SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

YEAR 11 PRELIMINARY HSC

ASSESSMENT TASK II

JULY 2011

General Instructions:

- Working time allowed – 70 minutes.
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working should be shown.
- Start each question on a new page.
- Attempt all questions.

NAME: ________________________________

TEACHER: ______________________________

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	TOTAL
/10	/9	/9	/9	/10	/10	/57

Question 1 **Marks**

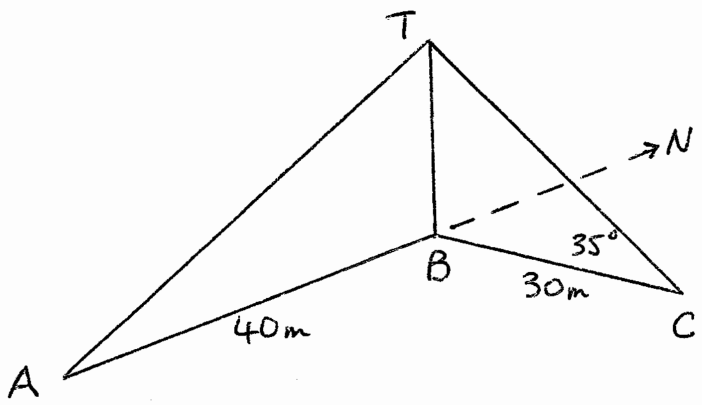
a) i) Convert 40° to radians. 1

ii) Solve $\sin\theta = -\frac{\sqrt{3}}{2}$ $for\ 0 \leq x \leq 2\pi$ 2

b) i) Write the expansion of $\cos(A + B)$. 1

ii) Hence find the value of cos 75° in simplest exact form. 2

c)



Points A,B,C are on level, horizontal ground.

A vertical tower BT is observed due north of A at a distance of 40 m. It is also observed from C, 30 m away and on a bearing of 050° from B, with an angle of of elevation of 35°.

i) Find BT. 1

ii) Find ∠BAT (to 1 dec. place). 1

iii) Find AC (to 1 dec. place). 2

Question 2 (start a new page)

a) Solve $\cos 2\theta = \cos\theta \quad for \quad 0° \leq \theta \leq 360°$. 3

b) If $\sin\theta = 0.3$, where θ is acute, find without a calculator, the value of $\sin 2\theta$. 2

Leave your answer in exact form.

c) Use the "t results" to :

i) solve $\cos\theta - \sin\theta = 1$ for $0° \leq \theta \leq 360°$, 3

ii) find the exact value of $\dfrac{2\tan 22.5°}{1+tan^2 22.5°}$ 1

Question 3 (start a new page)

a) Given that $3\,sin\,\theta - 2\cos\theta = A\sin(\theta - \alpha)$,

i) find A and α. 2

ii) Hence, solve $3\sin\theta - 2\cos\theta = 1$ for $0° \leq \theta \leq 360°$. 2

(answer to 1 decimal place)

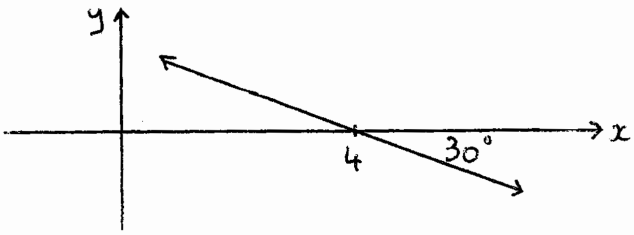
iii) Find the first positive θ such that $3\sin\theta - 2\cos\theta$ has a maximum value. 1

b) i) Write the expansion of $\tan 2\theta$. 1

ii) Hence, find the value of $\tan 22.5°$ in simplest exact form. 3

Question 4 (start a new page)

a) Find the equation of this line: 2



b) Point P divides the interval A(-2, 3), B(6, -1) externally in the ratio 1:3.

i) Show the relative positions of A, B, P using the given ratio. There is no need to plot A and B on a number plane. 1

ii) Find the coordinates of P. 2

c) Find the acute angle, correct to 1 decimal place, between:

i) the lines $y = 3x + 2$ and $2x + 2y - 5 = 0$, 2

ii) the line $y = 3x + 2$ and the y axis. 2

Question 5

a) Given $A(0,0)$, $B(4,-2)$, $C(3,3)$ and $D(9,7)$. P is the midpoint of AB and Q is the midpoint of CD.

i) Find the equation of the perpendicular bisector of PQ. 3

Give your answer in general form.

ii) Find the coordinates of a point E such that ABCE is a parallelogram. 1

b) Differentiate: i) $y = \frac{1}{\sqrt{4x-2}}$ 2

ii) $y = \frac{2x+1}{(4x-9)^4}$ Leave your answer fully simplified. 3

c) Evaluate $\lim_{x \to \infty} \frac{x^2-5x+2}{2x^2+3x}$ 1

Question 6 (start a new page)

a) Explain the essential geometrical distinction between $\frac{f(x+h)-f(x)}{h}$ 1

and $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

b) Use first principles to find the derivative of $y = \frac{1}{x^2}$. 3

c) Find the equation of the normal to the curve $y = \frac{1}{2x}$ at the point where $x = 1$. 3

d) Find the values of k such that the line $3x - 4y + k = 0$ intersects twice with the circle $(x-4)^2 + y^2 = 4$. 3

END OF TEST

SOLUTIONS

(1) a) i) $\frac{40\pi}{180} = \underline{\underline{\frac{2\pi}{9}}}$

ii) $\theta = 60°$ (3rd, 4th quads)

$= 240°, 300°$

$= \underline{\underline{\frac{4\pi}{3}, \frac{5\pi}{3}}}$

b) i) $\cos A \cos B - \sin A \sin B$

ii) $\cos 75 = \cos(45+30)$

$= \cos 45 \cos 30 - \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

c) i) $\tan 35 = \frac{BT}{30}$

$BT = 30 \tan 35$

$= \underline{\underline{21}}$

ii) $\tan A = \frac{30 \tan 35}{40}$

$\therefore \underline{\underline{A = 27.7°}}$

iii) $AC^2 = 40^2 + 30^2 - 2 \times 40 \times 30 \times \cos 130°$

$= 4042.7$

$\therefore \underline{\underline{AC = 63.6\text{ m}}}$.

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(2) a) $2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$

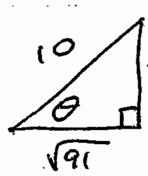
$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ or 1

$\theta = 60°$ (2nd, 3rd quads), $0°, 360°$

$= \underline{\underline{120°, 240°, 0°, 360°}}$

b) $\sin 2\theta = 2\sin\theta\cos\theta$



$= 2 \times \frac{3}{10} \times \frac{\sqrt{91}}{10}$

$= \frac{6\sqrt{91}}{100}$

or $\frac{3\sqrt{91}}{50}$

or $0.6\sqrt{0.}$

c) i) $\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} = 1$

$\therefore \cancel{1} - t^2 - 2t = \cancel{1} + t^2$

$\therefore -2t^2 - 2t = 0$

$-2t(t+1) = 0$

$t = 0$ or -1

$\tan\frac{\theta}{2} = 0$ or -1

$\frac{\theta}{2} = 0°, 180°, \cancel{360°}, 135°, \cancel{315}$

$\therefore \underline{\underline{\theta = 0°, 360°, 270°}}$

ii) $\frac{2t}{1+t^2} = \sin\theta$ $\left(t = \tan\frac{45}{2}\right)$ $\theta = 45°$

$= \sin 45$

$= \underline{\underline{\frac{1}{\sqrt{2}}}}$

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3) a) i) $\underline{\underline{A = \sqrt{13}}}$

$3\sin\theta - 2\cos\theta = \sqrt{13}\sin(\theta - \alpha)$

$\frac{3}{\sqrt{13}}\sin\theta - \frac{2}{\sqrt{13}}\cos\theta = \sin(\theta - \alpha)$

$= \sin\theta\cos\alpha - \cos\theta\sin\alpha$

$\therefore \left.\begin{aligned}\cos\alpha &= \frac{3}{\sqrt{13}} \\ \sin\alpha &= \frac{2}{\sqrt{13}}\end{aligned}\right\}$ 1st quad. $\underline{\underline{\alpha = 33.7°}}$

ii) $\sqrt{13}\sin(\theta - 33.7°) = 1$

$\sin(\theta - 33.7°) = \frac{1}{\sqrt{13}}$

$\therefore \theta - 33.7 = 16.1°$ (1st, 2nd quad.)

$= 16.1°$ or $163.9°$

$\therefore \underline{\underline{\theta = 49.8° \text{ or } 197.6°}}$

iii) max. value is when $\sin(\theta - 33.7°) = 1$

$\therefore \theta - 33.7° = 90°$

$\therefore \underline{\underline{\theta = 123.7°}}$

b) i) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

ii) $\tan 45° = \frac{2\tan 22.5°}{1-\tan^2 22.5°}$

$1 - \tan^2 22.5 = 2\tan 22.5$

$\tan^2 22.5 + 2\tan 22.5 - 1 =$

$\tan 22.5 = \frac{-2 \pm \sqrt{4+4}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

But tan (acute) > 0

$\therefore \underline{\underline{\tan 22.5° = -1 + \sqrt{2}}}$

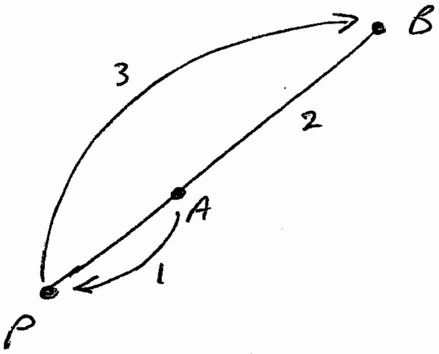
(4) a) $m = \tan 150^\circ$

$= -\frac{1}{\sqrt{3}}$

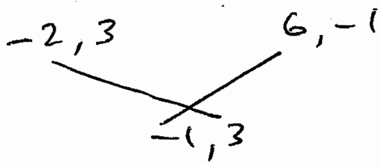
$\therefore$ eqn. of line is $y - 0 = -\frac{1}{\sqrt{3}}(x - 4)$

$\therefore \underline{\underline{y = \frac{-x}{\sqrt{3}} + \frac{4}{\sqrt{3}}}}$ (or $x + \sqrt{3}y - 4 = 0$)

b) i)



ii)



$x = \frac{-6-6}{2}$, $y = \frac{9+1}{2}$

$\therefore \underline{\underline{P \text{ is } (-6, 5)}}$

c) i) $m_1 = 3$, $m_2 = -1$

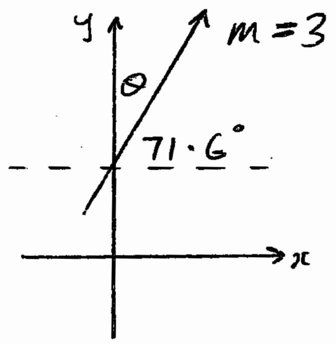
$\tan\theta = \left|\frac{3-(-1)}{1+3(-1)}\right|$

$= \left|\frac{4}{-2}\right|$

$= 2$

$\therefore \underline{\underline{\theta = 63.4^\circ}}$

ii)



$\therefore \underline{\underline{\theta = 18.4^\circ}}$

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(5) a) $P = (2, -1)$

$Q = (6, 5)$

grad. of PQ, $m_1 = \frac{6}{4} = \frac{3}{2}$

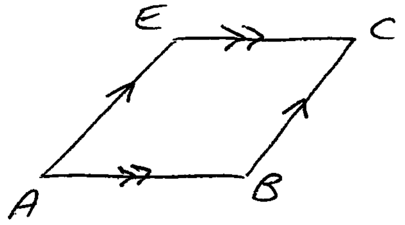
$\therefore m_2 = -\frac{2}{3}$ and use m.p. $(4, 2)$

$\therefore$ eqn. perp. bisector is $y - 2 = -\frac{2}{3}(x - 4)$

$3y - 6 = -2x + 8$

$\therefore \underline{\underline{2x + 3y - 14 = 0}}$.

ii) E is $\underline{\underline{(-1,5)}}$



b) i) $y = (4x-2)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = -\tfrac{1}{2}(4x-2)^{-\frac{3}{2}} \times 4$$

$$= \underline{\underline{-2(4x-2)^{-\frac{3}{2}}}}$$

(or equiv.)

ii) $$\frac{dy}{dx} = \frac{2(4x-9)^4 - 4(4x-9)^3 \times 4 \times (2x+1)}{(4x-9)^8}$$

$$= \frac{2(4x-9)^4 - 16(2x+1)(4x-9)^3}{(4x-9)^8}$$

$$= \frac{2(4x-9) - 16(2x+1)}{(4x-9)^5}$$

$$= \frac{8x - 18 - 32x - 16}{(4x-9)^5}$$

$$= \underline{\underline{\frac{-24x - 34}{(4x-9)^5}}}$$

c) $$\lim_{x\to\infty} \frac{1 - \frac{5}{x} + \frac{2}{x^2}}{2 + \frac{3}{x}}$$

$$= \underline{\underline{\frac{1}{2}}}.$$

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6 a) First is gradient of secant (2 points)
Second is gradient of curve, or tangent to curve (1 point)

b) $$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{x^2 - (x+h)^2}{x^2(x+h)^2} \times \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{x^2(x+h)^2} \times \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x - h)}{x^2(x+h)^2} \times \frac{1}{\cancel{h}}$$

$$= \frac{-2x}{x^4} = \frac{-2}{x^3}$$

c) $y = \frac{1}{2}x^{-1}$

$$\frac{dy}{dx} = -\frac{1}{2}x^{-2}$$

$$= \frac{-1}{2x^2}$$

when $x = 1$, $m_T = -\frac{1}{2}$

$m_N = 2$

and point is $(1, \frac{1}{2})$

$\therefore$ eqn. of normal is $y - \frac{1}{2} = 2(x-1)$

$$= 2x - 2$$

$$\therefore \underline{\underline{y = 2x - 1\tfrac{1}{2}}}$$

d) p.d. of $3x - 4y + k = 0$ from $(4, 0) < 2$ units (radius)

$$\therefore \frac{|3 \times 4 + 0 + k|}{\sqrt{3^2 + 4^2}} < 2$$

$$\left|\frac{12 + k}{5}\right| < 2$$

$$|12 + k| < 10$$

$12 + k < 10$ or $-12 - k < 10$

$k < -2$ or $-k < 22$

$k > -22$

$$\therefore \underline{\underline{-22 < k < -2}}$$