SYDNEY TECHNICAL HIGH SCHOOL

EXTENSION 1

MATHEMATICS

PRELIMINARY ASSESSMENT TASK 2

JULY 2012

Time Allowed: **70 minutes**

Instructions:

- Write using blue or black pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- **PLEASE START EACH NEW QUESTION ON A NEW PAGE.**

Name:________________________ **Teacher:** ________________________

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
/10	/10	/10	/10	/10	/10	/60

Question 1 (10 marks)

a) What is the value of $tan(\frac{-3\pi}{4})$? 1

b) Find the exact value of $\cos 105^o$ 2

c) Find the coordinates of the point which divides the interval AB, with A(1, 4) and B(5, 2), externally in the ratio 1 : 3. 2

d) Differentiate

i. $\frac{4}{\sqrt{x}}$ 1

ii. $(x^2 - 1)^{10}$ 2

iii. $\frac{x}{x+1}$ 2

Question 2 START A NEW PAGE (10 marks)

a) If $\sin\alpha = \frac{3}{4}$, $0 < \alpha < 90^o$ and $\sin\beta = \frac{2}{3}$, $90^o < \beta < 180^o$, find the exact values of :

i. $\tan 2\alpha$ 2

ii. $\sin(\alpha - \beta)$ 2

b) Find $\frac{dy}{dx}$ if $y = 2x(x+4)^8$ 2

c) Solve the equation $\sin 2x = \tan x$ for $0 \le x \le \pi$ 4

Question 3 START A NEW PAGE (10 MARKS)

a) If α and β are the roots of $3x^2 - 4x + 8 = 0$, find the value of :

i. $\alpha\beta$ 1

ii. $(\alpha - \beta)^2$ 2

b)

i. Show that $\frac{1+\cos 2A}{\sin 2A} = \cot A$ 2

ii. Hence, find the exact value of $\cot 15^\circ$ 1

c) The curve $y = ax^3 + bx$ cuts the x axis at $x = 1$ and the gradient of the tangent at this point is 4.

i. Find the values of a and b 2

ii. Find the acute angle between the tangent at $x = 1$ and the line $4x - 3y + 2 = 0$ 2

Question 4 START A NEW PAGE (10 MARKS)

a) For the function $y = \frac{1}{1+x^2}$

i. Find the derivative 2

ii. Determine the equation of the normal at the point where $y = \frac{1}{2}$ and $x < 0$ 3

iii. Where does this normal cut the x axis? 1

b) Given $\tan\frac{\theta}{2} = t$

i. Express $\sec\theta + \tan\theta$ in terms of t
(express your answer in its simplest form) 2

ii. Hence, or otherwise, solve $\sec\theta + \tan\theta = 2$, correct to nearest degree for $0^o < \theta < 90^o$ 2

Question 5 START A NEW PAGE (10 MARKS)

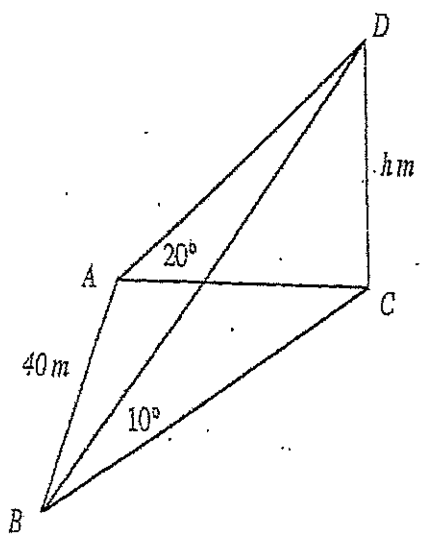
a) Show that $(\sin A - \cos A)^2 = 1 - \sin 2A$ 2

b)

i. Express $\cos\theta - \sqrt{3}\sin\theta$ in the form $R\cos(\theta + \alpha)$ for $R > 0$ 2

ii. Hence, or otherwise, solve $\cos\theta - \sqrt{3}\sin\theta = 1$ for $0 \le \theta \le 2\pi$ 2

c) A vertical flagpole CD of height *h* metres, stands with its base *C* on horizontal ground. *A* is a point on the ground due West of *C* and *B* is a point on the ground *40* metres due South of *A*. From *A* and *B* the angles of elevation of the top *D* of the flagpole are 20° and 10° respectively.



i. Use triangle ADC to show that AC = $h\tan 70^\circ$ 1

ii. Write a similar expression for BC 1

iii. Find the height of the flagpole correct to the nearest metre. 2

Question 6 START A NEW PAGE (10 MARKS)

a) For what value (or values) of k, will the quadratic equation

$(k+4)x^2 - 3kx - 4(k-2) = 0$ have two roots which differ by 1.

(hint let the roots be α and $\alpha + 1$) 4

b)

i. State the conditions for the quadratic expression $ax^2 + bx + c$ to be negative definite. 1

ii. Hence, or otherwise, show that the expression $(k^2 + k)x^2 - (2k-6)x + 2$ can never be negative definite 3

iii. Find the range of values of k for which the expression is positive definite 2

End of examination

Student Name: Ext 1 Prelim July Teacher Name: 2012 ANSWERS

Question 1

a) $\tan\left(\frac{-3\pi}{4}\right)$

$= \tan(-135^\circ)$

$= 1$ ✓

b) $\cos 105^\circ$

$\cos(60^\circ + 45^\circ)$ ✓

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$ ✓

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

c) $(1, 4)\ (5, 2)$

$-1 : 3$

$pt = \left(\frac{3 + -5}{2}, \frac{12 + -2}{2}\right)$

$= (-1, 5)$ ✓✓

d) $\frac{d}{dx}\left(\frac{4}{\sqrt{x}}\right) = -2x^{-3/2}$

$= \frac{-2}{\sqrt{x^3}}$ ✓

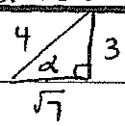
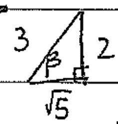
II. $\frac{d}{dx}(x^2 - 1)^{10}$

$= 20x(x^2 - 1)^9$ ✓✓

III. $\frac{dy}{dx} = \frac{(x+1)(1) - (x)(1)}{(x+1)^2}$

$= \frac{1}{(x+1)^2}$ ✓✓

Question 2

a)  

1st quad 2nd quad

I. $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{2 \times 3/\sqrt{7}}{1 - 9/7}$

$= \frac{6}{\sqrt{7}} \div \frac{-2}{7}$

$= -3\sqrt{7}$ ✓✓

II. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$= \frac{3}{4} \times \frac{-\sqrt{5}}{3} - \frac{2}{3} \times \frac{\sqrt{7}}{4}$

$= \frac{-3\sqrt{5} - 2\sqrt{7}}{12}$ ✓✓

b) $\frac{dy}{dx} = vu' + uv'$

$= (x+4)^8 . 2 + 2x\left[8(x+4)^7\right]$

$= 2(x+4)^7(x + 4 + 8x)$

$= 2(9x + 4)(x+4)^7$ ✓✓

c) $\sin 2x = \tan x$ 

$2\sin x \cos x = \frac{\sin x}{\cos x}$ ✓

$2\sin x \cos^2 x - \sin x = 0$

$\sin x\,(2\cos^2 x - 1) = 0$ ✓

$\sin x = 0 \qquad \cos^2 x = \frac{1}{2}$

 $\cos x = \pm\frac{1}{\sqrt{2}}$

✓

$x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}$

ie

$x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$ ✓

Student Name: ______________________ Teacher Name: ______________________

Question 3

a) $3x^2-4x+8=0$

i. $\alpha\beta = c/a = 8/3$ ✓

ii. $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$ ✓

$= (4/3)^2 - 4(8/3)$

$= -8\,8/9$ ✓

b) LHS $= \dfrac{1+\cos 2A}{\sin 2A}$

$= \dfrac{1+(2\cos^2 A - 1)}{2\sin A\cos A}$ ✓

$= \dfrac{2\cos^2 A}{2\sin A\cos A}$

$= \dfrac{\cos A}{\sin A}$ ✓

$= \cot A$

$=$ RHS

ii. $\cot 15° = \dfrac{1+\cos 30°}{\sin 30°}$

$= \dfrac{1+\sqrt{3}/2}{1/2}$

$= 2+\sqrt{3}$ ✓

c) $y = ax^3 + bx$ — ① $(1,0)$ $M_T = 4$

$y' = 3ax^2 + b$ — ②

at $(1,0)$ $y' = 4$

$\therefore$ $0 = a + b$ and

$4 = 3a + b$ $-$

$2a = 4$

$a = 2$ ✓

$b = -2$ ✓

ii $4x - 3y + 2 = 0$

$3y = 4x + 2$

$y = \frac{4}{3}x + 2/3$

$M_1 = 4$, $M_2 = 4/3$

$\tan\alpha = \left|\dfrac{M_1 - M_2}{1 + M_1 M_2}\right|$ ✓

$= \left|\dfrac{4 - 4/3}{1 + 4\times 4/3}\right|$

$\alpha = 22°50'$ ✓

Student Name: ____________ Teacher Name: ____________

Question 4.

a) i. $y' = -1(x^2+1)^{-2} \cdot 2x$

$= \dfrac{-2x}{(x^2+1)^2}$ ✓✓

ii. when $y = 1/2$ and $x < 0$

$\frac{1}{2} = \frac{1}{1+x^2}$ then $x = -1$

then $M_T = 1/2$ ✓

$M_N = -2$ ✓

point $(-1, \frac{1}{2})$ and $M_N = -2$

$y - \frac{1}{2} = -2(x+1)$

$2y - 1 = -4(x+1)$

$2y - 1 = -4x - 4$ ✓

$4x + 2y + 3 = 0$

iii. x-int $y = 0$ $\therefore x = -3/4$

b) $\tan\frac{\theta}{2} = t$

$\cos\theta = \dfrac{1-t^2}{1+t^2}$

$\tan\theta = \dfrac{2t}{1-t^2}$

$\therefore \sec\theta + \tan\theta$

$= \dfrac{1+t^2}{1-t^2} + \dfrac{2t}{1-t^2}$ ✓

$= \dfrac{1+2t+t^2}{1-t^2}$

$= \dfrac{(1+t)^2}{(1-t)(1+t)}$ $t \neq \pm 1$

$= \dfrac{1+t}{1-t}$ ✓

ii. Now $\sec\theta + \tan\theta = 2$

$\dfrac{1+t}{1-t} = 2$

$1 + t = 2 - 2t$

$3t = 1$

$t = 1/3$ ✓

$\therefore \tan\theta/2 = 1/3$

$\theta/2 = 18°26'$

$\theta = 36°52'$

$\doteqdot 37°$ ✓

Question 5

a. $(\sin A - \cos A)^2 = 1 - \sin 2A$

$LHS = (\sin A - \cos A)^2$

$= \sin^2 A - 2\sin A\cos A + \cos^2 A$

$= 1 - 2\sin A \cos A$

$= 1 - \sin 2A$

$= RHS.$ ✓✓

b) $\cos\theta - \sqrt{3}\sin\theta$

$R = \sqrt{1^2 + \sqrt{3}^2}$

$= \sqrt{4}$

$= 2$ ✓

$\tan\alpha = \frac{\sqrt{3}}{1}$

$\alpha = \frac{\pi}{3}$ ✓

$\therefore \cos\theta - \sqrt{3}\sin\theta = 2\cos(\theta + \frac{\pi}{3})$

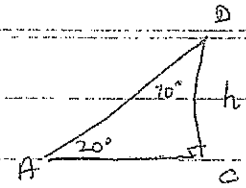
ii. Now $2\cos(\theta + \frac{\pi}{3}) = 1$

$\cos(\theta + \frac{\pi}{3}) = \frac{1}{2}$ ✓

$\theta + \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$\theta = 0, \frac{4\pi}{3}, 2\pi$ ✓

c. i. ΔADC

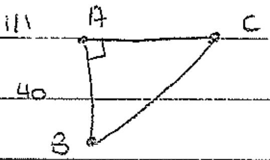


$\tan 70° = \frac{AC}{h}$

$AC = h\tan 70°$ ✓

ii. $BC = h\tan 80°$ ✓

iii.



$BC^2 = 40^2 + AC^2$

$h^2\tan^2 80° = 1600 + h^2\tan^2 70°$

$h^2(\tan^2 80° - \tan^2 70°) = 1600$ ✓

$h^2 = \frac{1600}{\tan^2 80° - \tan^2 70°}$

$h^2 = 65.0015...$

$h = 8.0623....$

height = 8 metres ✓

Question 6

a) $(K+4)x^2 - 3Kx - 4(K-2) = 0$

$2\alpha + 1 = \dfrac{3K}{K+4}$

$\alpha = \dfrac{K-2}{K+4}$ — ①

$\alpha(\alpha+1) = -\dfrac{4(K-2)}{K+4}$ — ②

Sub ① into ②

$\dfrac{(K-2)^2}{(K+4)^2} + \dfrac{(K-2)}{K+4} = \dfrac{-4K+8}{K+4}$

$(K-2)^2 + (K-2)(K+4) = -4(K-2)(K+4)$

$(K-2)^2 = -5(K-2)(K+4)$

$K = 2$ is a sol^n

also $K - 2 = -5(K+4)$

$K - 2 = -5K - 20$

$6K = -18$

$K = -3$

$\therefore K = 2$ or $K = -3$

b) i Negative definite

$a < 0,\ \Delta < 0$

ii. $\Delta = b^2 - 4ac$

$(2K-6)^2 - 4(K^2+K)(2) < 0$

$4K^2 - 24K + 36 - 8K^2 - 8K < 0$

$-4K^2 - 32K + 36 < 0$

$K^2 + 8K - 9 \not> 0$

$(K+9)(K-1) \not> 0$

$K < -9,\ K > 1$

but $K^2 + K < 0$

$K(K+1) < 0$

$-1 < K < 0$

with conclusion

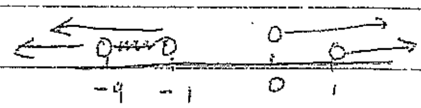
$\therefore$ can't be neg definite as $a < 0$, $\Delta < 0$ have no common values of k.

iii. Now $a > 0$ $\Delta < 0$

ie $K^2 + K > 0$

$K < -1,\ K > 0$

with $\Delta < 0$



$\therefore K < -9,\ K > 1$ is solution for positive definite