SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

YEAR 11 EXAMINATION

TASK 2 OF 3

JULY 2004

Time allowed: 70 minutes

Instructions:

- Show all necessary working
- Start each question on a new page
- Full marks may not be awarded for careless or badly arranged work
- Non-programmable calculators may be used
- This paper must be handed in on top of your answer sheets
- Answers must be written in blue or black pen
- Answers must be arranged in order and stapled securely. No claims for missing pages will be allowed.

Name:______________________________

Class:________________________

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/8	/8	/10	/10	/9	/8	/53

Question 1

(i) Sketch the parabola $y = x^2 + 5x$ on the number plane (2)

(ii) Find the equation of the axis of symmetry. (1)

(iii) Find the minimum value of the parabola. (1)

(iv) Solve the inequality $x^2 + 5x \geq 0$ (1)

(v) Using the formula $\dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ (3)

differentiate $y = x^2 + 5x$ from first principles.

Question 2

(a) Differentiate the following with respect to x

(i) $y = x^3 - 3x^2 + 2x - 1$ (1)

(ii) $y = \dfrac{3x^3 + x^2}{x}$ (1)

(iii) $y = \dfrac{3x}{5x + 1}$ (2)

(iv) $y = (6x - 1)^4$ (2)

(b) If $f(x) = 3x^2 + 2x - 1$

find (i) $f'(0)$ (1)

(ii) the gradient of the tangent at the point where $x = 1$. (1)

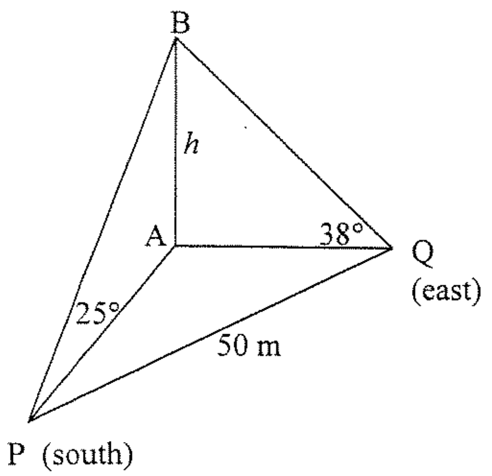
Question 3

a) Simplify $\dfrac{\sin 2\theta}{\sin\theta}$ (1)

b) Show that $\sin(\alpha-\beta)\cos\alpha-\cos(\alpha-\beta)\sin\alpha=-\sin\beta$ (2)

c) Solve $(\tan\alpha+1)(\tan\alpha+\sqrt{3})=0$ for $-180°\le\alpha\le180°$ (3)

d) From a point P due south of a vertical tower the angle of elevation of the top of the tower is $25°$ and from a point Q due east of the tower the angle of elevation is $38°$. The distance from P to Q is 50 m. (4)



(i) Show that $AP = h\tan 65°$

(ii) Express A Q in a similar way

(iii) Hence find h, the height of the tower.

Answer to the nearest metre.

Question 4

(a) Write the equation of the line $\dfrac{y-7}{x+1}=\dfrac{4}{3}$ in general form. (2)

(b) Write down the gradient and y – intercept of $\dfrac{x}{a}+\dfrac{y}{b}=c$ (2)

(c) Find the perpendicular distance from the point (2, -1) to the line $4x-3y+7=0$ (2)

(d) Find the equation of the line which is perpendicular to $y=2x-3$ and also passes through (2, -3). (2)

(e) Find the coordinates of A if P (7, 8) divides the join of A (p, q) and B (9, 10) internally in the ratio 5:2. (2)

Question 5

(a) Differentiate

(i) $y = \sqrt{x}$ (1)

(ii) $y = x^2\sqrt{x}$ (1)

(b) (i) Find the gradient of the tangent to the curve $y = \dfrac{x^3}{x^2 + 4}$ at the point where $x = 2$. (2)

(ii) Hence find the gradient of the normal at this point. (1)

(iii) Find the equation of the normal. (2)

(c) Show by a sketch of $y = 2x^2 - 4x + 5$ that it is a positive definite function. (2)

Question 6

(a) Find the equation of the line through the point of intersection of $2x + y + 3 = 0$ and $2x - y - 5 = 0$ and also passing through the point (1, 1). (2)

(b) Find the acute angle between the two lines given in part (a). Answer to the nearest minute. (2)

(c) Write $2\cos\theta + \sin\theta = 1$ in the form $R\sin(\theta + \alpha) = 1$. (3)

Hence solve $2\cos\theta + \sin\theta = 1$ for $0° \leq \theta \leq 360°$.

(d) Find the value of p for which $x^2 - 2x + p = 0$ has equal roots. (1)

End of Paper

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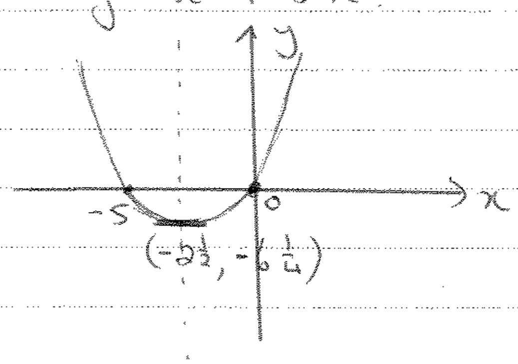
Question 1

(i) $y = x^2 + 5x$

2



1 (ii) $x = -2\frac{1}{2}$

1 (iii) Min value

$= -2\frac{1}{2}(-2\frac{1}{2} + 5)$

$= -6\frac{1}{4}$

1 (iv) $x^2 + 5x \geq 0 \Rightarrow x \geq 0$ and $x \leq -5$

(v) $\frac{dy}{dx} = \lim_{h \to 0} \left[\frac{(x+h)^2 + 5(x+h) - (x^2 + 5x)}{h}\right]$

$= \lim_{h \to 0} \left[\frac{\cancel{x^2} + 2hx + h^2 + \cancel{5x} + 5h - \cancel{x^2} - \cancel{5x}}{h}\right]$

3

$= \lim_{h \to 0} (2x + h + 5)$

$= 2x + 5.$

Question 2

a) (i) $y = x^3 - 3x^2 + 2x - 1$

$y' = 3x^2 - 6x + 2$

(ii) $y = \frac{3x^3 + x^2}{x}$

$= 3x^2 + x$

$y' = 6x + 1$

(iii) $y = \frac{3x}{5x+1}$

$y' = \frac{3(5x+1) - 5(3x)}{(5x+1)^2}$

$= \frac{3}{(5x+1)^2}$

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(iv) $y = (6x-1)^4$

$y' = 4(6x-1)^3 \times 6$

$= 24(6x-1)^3$

(b) $f(x) = 3x^2 + 2x - 1$

(i) $f'(x) = 6x + 2$

$\therefore f'(0) = 2.$

(ii) $f'(1) = 8$

$\therefore$ grad. tangent at $x=1$ is 8.

Question 3

a) $\frac{\sin 2\theta}{\sin\theta} = \frac{2\cancel{\sin\theta}\cos\theta}{\cancel{\sin\theta}} = 2\cos\theta.$

b) LHS $= \sin(\alpha-\beta)\cos\alpha - \cos(\alpha-\beta)\sin\alpha$

$= \sin(\alpha-\beta-\alpha)$ using $\sin(A-B)$ result.

$= \sin(-\beta)$

$= -\sin\beta$ (4th quadrant result)

$=$ RHS

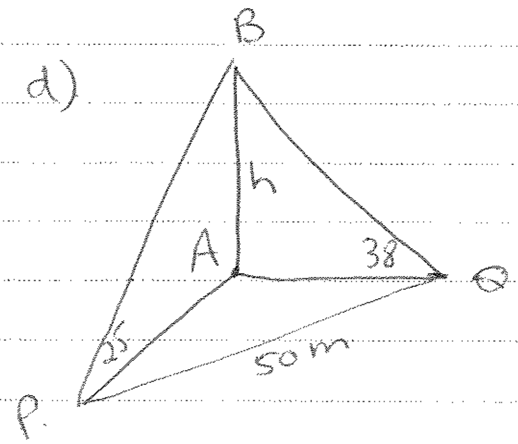
c) $(\tan\alpha + 1)(\tan\alpha + \sqrt{3}) = 0.$

$\Rightarrow \tan\alpha = -1$ or $\tan\alpha = -\sqrt{3}$

$\therefore \alpha = -45°, 135°, -60°, 120°$

ie $\alpha = -60°, -45°, 120°, 135°.$

d)



(i) $\angle PBA = 90 - 25$

$= 65°$ (ΔPBA rightangled)

$\therefore \tan 65° = \frac{PA}{BA}$

$= \frac{PA}{h}$

$\Rightarrow PA = h\tan 65°.$

(ii) $\angle ABQ = 90 - 38$

$= 52°$

$\therefore \tan 52^\circ = \frac{AQ}{AB} = \frac{AQ}{h}$

$\Rightarrow AQ = h \tan 52^\circ.$

(iii) By Pythagoras' theorem:

$AP^2 + AQ^2 = PQ^2$

$h^2 \tan^2 65^\circ + h^2 \tan^2 52^\circ = 50^2$

$h^2(\tan^2 65^\circ + \tan^2 52^\circ) = 2500$

$h^2 = \frac{2500}{\tan^2 65^\circ + \tan^2 52^\circ}$

$h = 20$ [illegible] m

Question 4

a) $\frac{y-7}{x+1} = \frac{4}{3}$

$3y - 21 = 4x + 4$

$\therefore 4x - 3y + 25 = 0.$

b) $bx + ay = abc$

$\therefore y = \frac{-bx + abc}{a}$

grad $= -\frac{b}{a}$

y intercept $= bc$

c) $d = \left| \frac{4\times 2 - 3\times -1 + 7}{\sqrt{16+9}} \right|$

$= \left| \frac{8+3+7}{5} \right| = \frac{18}{5} = 3\,^3/_5$ units.

d) $y = 2x - 3$ has gradient 2

$\therefore$ gradient perpendicular $= -\frac{1}{2}$

$\therefore y - (-3) = -\frac{1}{2}(x - 2)$

$2y + 6 = -x + 2$

$x + 2y + 4 = 0.$

e) Using

$x = \frac{nx_1 + mx_2}{m+n}$, $y = \frac{ny_1 + my_2}{m+n}$

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$$7 = \frac{2p + 5\times 9}{7} \quad \text{and} \quad 8 = \frac{2q + 5\times 10}{7}$$

$49 = 2p + 45$ $\qquad$ $56 = 2q + 50$

$2p = 4$ $\qquad$ $2q = 6$

$p = 2.$ $\qquad$ $q = 3$

<u>Question 5</u>

a) (i) $y = \sqrt{x} = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

(ii) $y = x^2\sqrt{x} = x^{5/2}$

$y' = 5/2\, x^{3/2}$

$= \frac{5x\sqrt{x}}{2}.$

b) (i) $y = \frac{x^3}{x^2+4}$

$$y' = \frac{3x^2(x^2+4) - 2x(x^3)}{(x^2+4)^2}$$

$$= \frac{3x^4 + 12x^2 - 2x^4}{(x^2+4)^2}$$

$$= \frac{x^4 + 12x^2}{(x^2+4)^2} = \frac{x^2(x^2+12)}{(x^2+4)^2}$$

$\therefore$ gradient tangent at $x = 2$

is $m = \frac{2^2(2^2+12)}{(2^2+4)^2} = \frac{64}{64} = 1.$

(ii) gradient normal $= -1.$

(iii) equation normal

$y - 1 = -1(x-2)$

$x + y - 3 = 0.$

c) $y = 2x^2 - 4x + 5$

$= 2(x^2 - 2x) + 5$

$= 2(x-1)^2 + 3$

Min. value of $(x-1)^2 = 0$ $\therefore$ Min. y value $= 3$

when $x = 1$

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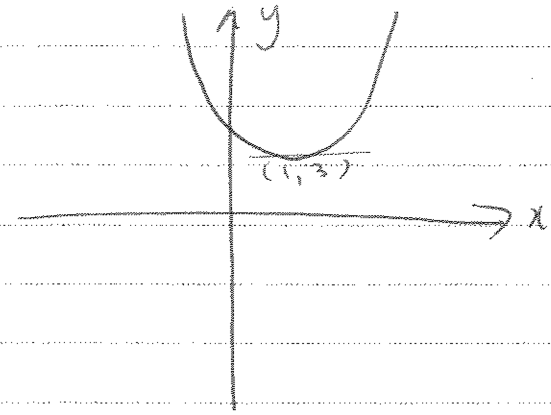

$\therefore$ Parabola has no real roots and all y values positive $\Rightarrow$ positive definite

Question 6

a) $2x + y + 3 + k(2x - y - 5) = 0$

$(1,1)$ satisfies

$\therefore 2 + 1 + 3 + k(2 - 1 - 5) = 0$

$6 - 4k = 0 \Rightarrow k = 3/2$

$\therefore 2x + y + 3 + 3/2(2x - y - 5) = 0$

$4x + 2y + 6 + 6x - 3y - 15 = 0$

$10x - y - 9 = 0$

OR

$2x + y + 3 = 0$ ①

$2x - y - 5 = 0$ ②

① + ② $4x - 2 = 0 \Rightarrow x = \frac{1}{2}$

In ① $1 + y + 3 = 0 \Rightarrow y = -4$

$\therefore \dfrac{y-1}{x-1} = \dfrac{1 - -4}{1 - 1/2}$

$= \dfrac{5}{(1/2)} = 10$

$\therefore y - 1 = 10x - 10$

$10x - y - 9 = 0$

(b) $2x + y + 3 = 0 \Rightarrow y = -2x - 3$

$m_1 = -2$

$2x - y - 5 = 0 \Rightarrow y = 2x - 5$

$m_2 = 2$

$\therefore \tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

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$$\tan\theta = \left|\frac{-2-2}{1+(-2)(2)}\right|$$

$$= \left|\frac{-4}{-3}\right| = \frac{4}{3}$$

$\therefore\ \theta(\text{acute}) = 53^\circ 8'$ (nearest minute)

c) $2\cos\theta + \sin\theta = 1$ $\qquad R = \sqrt{2^2+1^2}$

$= \sqrt{5}$

$\sqrt{5}\left(\frac{2}{\sqrt{5}}\cos\theta + \frac{1}{\sqrt{5}}\sin\theta\right) = 1$

$\sqrt{5}\sin(\theta+\alpha) = 1$ $\qquad$ where $\sin\alpha = \frac{2}{\sqrt{5}}$

$\cos\alpha = \frac{1}{\sqrt{5}} \Rightarrow \alpha = 63^\circ 26'$

$\therefore\ \sin(\theta+\alpha) = \frac{1}{\sqrt{5}}$

$\theta + \alpha = 26^\circ 34'$ or $153^\circ 26'$ (1 revolution)

$\theta + 63^\circ 26' = 26^\circ 34'$ or $153^\circ 26'$

$\therefore\ \theta = 26^\circ 34' - 63^\circ 26'$ or $153^\circ 26' - 63^\circ 26'$

$= -36^\circ 52'$ or 90°

$\therefore$ For $0^\circ \le \theta \le 360^\circ$

$\theta = 90^\circ$ or $323^\circ 8'$

d) $x^2 - 2x + p = 0$

Equal roots when $\Delta = 0$

ie $b^2 - 4ac = 0$

$\therefore\ 4 - 4(1)(p) = 0$

$4 = 4p \Rightarrow p = 1.$