SYDNEY TECHNICAL HIGH SCHOOL

Mathematics

Extension 1

Year 11
PRELIMINARY HSC ASSESSMENT TASK 2
JULY 2007

General Instructions

- Working time allowed – 70 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions
- Marks may not be awarded for careless or badly arranged work

NAME : _____

QUESTION 1	QUESTION 2	QUESTION 3	QUESTION 4	QUESTION 5	TOTAL

Question 1

Marks

a) Find the maximum value of $8 - 4x - x^2$ 1

b) Solve for x : $x(2x - 9) < 11$ 2

c) If A is acute and $\sin A = \frac{1}{3}$ find the exact value of

i) $\cos(90° - A)$ 1

ii) $\sin 2A$ 2

d) Evaluate $\lim_{x \to 2} \frac{3x^2 - 5x - 2}{x - 2}$ 1

e) Find the gradient of the curve $y = 2x^3 - 8x - 1$ at the point $(2, -1)$ 2

f) Find the coordinates of the point that divides the interval from $(-1, 6)$ to $(4, 3)$ externally in the ratio $5 : 2$. 2

Question 2 (start a new page) Marks

a) Find the acute angle between the lines $2x-4y-3=0$ 2

and $y=3x-4$.

b) For what values of k does the equation $(k+3)x^2-kx+1=0$

have i) equal roots 2

ii) real roots 1

c) A circle with centre $(-4,1)$ has one end of a diameter at $(-10,-4)$. 1

Find the coordinates of the other end of this diameter.

d) Find the equation of the tangent to $y=\sqrt{4x+1}$ at the point $(2,3)$ 3

e) Express $\dfrac{\sin x}{1+\cos x}$ in terms of t where $t=\tan(\dfrac{x}{2})$. 2

Question 3 (start a new page)

a) Evaluate $\lim\limits_{x\to\infty}\dfrac{8-2x^3}{2+x^2+x^3}$ 1

b) Show that $\dfrac{\sin 3\theta}{\sin\theta}+\dfrac{\cos 3\theta}{\cos\theta}=4\cos 2\theta$ 3

c) The line $ax+by+3=0$ is parallel to $3x+2y-4=0$ 3

and passes through the point $(1,-2)$. Find the values of a and b.

d) i) Find the exact value of $\tan 75°$ 2

ii) Use the above result to show that $\tan 75°+\cot 75°=4$ 2

Question 4 (start a new page) Marks

a) Differentiate the following with respect to x:

i) $y = \dfrac{1}{x^2}$ 1

ii) $y = \dfrac{4x^2}{x+1}$ 2

iii) $y = (x^2 - 1)(x+1)^4$ 2

b) Solve $\sin\theta = \sin 2\theta$ for $0° \le \theta \le 360°$ 3

c) Find the equation of the line which passes through the point of intersection of $x - 4y + 5 = 0$ and $2x - 3y - 1 = 0$ and the point $(2,3)$, giving your answer in general form. 3

Question 5 (start a new page)

a) The line $y = mx - 4$ is a tangent to the curve $y = x^2 - 8x$. 3

Find the possible values of m.

b) A and B have coordinates $(-1,7)$ and $(5,-2)$ respectively. 3

P divides the interval AB internally in the ratio $k:1$.

Find the value of k given that P lies on the line $5x - 4y - 1 = 0$

c) Simplify $\dfrac{\sin 2\alpha}{1 - \cos 2\alpha}$ 2

d) Solve the equation $4\sin\theta - 2\cos\theta = 3$ 3

for $0° \le \theta \le 360°$ giving answer correct to the nearest degree.

EXT 1 SOLUTIONS JULY 2007

Q1

a) max when $x = -2$

$\therefore$ max $= 12$

b) $2x^2 - 9x - 11 < 0$

$(2x - 11)(x + 1) < 0$

$-1 < x < 5\frac{1}{2}$

c) i) $\cos(90° - A)$

$= \sin A$

$= \frac{1}{3}$

ii) $\sin 2A$

$= 2 \sin A \cos A$

$= 2 \times \frac{1}{3} \times \frac{\sqrt{8}}{3}$

$= \frac{4\sqrt{2}}{9}$

d) $\lim_{x \to 2} \frac{(3x+1)(x-2)}{x-2}$

$= 7$

e) $y' = 6x^2 - 8$

when $x = 2$

$m = 6 \times 2^2 - 8$

$= 16$

f) $\left(\frac{20+2}{3}, \frac{15-12}{3} \right)$

$= \left(\frac{22}{3}, 1 \right)$

Q2

a) $m_1 = \frac{1}{2}$ $m_2 = 3$

$\tan \theta = \left| \frac{3 - \frac{1}{2}}{1 + 3 \times \frac{1}{2}} \right|$

$= 1$

$\therefore \theta = 45°$

b) i) $\Delta = 0$

$k^2 - 4(k+3)(1) = 0$

$k^2 - 4k - 12 = 0$

$(k-6)(k+2) = 0$

$k = 6, -2$

ii) $\Delta \geq 0$

$k \leq -2, \quad k \geq 6$

c) $\frac{-10 + x}{2} = -4$ $\frac{-4 + y}{2} = 1$

$x = 2$ $y = 6$

$\therefore (2, 6)$

d) $y' = \frac{1}{2}(4x+1)^{-\frac{1}{2}} \cdot 4$

when $x = 2$

$y' = \frac{2}{3}$

$\therefore y - 3 = \frac{2}{3}(x - 2)$

$2x - 3y + 5 = 0$

e) $\frac{2t}{1+t^2} \div \left(1 + \frac{1-t^2}{1+t^2} \right)$

$= \frac{2t}{1+t^2} \times \frac{1+t^2}{1+t^2+1-t^2}$

Q3

a) -2

b) LHS $= \dfrac{\sin 3\theta}{\sin\theta} + \dfrac{\cos 3\theta}{\cos\theta}$

$= \dfrac{\cos\theta \sin 3\theta + \sin\theta \cos 3\theta}{\sin\theta \cos\theta}$

$= \dfrac{2\sin 4\theta}{\sin 2\theta}$

$= \dfrac{4 \sin 2\theta \cos 2\theta}{\sin 2\theta}$

$= 4\cos 2\theta$

$=$ RHS

c) $-\dfrac{a}{b} = -\dfrac{3}{2}$

$\therefore \; 2a = 3b \quad ①$

sub $(1, -2)$

$a - 2b + 3 = 0$

$a - 2b = -3 \quad ②$

Solve ①, ② simultaneously

$a = 9, \quad b = 6$

d) i) $\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$= \dfrac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

ii) $\tan 75^\circ + \cot 75^\circ$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} + \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \dfrac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{8}{2}$

$= 4$

Q4

a) i) $y' = -2x^{-3}$

ii) $y' = \dfrac{(x+1)8x - 4x^2(1)}{(x+1)^2}$

$= \dfrac{4x^2 + 8x}{(x+1)^2}$

iii) $y' = 2x(x+1)^4 + (x^2-1)4(x+1)^3$

$= 2(x+1)^3(3x^2 + x - 2)$

b) $\sin\theta = \sin 2\theta$

$\sin\theta - 2\sin\theta\cos\theta = 0$

$\sin\theta(1 - 2\cos\theta) = 0$

$\theta = 0^\circ, 180^\circ, 360^\circ, 60^\circ, 300^\circ$

c) $x - 4y + 5 + k(2x - 3y - 1) = 0$

sub $(2, 3)$

$k = -\dfrac{5}{6}$

Q5

a) $y = mx - 4$

$y = x^2 - 8x$

Solve simultaneously

$x^2 - 8x = mx - 4$

$x^2 - (m+8)x + 4 = 0$

tangent $\Rightarrow$ 1 solution

$\Rightarrow \Delta = 0$

$(m+8)^2 - 4 \times 1 \times 4 = 0$

$m^2 + 16m + 48 = 0$

$(m+12)(m+4) = 0$

$m = -12, -4$

b) $P\left(\frac{1 \times -1 + k \times 5}{k+1}, \frac{1 \times 7 + k \times -2}{k+1}\right)$

$= \left(\frac{5k-1}{k+1}, \frac{7-2k}{k+1}\right)$

sub. P into $3x - 4y - 1 = 0$

$5\left(\frac{5k-1}{k+1}\right) - 4\left(\frac{7-2k}{k+1}\right) - 1 = 0$

$32k = 34$

$k = \frac{17}{16}$

c) $\frac{\sin 2\alpha}{1 - \cos 2\alpha} = \frac{2 \sin\alpha \cos\alpha}{1 - (1 - 2\sin^2\alpha)}$

$= \frac{\cos\alpha}{\sin\alpha}$

$= \cot\alpha$

d) $4\sin\theta - 2\cos\theta = 3$

$R \sin(\theta - \alpha) = 3$

$R = \sqrt{20}$ $\quad \tan\alpha = \frac{1}{2}$

$\alpha = 26.57°$

$\therefore \sqrt{20} \sin(\theta - \alpha) = 3$

$\sin(\theta - \alpha) = \frac{3}{\sqrt{20}}$

$\theta - \alpha = 42.13°, 137.87°$

$\theta = 69°, 164°$