2003 EXT 1 July Common

Question 1 (10 marks)

a) i) Write an expression for cos (A + B) (1)

ii) Hence find the exact value of cos 105° (2)

b) Find the coordinates of P (x, y) that divides the interval AB externally in the ratio 5:1 given A (0,2) and B (3,0) (2)

c) Find the acute angle between the lines x-2y – 6 = 0 and y = -3x + 4 to the nearest minute. (2)

d) Prove that the line 3x – y – 10 = 0 is a tangent to the circle $x^2 + y^2 = 10$ (3)

Question 2 (8 marks) Start a new page

a) Find i) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ (1)

ii) $\lim_{x \to \infty} \frac{2x}{x + 2}$ (2)

b) Solve $2\sin\frac{x}{2} = \cos\frac{x}{2}$ *for* $0° \leq x \leq 360°$ (3)

c) The minute and hour hands of a clock are 9cm and 6cm in length respectively. Find the distance between the ends of the hands when the time is 5 o'clock (to the nearest mm). (2)

Question 3 (10 marks) start a new page

a) Using $\tan\frac{\theta}{2} = t$ write $\frac{1}{2}\cot\frac{\theta}{2} - \cot\theta$ in terms of t in simplest form (2)

b) Differentiate with respect to x, $y = \frac{x}{3\sqrt{x}}$ (2)

c) Prove $\sin(\frac{\pi}{4} + A) - \sin(\frac{\pi}{4} - A) = \sqrt{2}\sin A$ (3)

d) Solve $\sin 2x + \cos x = 0$ *for* $0° \leq x \leq 360°$ (3)

Question 4 **(10 marks)** **Start a new page**

a) Differentiate

i) $\dfrac{1}{4-x^2}$ (2)

ii) $\dfrac{1-x^2}{1+x^2}$ (2)

b) i) Express $\sqrt{3}\cos x - \sin x$ in the form $A\cos(x+\vartheta)$ where $0° \le \vartheta \le 90°$ (2)

ii) Hence or otherwise solve $\sqrt{3}\cos x - \sin x = 1$ *where* $0° \le x \le 360°$ (2)

c) If $y = \sqrt{r^2 - x^2}$ where r is a constant, show that $\dfrac{dy}{dx} = \dfrac{-x}{y}$ (2)

Question 5 **(9 marks)** **Start a new page**

a) Find the coordinates of the points on the curve $y = 2x^3 - 9x^2 + 27$ where the tangent is parallel to the x axis. (3)

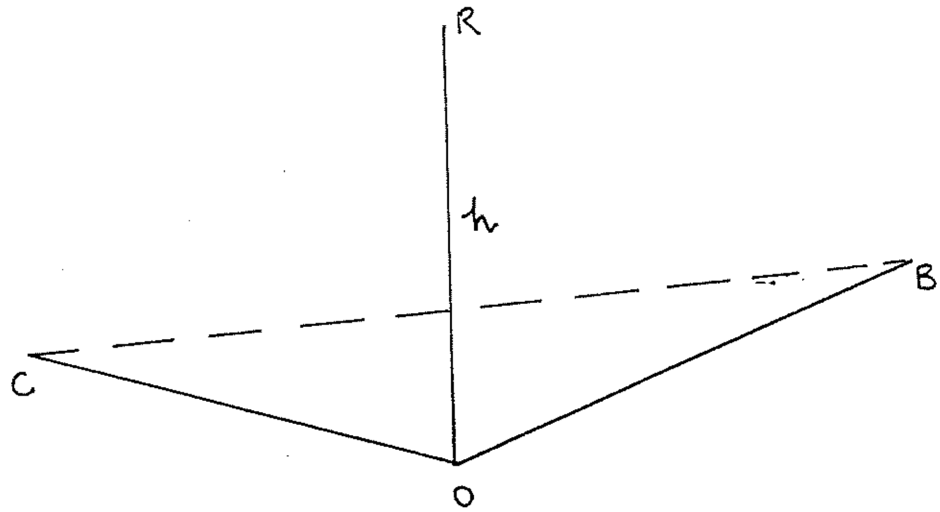
b) Given $\cos 3\vartheta = 4\cos^3\vartheta - 3\cos\vartheta$, solve the equation $\cos 3\vartheta + \cos\vartheta = 0$ for $-180° \le \vartheta \le 180°$ (3)

c) i) For the function $f(x) = x\sqrt{2-x}$ *find* $f'(x)$ (2)

ii) Hence solve $f'(x) = 0$ (1)

Question 6 (10 marks) Start a new page

a) From a tower OR, the bearings of two points C and B, on the same level ground as the base of the tower, are $300°$ *and* $015°$ respectively. The angles of elevation to the top of the tower are $11°$*and* $22°$ from Cand B respectively. BC is 150m. The tower OR has height h metres.



i) Redraw the above diagram showing all relevant information. (1)

ii) Find $\angle BOC$ (2)

iii) Find OC and OB in terms of h (3)

iv) Find the height of the tower to the nearest metre.

b) i) If $\sin(x+\vartheta) = k\sin(x-\vartheta)$ prove

$$(k-1)\tan x = (k+1)\tan\vartheta$$ (2)

ii) Hence solve $\sin(x+20°) = 2\sin(x-20°)$ *if* $0° \le x \le 360°$ (2)

YEAR 11 EXT 1   2003

SOLUTIONS 2003 EXT 1

QUESTION 1

a) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

b) $\cos 105^\circ = \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2}\cdot\frac{1}{\sqrt2} - \frac{\sqrt3}{2}\cdot\frac{1}{\sqrt2}$

$= \frac{1-\sqrt3}{2\sqrt2}$

$A(0, 2)$ $B(3, 0)$ $-5:1$

$\left(\frac{(1\times0)+(-5\times3)}{-4}, \frac{(1\times2)+(-5\times0)}{-4}\right)$

$P\left(\frac{15}{4}, -\frac{1}{2}\right)$

$x - 2y - 6 = 0 \quad \therefore y = \frac{x}{2} - 3$

let $m_1 = \frac{1}{2}$

$y = -3x + 4$ let $m_2 = -3$

$\tan\theta = \left|\frac{\frac{1}{2} - -3}{1 + \frac{1}{2}\times -3}\right|$

$\tan\theta = 7$

$\therefore \theta = 81^\circ 52'$ (to nearest min)

circle $x^2 + y^2 = 10$ has centre $(0,0)$ radius $= \sqrt{10}$

perp dist $(0,0)$ to $3x - y - 10 = 0$

$p = \left|\frac{-10}{\sqrt{9+1}}\right|$

$p = \frac{10}{\sqrt{10}}$ rationalise

$p = \sqrt{10}$ $\therefore$ since perp dist. is equal to radius $\Rightarrow$ tangent

QUESTION 2

a) i) $\lim_{x\to2} \frac{(x-2)(x+2)}{(x-2)} = 4$

ii) $\lim_{x\to\infty} \frac{2x}{x+2} = 2$

b) $2\sin\frac{x}{2} = \cos\frac{x}{2}$

$2\tan\frac{x}{2} = 1$

$\tan\frac{x}{2} = \frac{1}{2}$

$\frac{x}{2} = 26^\circ 34', \; 206^\circ 34'$

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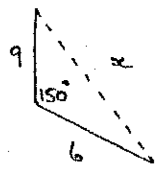
$x = 53^\circ 8'$ in domain

c)



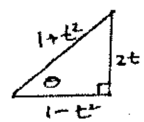
$x^2 = 9^2 + 6^2 - 2\times9\times6\cos150^\circ$

$\therefore x = 14.5$ cm (1 dec pl)

or 145 mm

QUESTION 3

a) $\frac{1}{2}\cot\frac{\theta}{2} - \cot\theta$



$= \frac{1}{2}\cdot\frac{1}{t} - \frac{(1-t^2)}{2t}$

$= \frac{1 - 1 + t^2}{2t}$

$= \frac{t^2}{2t}$

$= \frac{t}{2}$

b) $y = \frac{1}{3}x \cdot x^{n/2}$

$\therefore y = \frac{1}{3}x^{1/2}$

$\frac{dy}{dx} = \frac{1}{3}\times\frac{1}{2}x^{-1/2}$

$= \frac{1}{6\sqrt x}$

c) Prove $\sin\left(\frac{\pi}{4} + A\right) - \sin\left(\frac{\pi}{4} - A\right) = \sqrt2\sin A$

LHS $= \sin\frac{\pi}{4}\cos A + \cos\frac{\pi}{4}\sin A - \left(\sin\frac{\pi}{4}\cdot\cos A - \cos\frac{\pi}{4}\sin A\right)$

$= \frac{1}{\sqrt2}\cos A + \frac{1}{\sqrt2}\sin A - \frac{1}{\sqrt2}\cos A + \frac{1}{\sqrt2}\sin A$

$= \frac{2}{\sqrt2}\sin A$

$= \frac{2}{\sqrt2}\times\frac{\sqrt2}{\sqrt2}\sin A$

$= \frac{2\sqrt2}{2}\sin A$

$= \sqrt2\sin A$

$=$ RHS

d) $\sin 2x + \cos x = 0$

$2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\sin x = -\frac{1}{2}$

acute $x = 30$

$\therefore x = 90^\circ, 270^\circ$ $\therefore x = 210^\circ, 330^\circ$

Question 4

a) i) $\frac{d}{dx}(4-x^2)^{-1} = -1\times -2x(4-x^2)^{-2}$

$= \frac{2x}{(4-x^2)^2}$

ii) use quotient rule

$u = 1 - x^2$ $v = 1 + x^2$

$u' = -2x$ $v' = 2x$

$\frac{dy}{dx} = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2}$

$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$

$= \frac{-4x}{(1+x^2)^2}$

b) i) $A = \sqrt{3+1} = 2$

$2\left(\frac{\sqrt3}{2}\cos x - \frac{1}{2}\sin x\right)$

required form

$2(\cos x\cos\theta - \sin x\sin\theta)$

$\therefore \cos\theta = \frac{\sqrt3}{2}$ $\sin\theta = \frac{1}{2}$ S | A ✓ / T | C

$\therefore \theta = 30^\circ$

$\therefore \sqrt3\cos x - \sin x = 2\cos(x + 30^\circ)$

ii) $2\cos(x + 30^\circ) = 1$

$\cos(x + 30^\circ) = \frac{1}{2}$

$x + 30^\circ = 60^\circ, 300^\circ$

$x = 30^\circ, 270^\circ$

c) $y = (r^2 - x^2)^{1/2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}\times -2x\left(r^2 - x^2\right)^{-1/2}$

$= \frac{-x}{\sqrt{r^2 - x^2}}$

since $\frac{-x}{y} = \frac{-x}{\sqrt{r^2 - x^2}}$

$\therefore \frac{dy}{dx} = \frac{-x}{y}$

Question 5

a) $y = 2x^3 - 9x^2 + 27$

$\frac{dy}{dx} = 6x^2 - 18x$

$\frac{dy}{dx} = 0 \quad \therefore \quad 6x^2 - 18x = 0$

$6x(x-3) = 0$

$\therefore x = 0 \quad x = 3$

pts $(0, 27) \quad (3, 0)$

b) $\cos 3\theta + \cos\theta = 0$

$4\cos^3\theta - 3\cos\theta + \cos\theta = 0$

$4\cos^3\theta - 2\cos\theta = 0$

$2\cos\theta\,(2\cos^2\theta - 1) = 0$

$\therefore \cos\theta = 0 \qquad \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = \pm 90^\circ, \ \pm 45^\circ, \ \pm 135^\circ$

c) i) $f(x) = x\sqrt{2-x}$

$u = x \qquad v = \sqrt{2-x} = (2-x)^{1/2}$

$u' = 1 \qquad v' = -\frac{1}{2}(2-x)^{-1/2}$

$= \frac{-1}{2\sqrt{2-x}}$

$f'(x) = \sqrt{2-x} \quad - \frac{x}{2\sqrt{2-x}}$

ii) $\sqrt{2-x} \quad \frac{-x}{2\sqrt{2-x}} = 0$

$\frac{2(2-x) - x}{2\sqrt{2-x}} = 0$

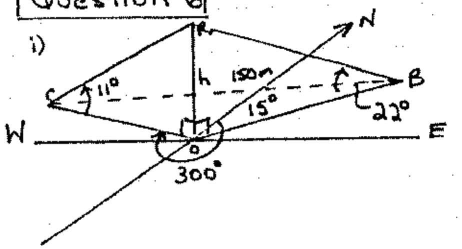
$4 - 2x - x = 0$

$4 - 3x = 0$

$\therefore \quad x = \frac{4}{3}$

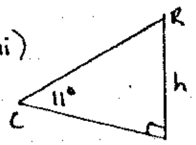
Question 6

i)



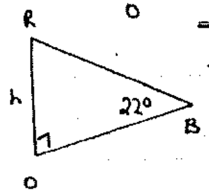
ii) $\angle BOC = 75^\circ$

iii)



$\tan 11^\circ = \frac{h}{OC}$

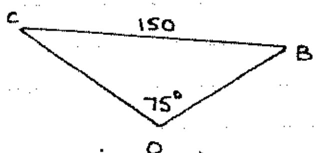
$OC = \frac{h}{\tan 11^\circ}$



$\tan 22^\circ = \frac{h}{OB}$

$OB = \frac{h}{\tan 22^\circ}$

iv)



use cosine rule

$150^2 = \frac{h^2}{\tan^2 22} + \frac{h^2}{\tan^2 11^\circ} - \left(2\frac{h}{\tan 22^\circ} \cdot \frac{h}{\tan 11^\circ}\cos 75^\circ\right)$

$150^2 = h^2\left[\frac{1}{\tan^2 22^\circ} + \frac{1}{\tan^2 11^\circ} - \frac{2\cos 75^\circ}{\tan 22^\circ \tan 11^\circ}\right]$

$h = 29\text{m}$ (nearest metre)

b) i) $\sin(x+\theta) = k\sin(x-\theta)$

$\sin x\cos\theta + \cos x\sin\theta = k[\sin x\cos\theta - \cos x\sin$

$\sin x.\cos\theta + \cos x\sin\theta = k\sin x.\cos\theta - k\cos x\sin\theta$

$\div$ by $\sin x.\cos\theta$

$1 + \frac{\cos x\sin\theta}{\sin x.\cos\theta} = k - k\frac{\cos x\sin\theta}{\sin x\cos\theta}$

$1 + \frac{\tan\theta}{\tan x} = k - k\frac{\tan\theta}{\tan x}$

$\tan x + \tan\theta = k\tan x - k\tan\theta$

$\tan\theta + k\tan\theta = k\tan x - \tan x$

$(k+1)\tan\theta = (k-1)\tan x$

ii) $\sin(x+20^\circ) = 2\sin(x-20^\circ) \qquad \therefore k = 2 \quad \theta = 20^\circ$

find x

$3\tan 20^\circ = \tan x$

$1.091\ldots = \tan x$

$\therefore x = 47^\circ 31', \ 227^\circ 31'$

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