SYDNEY TECHNICAL HIGH SCHOOL

EXTENSION 1

MATHEMATICS

PRELIMINARY ASSESSMENT TASK 2

2009

Time Allowed: **70 minutes**

Instructions:

- Write using blue or black pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- PLEASE START EACH NEW QUESTION ON A NEW PAGE.

Name:____________________________ **Teacher:** __________________________

Q1	Q2	Q3	Q4	Q5	Q6
/8	/8	/8	/8	/9	/10

Total
/51

Question 1 (8 marks)

a) Find the acute angle between the lines $x - y - 1 = 0$ and $y = 1 - 2x$, correct to the nearest degree. (2)

b) i) Expand $sin(A + B)$

ii) Write down the exact value of $sin 105°$ (3)

c) Find the distance between the parallel lines $y = 2x + 3$ and $y = 2x - 1$ (3)

Question 2 (8 marks) START A NEW PAGE

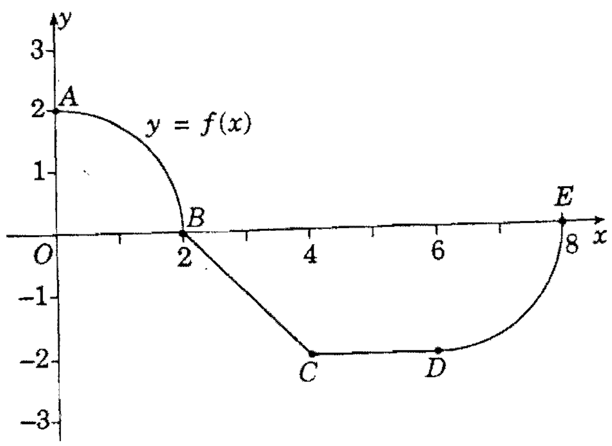
a) Differentiate

i) $y = \frac{1}{x^2} + \sqrt{x}$ (express with positive indices) (2)

ii) $y = \frac{x^2}{x-2}$ (2)

b) Express the derivative of $y = x\sqrt{2x + 1}$ as a single fraction. (3)

c)



The graph of the function f consists of a quarter circle AB, a straight line segment BC, a horizontal straight line segment CD, and a quarter circle DE as shown.

For what values of x satisfying $0 < x < 8$ is the function f NOT differentiable? (1)

Question 3 (8 marks) START A NEW PAGE

a) Use differentiation by first principles to find the gradient of the tangent to the curve $y = 3x^2 - 4x$ at $x = 1$. (3)

b) The quadratic equation $2x^2 + 4x + 1 = 0$ has roots α and β. Find:

i) $\alpha + \beta$

ii) $(\alpha - \beta)^2$

iii) the quadratic equation with roots α^2 and β^2 (5)

Question 4 (8 marks) START A NEW PAGE

a) The point P (17,36) divides the line joining A (2,1) and B (5,8) *externally* in the ratio $m:n$.

Find the values of m and n. (3)

b) Show that $\frac{cosx+sinx}{cosx-sinx} = \frac{sin2x+1}{cos2x}$ (2)

c) Solve for θ: $sin2\theta = cos^2\theta,\ \ 0 \le \theta \le 360°$ (3)

(answers to the nearest minute, where appropriate)

Question 5 (9 marks) START A NEW PAGE

a) Find the value of m for which $y = mx$ is a tangent to the curve $y = \frac{x-1}{x}$ (2)

b) i) Express $2cosx + 2\sqrt{3}\ sinx$ in the form $Rcos(x-\propto)$, where $R > 0$ and $0° \le \alpha \le 90°$ (2)

ii) Find the two **non-zero** solutions to: $2cosx + 2\sqrt{3}\ sinx = 2,\ 0 \le x \le 360°$ (2)

c) Solve for x: $9^x - 6(3^x) - 27 = 0$ (3)

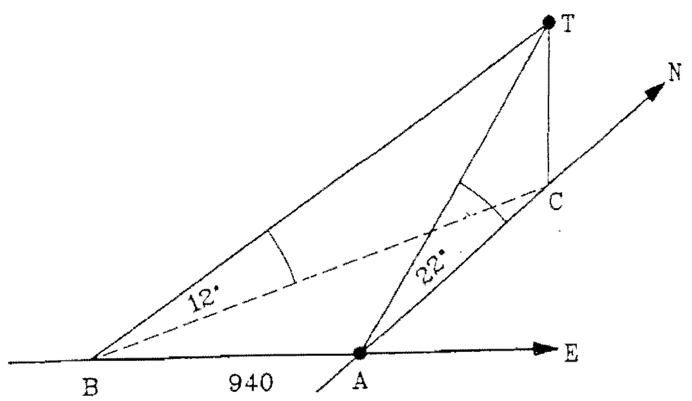
Question 6 (10 marks) START A NEW PAGE

a) i) Prove that the equation $3px^2 = 2px + 3qx - 2q$, where p and q are rational, has rational roots for all values of p and q. (3)

b) i) If $tan\frac{\theta}{2} = t$, express $\frac{sin\theta}{cos\theta+1}$ in terms of t (2)

ii) Hence, solve $\frac{sin\theta}{cos\theta+1} = \sqrt{3}$, $0° \leq \theta \leq 360°$ (1)

c) (4)



The angle of elevation of the top of a tower (T) from point A, which lies South of the tower, is 22 degrees. From point B, which lies 940 metres West of point A, the angle of elevation of the top of the tower (T) is 12 degrees. C is the base of the tower.

i) Show that $AC = h \tan 68°$ where $h = CT$ (the height of the tower).

ii) Find the height of the tower (correct to the nearest metre).

Answers

Question 1

a) $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\dfrac{1+2}{1+1\times -2}\right|$

$\tan\theta = 3$ (2)

$\theta = 72°$

b)

i. $\sin(A+B)$ (1)

$= \sin A\cos B + \sin B\cos A$

ii. $\sin 105°$

$= \sin(60+45)$ (2)

$= \sin 60°\cos 45° + \sin 45°\cos 60°$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$= \dfrac{\sqrt{6}+\sqrt{2}}{4}$

c) On $y = 2x+3$ pt $(0,3)$

$2x - y - 1 = 0$

$d_\perp = \dfrac{|2(0) - 3 - 1|}{\sqrt{4+1}}$

$= \dfrac{4}{\sqrt{5}}$ (2)

Question 2.

a) i. $y' = -2x^{-3} + \frac{1}{2}x^{-1/2}$

$y' = \dfrac{-2}{x^3} + \dfrac{1}{2\sqrt{x}}$ (2)

ii. $y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(x-2)(2x) - (x^2)(1)}{(x-2)^2}$

$= \dfrac{2x^2 - 4x - x^2}{(x-2)^2}$

$= \dfrac{x^2 - 4x}{(x-2)^2}$ (2)

b) $y' = vu' + uv'$

$= \sqrt{2x+1}\cdot 1 + x \cdot \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2$

$= \sqrt{2x+1} + \dfrac{x}{\sqrt{2x+1}}$

$= \dfrac{2x+1+x}{\sqrt{2x+1}}$

$= \dfrac{3x+1}{\sqrt{2x+1}}$ (here enough)

(3)

OR $= \dfrac{(3x+1)\sqrt{2x+1}}{2x+1}$

c) B and C. (1)

ie $x = 2$ $x = 4$

Question 3

a) $dy/dx =$

$\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \dfrac{3(x+h)^2 - 4(x+h) - (3x^2 - 4x)}{h}$

$\lim_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 - 4x - 4h - 3x^2 + 4x}{h}$

$\lim_{h \to 0} \dfrac{h(6x + 3h - 4)}{h}$ (3)

$= 6x - 4$ at $x = 1$

$m_T = 2.$

b) $\alpha + \beta = -b/a$

$= -4/2 = -2$ (1)

ii. $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= (\alpha+\beta)^2 - 4\alpha\beta$

$= (-2)^2 - 4 \times 1/2$

$= 4 - 2$

$= 2$ (2)

(iii) $x^2 - (\alpha^2 + \beta^2)x + (\alpha\beta)^2 = 0$

$x^2 - \left[(\alpha+\beta)^2 - 2\alpha\beta\right]x + \left(\frac{1}{2}\right)^2 = 0$

$x^2 - \left[4 - 2\cdot\frac{1}{2}\right]x + \frac{1}{4} = 0$

$x^2 - 3x + 1/4 = 0$

or

$4x^2 - 12x + 1 = 0$ (2)

Question 4

a. $(2,1)$ $(5,8)$

$-m : n$ $\rightarrow \dfrac{m}{n}$

$\dfrac{2n - 5m}{n - m} = 17$

$2n - 5m = 17n - 17m$

$\dfrac{-15n}{-12} = \dfrac{-12m}{-12}$

$\dfrac{5}{4} = \dfrac{m}{n}$ (3)

$\therefore m = 5$

$n = 4.$

b.

LHS $= \dfrac{\cos x + \sin x}{\cos x - \sin x} \times \dfrac{\cos x + \sin x}{\cos x + \sin x}$

$= \dfrac{(\cos x + \sin x)^2}{\cos^2 x - \sin^2 x}$

$= \dfrac{\cos^2 x + 2\sin x\cos x + \sin^2 x}{\cos 2x}$

$= \dfrac{\sin 2x + 1}{\cos 2x}$ (2)

$=$ RHS.

Question 4c

c) $\sin 2\theta = \cos^2\theta$

$2\sin\theta\cos\theta - \cos^2\theta = 0$

$\cos\theta\left[2\sin\theta - \cos\theta\right] = 0$

$\Rightarrow \cos\theta = 0$; $\theta = 90^\circ, 270^\circ$

$\Rightarrow 2\sin\theta = \cos\theta$

$\tan\theta = 1/2 \quad \theta = 26^\circ 34', 206^\circ 34'$

ie $\theta = 26^\circ 34', 90^\circ, 206^\circ 34', 270^\circ$

Question 5

a) $mx = \frac{x-1}{x}$

$mx^2 - x + 1 = 0 \quad \Delta = 0$ tangent

$1 - 4m \times 1 = 0$

$1 = 4m$

$m = \frac{1}{4}$ (2)

b) $R = \sqrt{2^2 + (2\sqrt{3})^2} = 4$

i) $\tan\alpha = \frac{2\sqrt{3}}{2} \quad \alpha = 60^\circ$

$\therefore 4\cos(x - 60^\circ)$ (2)

ii) $4\cos(x - 60^\circ) = 2$

$\cos(x - 60^\circ) = \frac{1}{2}$

$x - 60^\circ = 60^\circ, 300^\circ, -60^\circ$

$x = 0^\circ, 120^\circ, 360^\circ$ (2)

$\therefore$ non zero soln's

$x = 120^\circ, 360^\circ$

c. let $3^x = m$

$\therefore m^2 - 6m - 27 = 0$

$(m-9)(m+3) = 0$

$m = 9 \quad m = -3$

$\therefore 3^x = 9 \quad , \quad 3^x = -3$

$x = 2 \quad$ No Sol

ie $x = 2$ only. (3)

Question 6

a) $3px^2 - 2px - 3qx + 2q = 0$

$a = 3p \quad b = -(2p + 3q)$

$c = 2q$

$\Delta = b^2 - 4ac$

$= (2p+3q)^2 - 4(3p)(2q)$

$= 4p^2 + 12pq + 9q^2 - 24pq$

$= 4p^2 - 12pq + 9q^2$ (3)

$= (2p - 3q)^2$

$\therefore$ if p and q are rational Δ is a perfect square and the equation has rational roots.

b)

i) $\frac{2t}{1+t^2} \div \left[\frac{1-t^2}{1+t^2} + 1\right]$

$\frac{2t}{1+t^2} \div \left[\frac{1-t^2+1+t^2}{1+t^2}\right]$

$\frac{2t}{1+t^2} \times \frac{1+t^2}{2}$ (2)

$= t$

ii) $\frac{\sin\theta}{\cos\theta + 1} = \sqrt{3}$

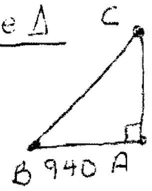
$t = \sqrt{3}$

$\therefore \tan\frac{\theta}{2} = \sqrt{3} \quad 0^\circ \le \frac{\theta}{2} \le 180^\circ$

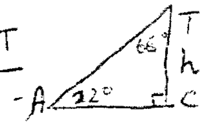
$\frac{\theta}{2} = 60^\circ$

$\theta = 120^\circ$

c) Base Δ



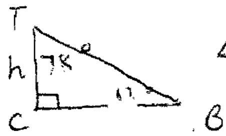
i) ΔACT



$\tan 68^\circ = \frac{AC}{h}$

$AC = h\tan 68^\circ$ *

ΔTCB



$\tan 78^\circ = \frac{BC}{h}$

$BC = h\tan 78^\circ$

(ii) $BC^2 = AC^2 + 940^2$

$h^2\tan^2 78^\circ = h^2\tan^2 68 + 940^2$

$940^2 = h^2\left[\tan^2 78 - \tan^2 68\right]$

$h^2 = \frac{940^2}{\tan^2 78^\circ - \tan^2 68^\circ}$

$h^2 = 55199.1615$

$h = 234.945\ldots$

$\therefore$ height of the tower is 235 m.