = (2x - 1)(4x - 1)^{\frac{1}{2}}$

$\frac{dy}{dx} = 2(4x - 1)^{\frac{1}{2}} + (2x - 1)\frac{1}{2}\cdot(4x - 1)^{-\frac{1}{2}}\cdot 4$

$= 2\sqrt{4x - 1} + \frac{2(2x - 1)}{\sqrt{4x - 1}}$

$= \frac{2(4x - 1) + 2(2x - 1)}{\sqrt{4x - 1}}$

$= \frac{12x - 4}{\sqrt{4x - 1}}$

c. $3\cos\theta - \sin\theta = 2$

$\sqrt{10}\cos(\theta + \alpha) = 2$

$\tan\alpha = \frac{1}{3}$

$\alpha = 18°26'$

$\therefore \cos(\theta + \alpha) = \frac{2}{\sqrt{10}}$

$\theta + \alpha = 50°46', 309°14'$

$\theta = 32°, 291°$