Name: File

Teacher:__________________

SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

Extension 1

Year 11

Preliminary HSC Assessment Task 2

July 2008

General Instructions

- Working time allowed – 70 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions
- Marks may not be awarded for careless or badly arranged work

Question 1	Question 2	Question 3	Question 4	Question 5	TOTAL
/10	/10	/10	/10	/10	/50

Question 1 (10 marks)

a) Express $\frac{1}{1-\frac{m}{n}} \div \frac{1}{1-\frac{n}{m}}$ as a single fraction 2

b) Solve $\frac{1}{x-1} > \frac{1}{6}$ 2

c) i) Sketch $y = cos x°$ for $-180 \leq x \leq 180$ 1

ii) Sketch $y = \frac{1}{2}$ on the same axes as above 1

iii) Hence, solve $cos x° > \frac{1}{2}$ for $-180° \leq x \leq 180°$ 2

d) Given A(-4, -6) and B (6, -1) find the coordinates of P (x,y) such that P divides the interval AB externally in the ratio 3:2 2

Question 2 (10 marks) (Start a new page)

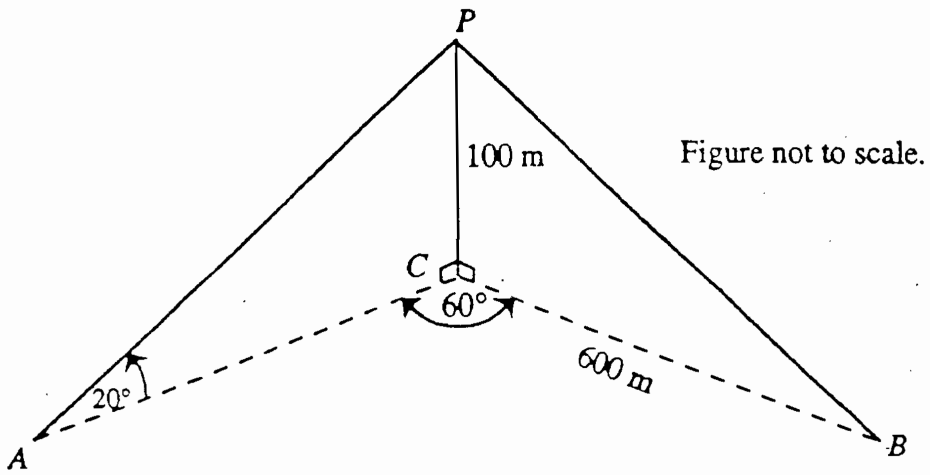
a) i) Write an expansion for $\sin(A + B)$ 1

ii) By using part i) find the exact value of $sin 75°$ 2

b) Differentiate $f(x) = 1 - 2x^2$ by the method of first principles 2

(Question 2 continues over)

c)



Two yachts A and B subtend an angle 60° at the base C of a cliff. From yacht A the angle of elevation of the point P, 100 metres vertically above C, is 20°. Yacht B is 600 metres from C.

i) Calculate the length AC in exact form. 1

ii) Calculate the distance between the two yachts. (to nearest metre). 1

d) The line $y = mx$ makes an angle of 45° with the line $y = 2x - 3$. Find the two possible values for m. 3

Question 3 (10 marks) (Start a new page)

a) Differentiate the following

i) $y = \frac{2x}{\sqrt{x}}$ 2

ii) $y = (3 - 2x^2)^4$ 2

iii) $y = \frac{x^2}{x+1}$ 2

b) Show that $\frac{d}{dx}\left(x\sqrt{x+1}\right) = \frac{3x+2}{2\sqrt{x+1}}$ 2

c) Simplify $\frac{1+sinx-cosx}{1+sinx+cosx}$ as far as possible in terms of t, where $t = tan\frac{x}{2}$

2

Question 4 (10 marks) (Start a new page)

a) Prove $\frac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$ 3

b) Solve

i) $\sin 2\theta = \cos\theta$ for $0° \leq \theta \leq 360°$ 2

ii) $\cos^2\theta - \sin^2\theta = 0.1$ for $0° \leq \theta \leq 180°$

(θ correct to the nearest minute) 2

c) i) Write $2\cos x + 3\sin x$ in the form $R\sin(x+\propto)$ where $\propto$ is acute and to the nearest minute, $R > 0$. 1

ii) Hence, solve $2\cos x + 3\sin x = 1$ for $0° \leq x \leq 360°$ 2

Question 5 (10 marks) (Start a new page)

a) Given $f(n) = 2^n$ and $g(n) = 3^n$, prove that

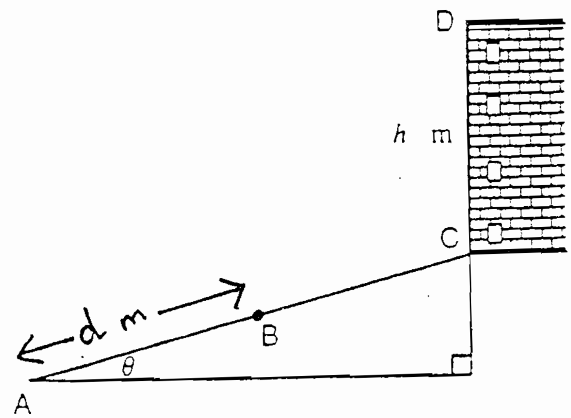
$$\frac{3f(n+1)+2g(n+1)}{6} = f(n) + g(n)$$ 2

b) i) Find the equation of the tangent to the semicircle $y = \sqrt{25 - x^2}$ at the point *P (3, 4)* 3

ii) Another tangent to the semicircle above at $Q(0,5)$, meets the tangent at *P*, in the point *R*. Find the coordinates of *R*. 1

(Question 5 continues over)

c)



A ramp, ABC rises at an angle of θ degrees. AB is d metres and, at C there is a building of height h metres. The angles of elevation of D from A and B are $(\theta + \propto)$ and $(\theta + \beta)$ respectively.

i) Find an expression for DB **1**

ii) Hence prove that $h = \dfrac{d\sin\propto . \sin\beta}{\sin(\beta - \propto)\cos\theta}$ **1**

iii) Deduce that, if $\theta = 15°, \propto = 45°$ and $\beta = 60°$ then $h = d\sqrt{6}$ **2**

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Question 1

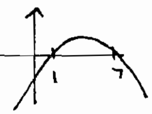
a) $\dfrac{1}{\frac{n-m}{n}} \div \dfrac{1}{\frac{m-n}{m}}$

$= \dfrac{n}{n-m} \times \dfrac{m-n}{m}$

$= \dfrac{-(n-m).n}{(n-m)\,m}$

$= -n/m$

b) $\dfrac{1}{x-1} > \dfrac{1}{6}$



$6(x-1) > (x-1)^2$

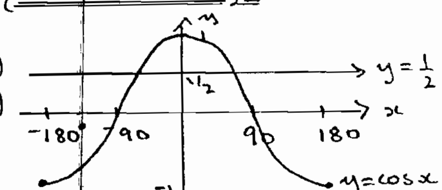
$6(x-1) - (x-1)^2 > 0$

$(x-1)(7-x) > 0$

$\{x : 1 < x < 7\}$

c) i)

ii)



iii) $\cos x = \frac{1}{2}$ at 60° and -60°

$\therefore$ for $\cos x > \frac{1}{2}$ $\quad -60^\circ < x < 60^\circ$

d) $A(-4,-6)$ $\quad B(6,-1)$

$-3 : 2$

$P\left(\dfrac{-8+18}{-1}, \dfrac{-12+3}{-1}\right)$

$P(26, 9)$

Question 2

a) i) $\sin(A+B) = \sin A\cos B + \cos A \sin B$

ii) $\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ . \sin 45^\circ$

$= \frac{1}{2}.\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

b) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{1-2(x+h)^2 - [1-2x^2]}{h}$

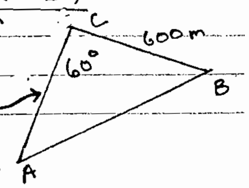
$= \lim\limits_{h\to 0} \dfrac{1-2x^2 - 4xh - 2h^2 - 1 + 2x^2}{h}$

$= \lim\limits_{h\to 0} \dfrac{h(-4x-2h)}{h}$

$f'(x) = -4x$

c) i) $\tan 20^\circ = \dfrac{100}{AC}$

$AC = \dfrac{100}{\tan 20^\circ}$



ii) $BC = 600$

$AB = \left(\dfrac{100}{\tan 20}\right)^2 + 600^2 - 2 \times \dfrac{100}{\tan 20} . 600 . \cos 6($

$AB = 520\text{m}$

d) gradients m and 2

$\tan 45^\circ = \left|\dfrac{m-2}{1+2m}\right|$

$1 = \dfrac{m-2}{1+2m}$ or $-1 = \dfrac{m-2}{1+2m}$

$1 + 2m = m - 2$ $\qquad -1-2m = m-2$

$m = -3$ $\qquad 1 = 3m$

$m = \frac{1}{3}$

Question 3

a) i) $y = \dfrac{2x}{\sqrt{x}} \Rightarrow y = 2x^1 . x^{-1/2}$

$y = 2x^{1/2}$

$\therefore \dfrac{d}{dx}(2x^{1/2}) = x^{-1/2}$

$= \dfrac{1}{\sqrt{x}}$

ii) $\dfrac{d}{dx}(3-2x^2)^4 = 4(-4x)(3-2x^2)^3$

$= -16x(3-2x^2)^3$

iii) $u = x^2$ $\quad v = x+1$

$u' = 2x$ $\quad v' = 1$

$\dfrac{d}{dx}\left(\dfrac{x^2}{x+1}\right) = \dfrac{2x(x+1) - x^2}{(x+1)^2}$

$= \dfrac{x^2+2x}{(x+1)^2}$

b) $u = x$ $\quad v = (x+1)^{1/2}$

$u' = 1$ $\quad v' = \frac{1}{2}(x+1)^{-1/2}$

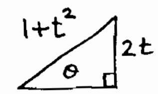
$= \dfrac{1}{2\sqrt{x+1}}$

$\dfrac{dy}{dx} = \sqrt{x+1} + \dfrac{x}{2\sqrt{x+1}}$

$= \dfrac{2(x+1) + x}{2\sqrt{x+1}}$

$\dfrac{d}{dx}\left(x\sqrt{x+1}\right) = \dfrac{3x+2}{2\sqrt{x+1}}$

c)



$= \dfrac{1+\sin x - \cos x}{1+\sin x + \cos x}$

$= \left[1 + \dfrac{2t}{1+t^2} - \left(\dfrac{1-t^2}{1+t^2}\right)\right] \div \left[1 + \dfrac{2t}{1+t^2} + \dfrac{1-t^2}{1+t^2}\right]$

$= \dfrac{1+t^2+2t-1+t^2}{1+t^2} \times \dfrac{1+t^2}{1+t^2+2t+1-t^2}$

$= \dfrac{2t^2+2t}{2t+2}$

$= \dfrac{2t(t+1)}{2(t+1)}$

$= t$

Question 4

a) LHS $= \dfrac{1-\cos 2A}{1+\cos 2A}$

$= \dfrac{1-(\cos^2 A - \sin^2 A)}{1+\cos^2 A - \sin^2 A}$

$= \dfrac{(1-\cos^2 A) + \sin^2 A}{(1-\sin^2 A) + \cos^2 A}$

$= \dfrac{2\sin^2 A}{2\cos^2 A}$

$= \tan^2 A$

$= \text{RHS}$

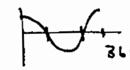
b) i) $\sin 2\theta = \cos\theta$

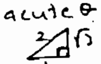
$2\sin\theta . \cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ $\qquad \sin\theta = \frac{1}{2}$

acute $\theta = 30^\circ$





$\therefore \theta = 90^\circ, 270^\circ$ $\qquad \theta = 30^\circ,$

ii) $\cos^2\theta - \sin^2\theta = 0.1$

$\cos 2\theta = 0.1$

acute $2\theta = 84^\circ 16'$

$\therefore 2\theta = 84^\circ 16', 275^\circ 44'$

$\theta = 42^\circ 8', 137^\circ 52'$

c) i) $R = \sqrt{4+9} = \sqrt{13}$

$\sqrt{13}\left\{\dfrac{2}{\sqrt{13}}\cos x + \dfrac{3}{\sqrt{13}}\sin x\right\}$

$\sin\alpha = \dfrac{2}{\sqrt{13}}$, $\cos\alpha =$

$\alpha = 33^\circ 41'$

$\therefore$ Form required

$\sqrt{13}\sin(x + 33^\circ 41')$

ii) $\sqrt{13}\sin(x + 33^\circ 41') = 1$

$\sin(x + 33^\circ 41') = \dfrac{1}{\sqrt{13}}$

$x + 33^\circ 41' = 16^\circ 6', 163^\circ 54',$

$x = 130^\circ 13', 342^\circ 25'$

Question 5

a)

$f(n) = 2^n \qquad g(n) = 3^n$

$$\text{LHS} = \frac{3f(n+1) + 2g(n+1)}{6}$$

$$= \frac{3 \cdot 2^{n+1} + 2 \cdot 3^{n+1}}{6}$$

$$= \frac{3^1 \cdot 2^n \cdot 2^1 + 2 \cdot 3^n \cdot 3^1}{6}$$

$$= \frac{6(2^n + 3^n)}{6}$$

$$= 2^n + 3^n$$

$$= f(n) + g(n)$$

$$= \text{RHS}$$

b) i) $y = \sqrt{25 - x^2} = (25 - x^2)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2} \cdot -2x(25 - x^2)^{-1/2}$$

$$= \frac{-x}{\sqrt{25 - x^2}}$$

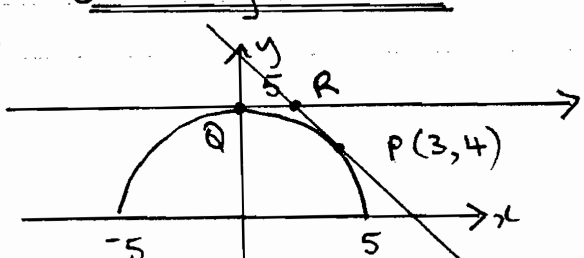
at $P(3, 4)$ $\quad m_T = -\frac{3}{4}$

eqn: $y - 4 = -\frac{3}{4}(x - 3)$

$$4y - 16 = -3x + 9$$

$$3x + 4y - 25 = 0$$

ii)

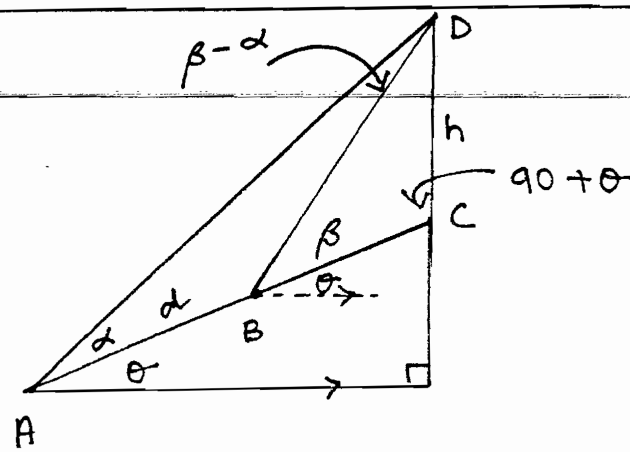


sub $y = 5$ in tang. eqn above

$R\left(\frac{5}{3}, 5\right) \qquad 3x - 5 = 0$

$\therefore x = 5/3$

c)



i) In ΔABD

$$\frac{BD}{\sin\alpha} = \frac{d}{\sin(\beta - \alpha)}$$

$$BD = \frac{d\sin\alpha}{\sin(\beta - \alpha)}$$

ii) In ΔBDC

$$\frac{h}{\sin\beta} = \frac{BD}{\sin(90 + \theta)}$$

$$h = \frac{BD\sin\beta}{\sin(90 + \theta)}$$

note: $\sin(90 + \theta) = \cos\theta$

$$\therefore h = \frac{d\sin\alpha\sin\beta}{\sin(\beta - \alpha) \cdot \cos\theta}$$

iii) $h = d\,\dfrac{\sin 45^\circ \cdot \sin 60^\circ}{\sin 15^\circ \cdot \cos 15^\circ}$

since

$\sin 15^\circ = \sin 60^\circ \cdot \cos 45^\circ - \cos 60^\circ \sin 45^\circ$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$\cos 15^\circ = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 4$

$$= \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

$$h = d\left[\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \div \frac{\sqrt{3} - 1}{2\sqrt{2}} \cdot \frac{1 + \sqrt{3}}{2\sqrt{2}}\right]$$

$$h = d\left[\frac{\sqrt{3}}{2\sqrt{2}} \times \frac{8}{2}\right]$$