Extension 1
MATHEMATICS

YEAR 11
PRELIMINARY ASSESSMENT TASK 2
July 2005

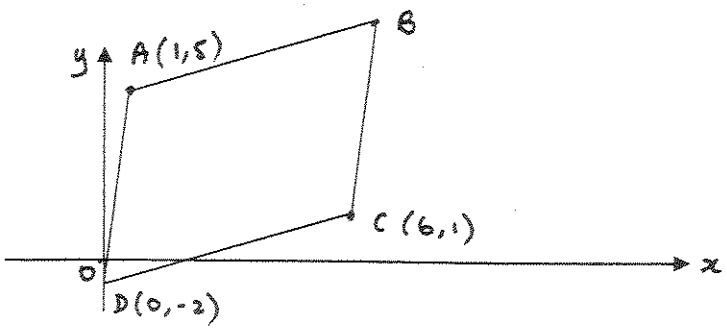
Question 1 10 marks

a. Give the exact value of 4
   i. $\cot 300^0$
   ii. $2\sin 15^0 \cos 15^0$

b. Differentiate $\dfrac{3x^2 + x}{x}$ 1

c. Evaluate $\lim\limits_{x \to \infty} \dfrac{4x^3 + 2x^2 - 1}{3 - 2x^2 + x^3}$ 1

d. ABCD is a parallelogram 2
   i. Find the midpoint of diagonal AC
   ii. Find the coordinates of the point B



e. Let A be the point (-1, 2) and B be the point (4, 9). Find the coordinates of the point P which divides the interval AB internally in the ratio 3 : 2 2

Question 2 10 marks **START A NEW PAGE**

a. Find the acute angle, α, between the two lines, $2x - y + 4 = 0$ and $y = 1 - x$. Give your answer correct to the nearest minute. 3

b. Find the exact value of $\sin 105^0$ 2

c. Find $\dfrac{d}{dt}\left(at^3 - 2a + t\right)$ 1

d. If $y = (x^2 + 1)^5$, find $\frac{dy}{dx}$ 2

e. Differentiate $f(x) = \frac{x^2}{1 - x^2}$ 2

Question 3 10 marks **START A NEW PAGE**

a. Find the gradient of the normal to the curve $y = x\sqrt{x}$ at the point (4, 8) 2

b. Find the equation of the tangent to the curve $y = x^2 + \frac{5}{x} - 2$ at the point P (1, 4) 2

c. Given that $0 < x < 45^0$, prove that 3

$$\tan(x + 45^0) = \frac{\cos x + \sin x}{\cos x - \sin x}$$

d. Find all angles , ϑ, with $0^0 \leq \vartheta \leq 360^0$, which $\sin 2\vartheta = \sin \vartheta$. 3

Question 4 10 marks **START A NEW PAGE**

a. 4

i. Differentiate $f(x) = \sqrt{x}$ by first principles

given $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

ii. For what values of x is $\frac{d}{dx}(\sqrt{x})$ undefined?

b. 4

i. Show that $(\sin x - \cos x)^2 = 1 - \sin 2x$

ii. Hence, or otherwise, find the value of $\sin 15^0 - \cos 15^0$, in simplest exact form.

c. If $\cos x = \frac{3}{4}$ and $\sin x < 0$, find the exact value of $\sin 2x$ 2

Question 5 10 marks **START A NEW PAGE**

a. 5

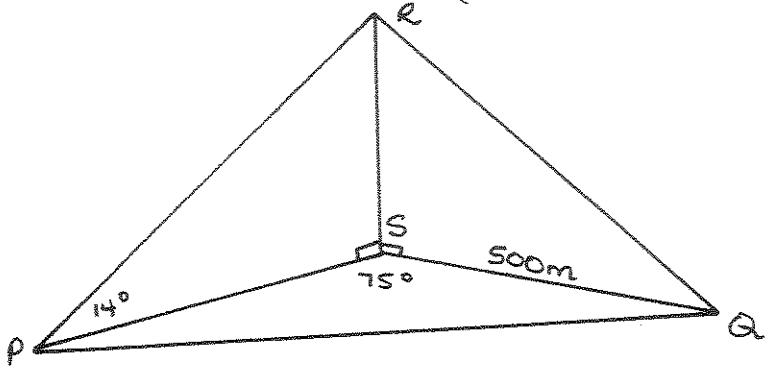
i. Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$ if $R > 0$ and $0 < \alpha < 90^0$

ii. Hence, or otherwise, solve $\cos x - \sqrt{3}\sin x = 2$, for $0^0 \le x \le 360^0$

b. 5

Two houses, P and Q, lie in the same plane as S, the foot of a tower RS. The angle of elevation from P to the top of the tower is 14^0 and the angle of depression from the tower to Q is 18^0.

It is known that the distance from S to Q is 500 metres.



i. Find an expression for the height of the tower.

ii. Find the distance from P to S correct to the nearest centimetre.

iii. Given that P and Q subtend an angle of 75^0 at S, how far apart are the two houses, correct to the nearest metre?

Question 6 10 marks **START A NEW PAGE**

a. 5

i. If $\tan\frac{\theta}{2} = t$, state the results for $\sin\theta$ and $\cos\theta$

ii. Using these results show that $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$

iii. Hence, find the exact value of $\tan 15^0$

b. 5

i. Show that the perpendicular distance from the point (4, 5) to the line $y = mx$ is given by,

$$d = \frac{|4m-5|}{\sqrt{m^2+1}}$$

ii. The line $y = mx$ is a tangent to the circle $(x-4)^2 + (y-5)^2 = 4$, explain why $\frac{|4m-5|}{\sqrt{m^2+1}} = 2$

iii. Hence, show that m satisfies the equation $12m^2 - 40m + 21 = 0$

2005 - Year 11 EXTENSION ONE TASK 2.

Question 1

a. i. $\cot 300° = \dfrac{1}{-\tan 60°}$

$= -1/\sqrt{3}$ ✓

ii. $2\sin 15°\cos 15° = \sin 2(15°)$

$= \sin 30°$

$= 1/2$ ✓

b. $\frac{d}{dx}(3x+1) = 3$ ✓

c. Lim $= 4$ ✓

d. i. $(7/2, 3)$ ✓

ii. $\dfrac{0+x}{2} = \dfrac{7}{2}$ $\quad \dfrac{-2+y}{2} = 3$

$x = 7$ $\quad -2+y = 6$

$y = 8$

$B(7, 8)$ ✓

2. $A(-1, 2)$ $B(4, 9)$

$3 : 2$

P is $\left(\dfrac{-1\times 2 + 4\times 3}{5}, \dfrac{2\times 2 + 9\times 3}{5}\right)$ ✓ ✓

$= (2, 31/5)$

b) $\sin 105° = \sin(45+60)$ ✓

$= \sin 45\cos 60 + \cos 45 \sin 60$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2} + \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}}$ ✓

$= \dfrac{\sqrt{2}+\sqrt{6}}{4}$ ✓

c) $\frac{d}{dt}(at^3 - 2a + t) = 3at^2 + 1$

d) $\dfrac{dy}{dx} = 5(x^2+1)^4 . 2x$

$= 10x(x^2+1)^4$

e) $u = x^2 \quad u' = 2x$

$v = 1-x^2 \quad v' = -2x$

$f'(x) = \dfrac{(1-x^2).2x - x^2.(-2x)}{(1-x^2)^2}$

$= \dfrac{2x - 2x^3 + 2x^3}{(1-x^2)^2}$

$= \dfrac{2x}{(1-x^2)^2}$ ✓

Question 2

a) $y = 2x+4 \quad y = 1-x$ ✓

$m_1 = 2 \quad m_2 = -1$

$\tan\alpha = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$ ✓

$= \left|\dfrac{2 - -1}{1 + 2\times -1}\right|$

$= \left|\dfrac{3}{-1}\right|$

$\tan\alpha = 3$

$\alpha = 71°34'$. ✓

Question 3

a. $y = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$= \frac{3\sqrt{x}}{2}$ ✓ at $x = 4$

$M_{tangent} = \frac{3.\sqrt{4}}{2}$

$= 3$

$\therefore M_N = -1/3$ ✓

i. $y' = 2x - \frac{5}{x^2}$ at $x = 1$

$M_T = 2 \times 1 - \frac{5}{1}$

$= 2 - 5$

$= -3$ ✓

$\therefore y - 4 = -3(x - 1)$ ✓

$y = -3x + 7$

$\therefore$ LHS $= \tan(x + 45)$

$= \frac{\tan x + \tan 45}{1 - \tan x . \tan 45}$ ✓

$= \frac{\tan x + 1}{1 - \tan x . 1}$

$= \frac{\sin x/\cos x + 1}{1 - \sin x/\cos x}$ ✓ $\left(\times \frac{\cos x}{\cos x}\right)$

$= \frac{\sin x + \cos x}{\cos x - \sin x}$

$= \frac{\cos x + \sin x}{\cos x - \sin x}$ ✓

$=$ RHS

d) $\sin 2\theta = \sin\theta$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$ ✓

$\sin\theta = 0$ $\quad \cos\theta = 1/2$

$\theta = 0, 180°, 360°$ ✓ $\quad \theta = 60°, 300°$ ✓

$\therefore \theta = 0, 60°, 180°, 300°, 360°$

Question 4

a. $\frac{dy}{dx} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$ ✓

$= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$ ✓

$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$

$= \frac{1}{\sqrt{x} + \sqrt{x}}$

$= \frac{1}{2\sqrt{x}}$ ✓

ii. undefined when $x \leq 0$ ✓

b.

i. LHS $= (\sin x - \cos x)^2$

$= \sin^2 x - 2\sin x\cos x + \cos^2 x$ ✓

$= 1 - 2\sin x\cos x$

$= 1 - \sin 2x$ ✓

ii. using result in part (i)

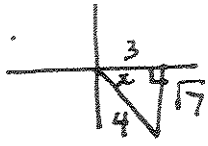
$(\sin 15° - \cos 15°)^2 = 1 - \sin 30°$

$\sin 15° - \cos 15° = \pm\sqrt{1 - 1/2}$

$= \pm\sqrt{1/2}$

take neg $= -1/\sqrt{2}$. ✓✓ (one for the neg)

as $\cos 15° > \sin 15°$

c. 

4th quad ✓

$\sin 2x = 2\sin x\cos x$

$= 2 \times -\frac{\sqrt{7}}{4} \times \frac{3}{4} = -\frac{3\sqrt{7}}{8}$ ✓

Question 5

a. i. $R = \sqrt{1+\sqrt{3}^2} = 2$ ✓

$\tan\alpha = \sqrt{3}$

$\alpha = 60°$ ✓

$\therefore 2\cos(x+60°)$

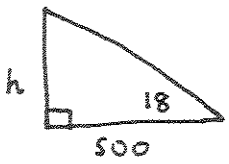
ii. $2\cos(x+60°) = 2$ ✓

$\cos(x+60°) = 1$

$x+60° = 0, 360°$ ✓

$\therefore x = 300°$ ✓

b. i.



$\frac{h}{500} = \tan 18°$ ✓

$h = 500\tan 18°$

ii.

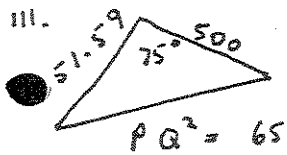
$\tan 14° = \frac{h}{PS}$ ✓

$PS = h \div \tan 14$

$= \frac{500\tan 18°}{\tan 14°}$ ✓

$= 651.59\ m$

iii.



$PQ^2 = 651.59^2 + 500^2 - 2\times 651.59 \times 500 \times \cos 75°$ ✓

$PQ^2 = 505925.62..$

$PQ = 711.284....$

$= 711\ m$ ✓

Question 6

a. $\sin\theta = \frac{2t}{t^2+1}$ ✓

$\cos\theta = \frac{1-t^2}{1+t^2}$ ✓

ii. $\frac{\sin\theta}{1+\cos\theta}$

$= \frac{\frac{2t}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}$ ✓ $\times \frac{1+t^2}{1+t^2}$

$= \frac{2t}{1+t^2+(1-t^2)}$ ✓

$= \frac{2t}{2}$

$= t$

$= \tan\theta/2$

$=$ RHS.

iii. $\tan 15° = \tan(30/2)$

$= \frac{1-\cos 30}{\sin 30}$

$= \left(1-\frac{\sqrt{3}}{2}\right)\times\frac{2}{1}$

$= 2-\sqrt{3}$ ✓

b. $mx - y = 0$

i. $d_\perp = \left|\frac{m.4 - 5 + 0}{\sqrt{m^2+(-1)^2}}\right|$ ✓

$= \frac{|4m-5|}{\sqrt{m^2+1}}$

ii. $C = (4,5)\quad r = 2$

$\therefore$ perp. dist from centre to circumf = radius.

$\therefore \frac{|4m-5|}{\sqrt{m^2+1}} = 2$

iii. $\frac{|4m-5|}{\sqrt{m^2+1}} = 2$

$\frac{\sqrt{(4m-5)^2}}{\sqrt{m^2+1}} = 2$

$(4m-5)^2 = 4(m^2+1)$

$16m^2 - 40m + 25 = 4m^2 + 4$

$\therefore 12m^2 - 40m + 21 = 0$