

2003 EXT 1 July Common

Question 1 (10 marks)

- a) i) Write an expression for $\cos(A + B)$ (1)
ii) Hence find the exact value of $\cos 105^\circ$ (2)
- b) Find the coordinates of P (x, y) that divides the interval AB externally in the ratio 5:1 given A (0,2) and B (3,0) (2)
- c) Find the acute angle between the lines $x - 2y - 6 = 0$ and $y = -3x + 4$ to the nearest minute. (2)
- d) Prove that the line $3x - y - 10 = 0$ is a tangent to the circle $x^2 + y^2 = 10$ (3)

Question 2 (8 marks) Start a new page

- a) Find i) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$ (1)
ii) $\lim_{x \rightarrow \infty} \frac{2x}{x + 2}$ (2)
- b) Solve $2 \sin \frac{x}{2} = \cos \frac{x}{2}$ for $0^\circ \leq x \leq 360^\circ$ (3)
- c) The minute and hour hands of a clock are 9cm and 6cm in length respectively. (2)
Find the distance between the ends of the hands when the time is 5 o'clock
(to the nearest mm).

Question 3 (10 marks) start a new page

- a) Using $\tan \frac{\theta}{2} = t$ write $\frac{1}{2} \cot \frac{\theta}{2} - \cot \theta$ in terms of t in simplest form (2)
- b) Differentiate with respect to x, $y = \frac{x}{3\sqrt{x}}$ (2)
- c) Prove $\sin(\frac{\pi}{4} + A) - \sin(\frac{\pi}{4} - A) = \sqrt{2} \sin A$ (3)
- d) Solve $\sin 2x + \cos x = 0$ for $0^\circ \leq x \leq 360^\circ$ (3)

Question 4 (10 marks) Start a new page

a) Differentiate

i) $\frac{1}{4-x^2}$ (2)

ii) $\frac{1-x^2}{1+x^2}$ (2)

b) i) Express $\sqrt{3} \cos x - \sin x$ in the form $A \cos(x + \vartheta)$ where $0^\circ \leq \vartheta \leq 90^\circ$ (2)

ii) Hence or otherwise solve $\sqrt{3} \cos x - \sin x = 1$ where $0^\circ \leq x \leq 360^\circ$ (2)

c) If $y = \sqrt{r^2 - x^2}$ where r is a constant, show that $\frac{dy}{dx} = \frac{-x}{y}$ (2)

Question 5 (9 marks) Start a new page

a) Find the coordinates of the points on the curve $y = 2x^3 - 9x^2 + 27$ where the tangent is parallel to the x axis. (3)

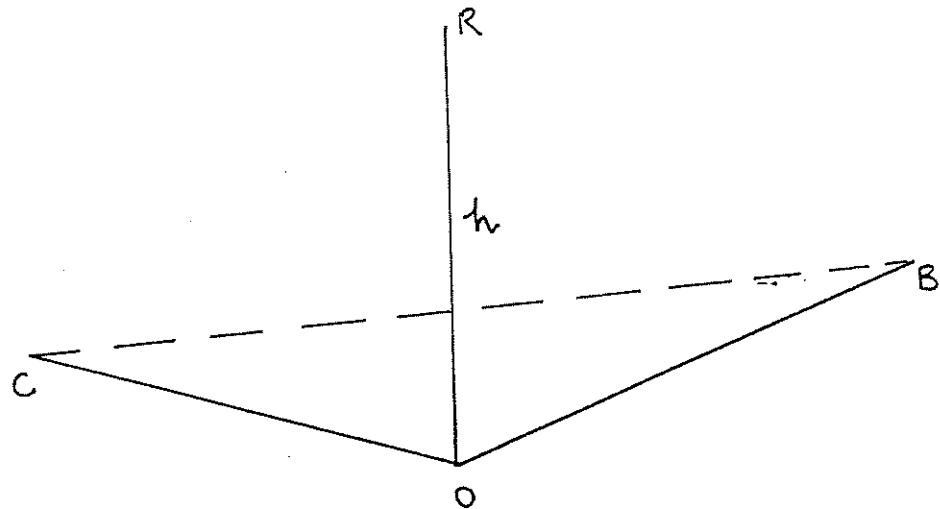
b) Given $\cos 3\vartheta = 4 \cos^3 \vartheta - 3 \cos \vartheta$, solve the equation $\cos 3\vartheta + \cos \vartheta = 0$ for $-180^\circ \leq \vartheta \leq 180^\circ$ (3)

c) i) For the function $f(x) = x\sqrt{2-x}$ find $f'(x)$ (2)

ii) Hence solve $f'(x) = 0$ (1)

Question 6 (10 marks) Start a new page

- a) From a tower OR, the bearings of two points C and B, on the same level ground as the base of the tower, are 300° and 015° respectively. The angles of elevation to the top of the tower are 11° and 22° from C and B respectively. BC is 150m. The tower OR has height h metres.



- i) Redraw the above diagram showing all relevant information. (1)
- ii) Find $\angle BOC$ (2)
- iii) Find OC and OB in terms of h (2)
- iv) Find the height of the tower to the nearest metre. (3)

- b) i) If $\sin(x + \theta) = k \sin(x - \theta)$ prove

$$(k - 1) \tan x = (k + 1) \tan \theta \quad (2)$$
- ii) Hence solve $\sin(x + 20^\circ) = 2 \sin(x - 20^\circ)$ if $0^\circ \leq x \leq 360^\circ$ (2)

Solutions

2003 Ext 1

QUESTION 1

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QUESTION 2

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$$i) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos 105^\circ = \cos(60^\circ + 45^\circ)$$

$$= 2\cos^2 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

$$A(\frac{1-\sqrt{3}}{2}, 0)$$

$$B(\frac{1+\sqrt{3}}{2}, 0)$$

$$(-\frac{1+\sqrt{3}}{4}, 0), (\frac{1+\sqrt{3}}{4}, 0)$$

$$P(\frac{15}{16}, -\frac{1}{2})$$



$$x - 2y - 6 = 0 \quad \text{let } m_1 = \frac{1}{2}$$

$$y = -3m_1 + 4 \quad \text{let } m_2 = -3$$

$$\tan \theta = \frac{1/2 - (-3)}{1 + \frac{1}{2} \times 3}$$

$$\tan \theta = 7 \quad \therefore \theta = 81^\circ 52' \quad (\text{to nearest min})$$

circle $x^2 + y^2 = 10$ has

centre $(0,0)$ radius = $\sqrt{10}$

$$\text{perp dist } (0,0) \text{ to } 3x - y - 10 = 0$$

$$r = \left| \frac{-10}{\sqrt{9+1}} \right| = \frac{10}{\sqrt{10}} = \frac{10}{\sqrt{10}} = \frac{10}{\sqrt{10}} = \frac{10}{\sqrt{10}}$$

$$r = \frac{10}{\sqrt{10}} \quad \text{rationalise}$$

$$r = \sqrt{10} \quad \therefore \text{sine perp dist.}$$

is equal to radius \Rightarrow tangent

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QUESTION 4

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QUESTION 5

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QUESTION 125

Question 5

$$a) y = 2x^3 - 9x^2 + 27$$

$$\frac{dy}{dx} = 6x^2 - 18x$$

$$\frac{dy}{dx} = 0 \therefore 6x^2 - 18x = 0 \\ 6x(x-3) = 0$$

$$\therefore x = 0 \quad x = 3$$

i) $(0, 27)$ (3, 0)

$$b) \cos 3\theta + \cos \theta = 0 \\ 4\cos^3 \theta - 3\cos \theta + \cos \theta = 0 \\ 4\cos^3 \theta - 2\cos \theta = 0 \\ 2\cos \theta (2\cos^2 \theta - 1) = 0 \\ \therefore \cos \theta = 0 \quad \cos \theta = \pm \frac{1}{2} \\ \theta = \pm 90^\circ \quad \pm 45^\circ, \pm 135^\circ$$

$$c) i) f(x) = x \sqrt{2-x}$$

$$u = x \quad v = \sqrt{2-x} = (2-x)^{\frac{1}{2}}$$

$$u' = 1 \quad v' = -\frac{1}{2}(2-x)^{-\frac{1}{2}}$$

$$= \frac{-1}{2}\frac{1}{\sqrt{2-x}}$$

$$f'(x) = \sqrt{2-x} - \frac{x}{2\sqrt{2-x}}$$

$$ii) \sqrt{2-x} - \frac{x}{2\sqrt{2-x}} = 0$$

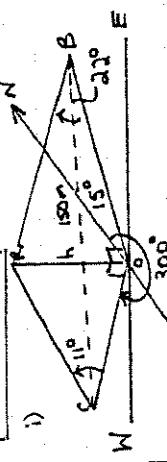
$$\frac{2(2-x) - x}{2\sqrt{2-x}} = 0$$

$$4 - 2x - x = 0$$

$$4 - 3x = 0$$

i. $x = \frac{4}{3}$

Question 6



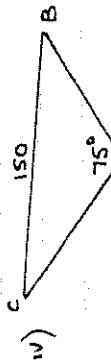
$$ii) \angle BAC = 75^\circ$$

$$iii) \tan 11^\circ = \frac{h}{OC}$$

$$OC = \frac{h}{\tan 11^\circ}$$

$$\tan 22^\circ = \frac{h}{OB}$$

$$OB = \frac{h}{\tan 22^\circ}$$



use cosine rule

$$150^2 = h^2 + \frac{h^2}{\tan^2 22} - \frac{2h^2}{\tan 22 \tan 11}$$

$$(2 \frac{h}{\tan 22} \cdot \frac{h}{\tan 11} \cos 75^\circ)$$

$$150^2 = h^2 \left[\frac{1}{\tan^2 22} + \frac{1}{\tan^2 11} - \frac{2 \cos 75^\circ}{\tan 22 \tan 11} \right]$$

$$h = 29 \text{ m (nearest metric)}$$

Question 6



$$i) \angle BAC = 75^\circ$$

$$ii) \angle BAC = 75^\circ$$

$$iii) \tan 11^\circ = \frac{h}{OC}$$

$$iv) \tan 22^\circ = \frac{h}{OB}$$

$$v) \tan 11^\circ = \frac{h}{OC}$$

$$vi) \tan 22^\circ = \frac{h}{OB}$$

$$vii) \tan 75^\circ = \frac{h}{BC}$$

$$viii) \tan 75^\circ = \frac{h}{AC}$$

$$ix) \tan 75^\circ = \frac{h}{h}$$

$$x) \tan 75^\circ = 1$$

$$xi) 1 = \frac{h}{h}$$

$$xii) h = h$$

$$xiii) h = h$$

$$xiv) h = h$$

$$xv) h = h$$

$$xvi) h = h$$

$$xvii) h = h$$

$$xviii) h = \frac{4}{3}$$