SYDNEY TECHNICAL HIGH SCHOOL

EXTENSION 1

MATHEMATICS

PRELIMINARY ASSESSMENT TASK 2

2010

Time Allowed: **70 minutes**

Instructions:

- Write using blue or black pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- PLEASE START EACH NEW QUESTION ON A NEW PAGE

Name: _____ **Teacher:** _____

Q1 /10	Q2 /10	Q3 /10	Q4 /10	Q5 /10	Q6 /10

Total
/60

Marks

Question 1

a) Find the values of k such that $x^2 + (k+1)x + 4 = 0$ has real roots. 3

b) Evaluate $\lim_{x \to 3} \dfrac{2x^2 + x - 21}{x^2 - 3x}$ 2

c) (i) Differentiate $y = \sqrt{2x - 1}$ 2

(ii) Hence find the equation of the normal to the above function at $x = 1$. 3

Question 2

a) If $sinx = \frac{2}{3}$ and $cosy = \frac{1}{9}$, where angles x and y are acute, show

(i) $\cos(x - y) = \frac{\sqrt{5}}{3}$ 2

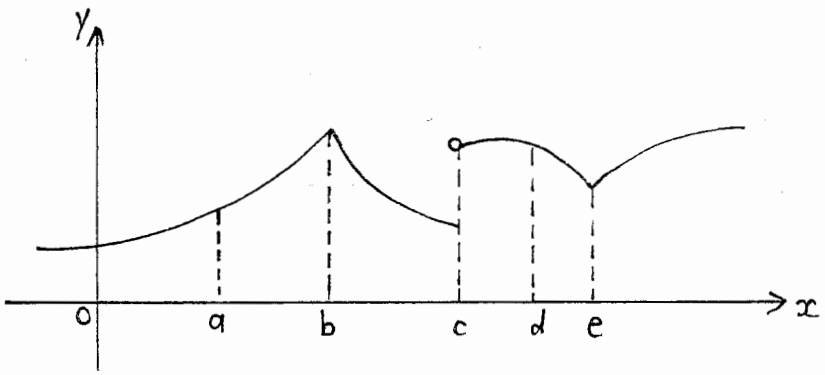
(ii) $cosy - cos2x = 0$ 2

b) Find the acute angle between the lines $x = 2$ and $y = \frac{1}{\sqrt{3}}x + 2$ 2

c) Let $t = \tan\frac{\theta}{2}$ and then use the 't' results to prove 3

$sec\theta - tan\theta = \frac{1}{sec\theta + tan\theta}$

d) 1



For which x value(s) is the function not differentiable?

Question 3 **Marks**

a) If α and β are the roots of the equation $2x^2 - 13x + 9 = 0$,

(i) Show that $(\alpha + 1)(\beta + 1) = 12$ 2

(ii) Find the quadratic equation with integer coefficients which has roots $\alpha + 1$ and $\beta + 1$. 2

b) Show that the line $y = mx - m^2$ is a tangent to the parabola $y = \frac{x^2}{4}$ for all real m. 2

c) Use differentiation by first principles to find the derivative of $f(x) = x^2 - 5x$. 3

d) If the discriminant of a quadratic equation has the value of 36, what conclusion can you make about the 2 distinct real roots of the equation? 1

Question 4

a) (i) Expand $\tan(A + B)$ 1

(ii) Find the exact value of $tan105°$ (do not rationalise denominator) 2

b) Differentiate $y = \frac{1+2x}{1-2x}$ 2

and give your answer as a fully simplified algebraic fraction.

c) If the difference between the roots $2x^2$-$12x + p + 2 = 0$ is equal to 2, find the value of p. 3

d) At what point(s) on the curve $y = x^2 - 2x - 1$ is the tangent parallel to the line $y = x$. 2

Question 5 **Marks**

a) Solve $2cos^2x = sin2x$ for $0 \leq x \leq \pi$. 3

b) Find the ratio in which the point $(\frac{7}{9}, \frac{-2}{9})$ divides the interval joining $(3, -6)$ and $(-2,7)$. 3

c) (i) Express $cosx + \sqrt{3}sinx$ in the form $Acos(x - \alpha)$ where $A > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$. 2

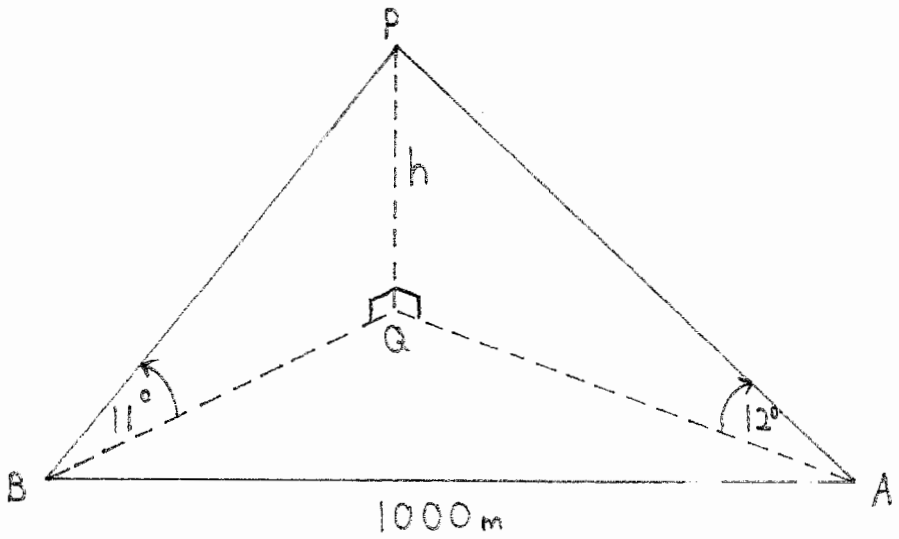
(ii) Hence solve $cosx + \sqrt{3}sinx = 1$ for $-\pi \leq x \leq \pi$. 2

Question 6 **Marks**

a) Solve $9^x - 7(3^x) - 18 = 0$ 2

b) Show that $(p+3)x^2 - px + 1$ can never be negative definite. 3

c)



The angle of elevation of a tower PQ, of height h metres, at a point A due east of it is $12°$. From another point B, the bearing of the tower is $015°T$ and the angle of elevation is $11°$. The points A and B are 1000m apart and on the same level as the base Q of the tower.

(i) By sketching a top view diagram show that $\angle AQB$ is $105°$. 1

(ii) Show that $AQ = h\tan 78$. 1

(iii) Calculate h to the nearest metre. 3

Teacher's Name: Student's Name/N°:

Prelim. Ass. Task 2 - Extension 1 - 2010

Question 1

a) Real roots if $\Delta \geq 0$

$(k+1)^2 - 4 \times 1 \times 4 \geq 0$

$k^2 + 2k + 1 - 16 \geq 0$

$k^2 + 2k - 15 \geq 0$

$(k-3)(k+5) \geq 0$

$k \geq 3, \quad k \leq -5$

b) $\lim_{x \to 3} \dfrac{2x^2 + x - 21}{x^2 - 3x}$

$= \lim_{x \to 3} \dfrac{(2x+7)(x-3)}{x(x-3)}$

$= \dfrac{13}{3}$

c) (a) $y = \sqrt{2x-1}$

$= (2x-1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2$

(ii) $y' = \dfrac{1}{\sqrt{2x-1}}$

At $x = 1$,

$y' = \dfrac{1}{\sqrt{1}} = 1$ (m of tan)

$\therefore m_{normal} = -1$

At $x = 1, \quad y = 1$

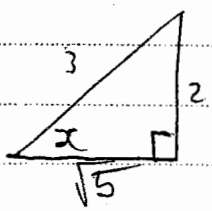
$\therefore y - 1 = -1(x-1)$

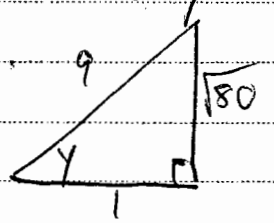
$y = -x + 2$

or $x + y - 2 = 0$

Question 2

a) (i) $\cos(x-y) = \cos x \cos y + \sin x \sin y$





$= \dfrac{\sqrt{5}}{3} \times \dfrac{1}{9} + \dfrac{2}{3} \times \dfrac{\sqrt{80}}{9}$

$= \dfrac{\sqrt{5} + 2\sqrt{80}}{27}$

$= \dfrac{\sqrt{5} + 2 \times 4\sqrt{}}{27}$

$= \dfrac{9\sqrt{5}}{27} = \dfrac{\sqrt{5}}{3}$

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c(ii) $\cos y - \cos 2x$

$= \cos y - (2\cos^2 x - 1)$

$= \frac{1}{9} - (2 \times (\frac{\sqrt{5}}{3})^2 - 1)$

$= \frac{1}{9} - (\frac{10}{9} - 1)$

$= \frac{1}{9} - \frac{1}{9}$

$= 0$

b) $m = \tan\theta$

$\frac{1}{\sqrt{3}} = \tan\theta$

$\theta = 30^\circ$

is angle made with x axis $\therefore$ 60° is the angle made with $x = 2$.

c) $\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$

$\frac{1+t^2}{1-t^2} - \frac{2t}{1-t^2} = \frac{1}{\frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}} \times \frac{1-t^2}{1-t^2}$

$\frac{t^2-2t+1}{1-t^2} = \frac{1-t^2}{1+t^2+2t}$

$\frac{(t-1)^2}{(1-t)(1+t)} = \frac{(1-t)(1+t)}{(t+1)^2}$

$\frac{(1-t)^2}{(1-t)(1+t)} = \frac{1-t}{1+t}$

$\frac{1-t}{1+t} = \frac{1-t}{1+t}$

d) $x = b, c, e$

Question 3

a)(i) $(\alpha+1)(\beta+1)$

$= \alpha\beta + (\alpha+\beta) + 1$

$= \frac{9}{2} + -\frac{13}{2} + 1$

$= 4\frac{1}{2} + 6\frac{1}{2} + 1$

$= 12$

c(ii) $x^2 - (\text{sum of roots}) + \text{produc}$

Sum of roots is

$\alpha + 1 + \beta + 1 = 2 + \frac{13}{2} = 8\frac{1}{2}$

$\therefore x^2 - 8\frac{1}{2}x + 12 = 0$

$2x^2 - 17x + 24 = 0$

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b) Solve simultaneously. Tangent if $\Delta = 0$

$mx - m^2 = \frac{x^2}{4}$

$4mx - 4m^2 = x^2$

$x^2 - 4mx + 4m^2 = 0$

$\Delta = (-4m)^2 - 4 \times 1 \times 4m^2$

$= 16m^2 - 16m^2$

$\Delta = 0$

∴ Line is a tangent

c) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 5(x+h) - x^2 + 5x}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$

$= \lim_{h \to 0} \frac{h(2x + h - 5)}{h}$

$= 2x - 5$

d) They are rational

Question 4

a) (i) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan 105 = \tan(60+45)$

$= \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$

$= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \times 1}$

$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

b) $y = \frac{1+2x}{1-2x}$

$y' = \frac{(1-2x) \times 2 - (1+2x) \times -2}{(1-2x)^2}$

$= \frac{2 - 4x + 2 + 4x}{(1-2x)^2}$

$= \frac{4}{(1-2x)^2}$

c) $2x^2 - 12x + p + 2 = 0$

Let roots be $\alpha, \alpha + 2$

$\alpha + \alpha + 2 = \frac{12}{2}$ $\quad \alpha(\alpha+2) = \frac{p+2}{2}$

$2\alpha + 2 = 6$

$\alpha = 2$ $\quad \therefore 2(2+2) = \frac{p+2}{2}$

$16 = p + 2$

$p = 14$.

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d) $y = x^2 - 2x - 1$

$y' = 2x - 2 = 1$ (gradient of $y = x$)

$2x = 3$

$x = \frac{3}{2}$

$y = (\frac{3}{2})^2 - 2 \times \frac{3}{2} - 1$

$= \frac{9}{4} - 4$

$y = -1\frac{3}{4}$ $\therefore$ Point is $(\frac{3}{2}, -1\frac{3}{4})$

Question 5

a) $2\cos^2 x = \sin 2x$

$2\cos^2 x = 2\sin x \cos x$

$\cos^2 x - \sin x \cos x = 0$

$\cos x(\cos x - \sin x) = 0$

$\cos x = 0$ or $\cos x = \sin x$

$x = \frac{\pi}{2}$ $\tan x = 1$

$x = \frac{\pi}{4}$

over domain $0 \le x \le \pi$

b) $(3, -6)$ $(-2, 7)$ $(\frac{7}{9}, \frac{2}{9})$ $k : l$?

x_1, y_1 x_2, y_2 x, y

$x = \frac{kx_2 + lx_1}{k + l}$

$\frac{7}{9} = \frac{k \times -2 + l \times 3}{k + l}$

$7(k + l) = (-2k + 3l)9$

$7k + 7l = -18k + 27l$

$25k = 20l$

$\therefore k : l$ is $4 : 5$

c(i) $\cos x + \sqrt{3}\sin x$

$A = \sqrt{(\sqrt{3})^2 + 1} = 2$

$\alpha = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore = 2\cos(x - \frac{\pi}{3})$

(ii) $2\cos(x - \frac{\pi}{3}) = 1$

$\cos(x - \frac{\pi}{3}) = \frac{1}{2}$

$x - \frac{\pi}{3} = \frac{\pi}{3}, -\frac{\pi}{3}$

$x = \frac{2\pi}{3}$ or 0.

Question 6

a) $9^x - 7(3^x) - 18 = 0$

$3^{2x} - 7(3^x) - 18 = 0$

$(3^x)^2 - 7(3^x) - 18 = 0$

$(3^x + 2)(3^x - 9) = 0$

$3^x = -2$ or $3^x = 9$

$x = 2$ or $3^x = -2$ no

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d) $y = x^2 - 2x - 1$

$y' = 2x - 2 = 1$ (gradient of $y = x$)

$2x = 3$

$x = \frac{3}{2}$

$y = (\frac{3}{2})^2 - 2 \times \frac{3}{2} - 1$

$= \frac{9}{4} + 3$

$y = 5\frac{1}{4}$ $\therefore$ Point is $(\frac{3}{2}, 5\frac{1}{4})$

Question 5

a) $2\cos^2 x = \sin 2x$

$2\cos^2 x = 2\sin x \cos x$

$\cos^2 x - \sin x \cos x = 0$

$\cos x(\cos x - \sin x) = 0$

$\cos x = 0$ or $\cos x = \sin x$

$x = \frac{\pi}{2}$ $\tan x = 1$

$x = \frac{\pi}{4}$

over domain $0 \leq x \leq \pi$

b) $(3, -6)$ $(-2, 7)$ $(\frac{7}{9}, \frac{2}{9})$ $k : l$?

$x_1\ y_1$ $x_2\ y_2$ $x\ y$

$x = \frac{kx_2 + lx_1}{k + l}$

$\frac{7}{9} = \frac{k \times -2 + l \times 3}{k + l}$

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c(ii) $2\cos(x - \frac{\pi}{3}) = 1$

$\cos(x - \frac{\pi}{3}) = \frac{1}{2}$

$x - \frac{\pi}{3} = \frac{\pi}{3}, -\frac{\pi}{3}$

$x = \frac{2\pi}{3}$ or 0.

Question 6

a) $9^x - 7(3^x) - 18 = 0$

$3^{2x} - 7(3^x) - 18 = 0$

$(3^x)^2 - 7(3^x) - 18 = 0$

$(3^x + 2)(3^x - 9) = 0$

$3^x = -2$ or $3^x = 9$

$x = 2$ or $3^x = -2$ no solution

Teacher's Name: **Student's Name/N°:**

b) $(p+3)x^2 - px + 1$

negative definite

if $p+3 < 0$ and $\Delta < 0$

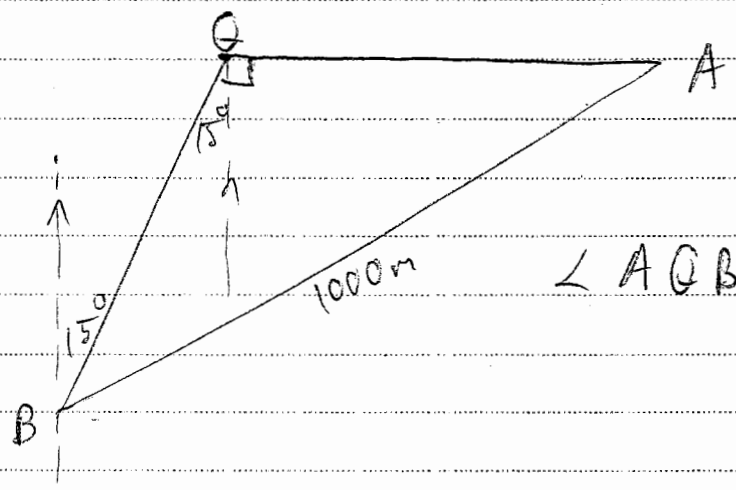
$p < -3$ $\quad p^2 - 4(p+3) < 0$

$p^2 - 4p - 12 < 0$

$(p+2)(p-6) < 0$

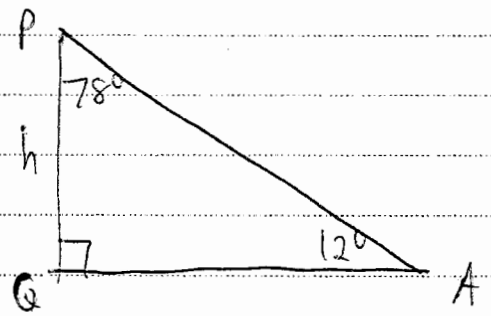
∴ Not possible and so can't be negative definite.

c)



$\angle AQB = 105^\circ$

c(i)



$\tan 78 = \frac{AQ}{h}$

$AQ = h \tan 78$

c(ii) Similarly

$BQ = h \tan 79$. By the cosine rule

$AB^2 = AQ^2 + BQ^2 - 2 \times AQ \times BQ \times \cos 105^\circ$

$1000^2 = h^2 \tan^2 78 + h^2 \tan^2 79 - 2h^2 \tan 79 \tan 78 \cos 105$

$h^2 = \frac{1000000}{\tan^2 78 + \tan^2 79 - 2 \tan 78 \tan 79 \cos 105}$

$h = 128 m$