Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 Preliminary Course

Extension 1 Mathematics

Assessment 2
July 2013

TIME ALLOWED: 75 minutes

Instructions:

- Write your name and class at the top of this page, and on your answer booklet.
- Both this question sheet and your answer booklet must be handed in.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated within each question are a guide only and may be varied at the time of marking
- Approved calculators may be used.
- PART A is worth 5 marks, and is to be answered on the multiple choice answer sheet, which is the first page in your answer booklet. It should take about 7 minutes. DO NOT DETACH IT.
- PART B is to be written in the answer booklet provided. ***Start each question on a new page.***

PART A: (5 Marks)

Answers to these multiple choice questions are to completed using the multiple choice answer sheet, which is the first page of your answer booklet

All questions are worth 1 mark

1	The function $f(x) = \frac{3x}{16+x^2}$ is: A. Odd B. Even C. Neither D. Cannot be determined
2	The line through the point (2, -5) and parallel to $3x - 2y = 6$ is: A. $3x - 2y - 16 = 0$ B. $3x + 2y + 4 = 0$ C. $2x - 3y - 19 = 0$ D. $2x + 3y + 11 = 0$
3	$sin^2 x cos^2 x =$ A. $\frac{1}{4} sin^2 \frac{x}{2}$ B. . $\frac{1}{4} sin^2 2x$ C. $4sin^2 \frac{x}{2}$ D. $4sin^2 2x$
4	The solution to $\frac{2}{x-1} \leq 1$ is: A. $1 \leq x \leq 3$ B. $x \leq 1$ or $x \geq 3$ C. $1 < x \leq 3$ D. $x < 1$ or $x \geq 3$
5	If $f(x) = x^2 + 2$ and $g(x) = \sqrt{x-2}$ which of the following is true? A. $f\{g(x)\} = x$ B. $g\{f(x)\} = x$ C. Both are true D. Neither is true

PART B

(START EACH QUESTION ON A NEW PAGE)

QUESTION 6: (10 Marks)

Marks

(a) Find the exact value of $cos\frac{\pi}{6}$ **1**

(b) If $2sinx = \sin(x + 90^o)$ find the value of $\tan x$ **2**

(c) (i) Fully factorise: $x^3 - 4x^2 - 16x + 64$ **1**

(ii) A curve is defined by $y \begin{cases} = \dfrac{x^3-4x^2-16x+64}{x-4}, & x \neq 4 \\ = 1, & x = 4 \end{cases}$

Sketch this curve **1**

(d) Find the acute angle between the lines x - 2y = 1 and x = 2, giving your answer to the nearest minute. **2**

(e) If $cosx = \frac{3}{5}$, $0 < x < 90^o$ and $siny = \frac{7}{25}$ $90^o < y < 180^o$ find the exact value of $\cos(x - y)$, giving your answer as a fraction. **3**

QUESTION 7: (9 Marks) Start a New Page

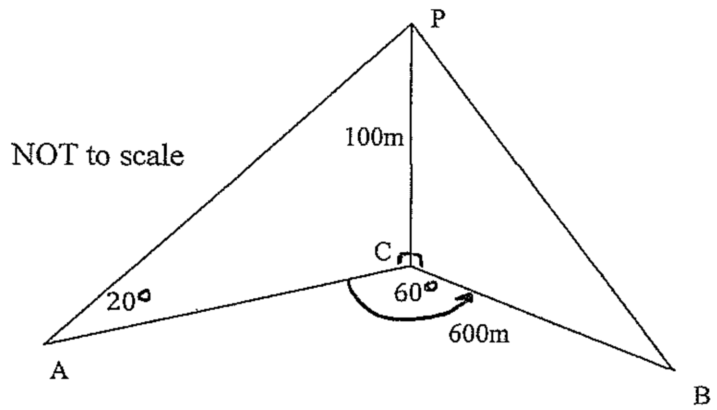
Marks

(a) Solve the inequality $x - 1 > \frac{2}{x}$ **2**

(b) Find the distance between the parallel lines: **2**

$2x - y + 2 = 0$ and $2x - y = 5$

(c)



Two yachts A and B subtend an angle of 60^o at the base of a cliff, C.
From yacht A, the angle of elevation of the point P, 100m vertically above C, is 20^o
Yacht B is 600m from C.

(i) Calculate the length AC to the nearest metre. **2**

(ii) Calculate the distance between the two yachts, to the nearest metre **2**

(d) Find $\frac{d}{dx} \; 3\sqrt{x}$ **1**

QUESTION 8: (9 Marks) Start a New Page

Marks

(a) (i) M is the point which divides the interval joining A (-1, 3) to B (6, 17) in the ratio 5 : 2. Find the co-ordinates of M. **1**

(ii) Find the equation of the line passing through M which is perpendicular to the line joining the points C (-1, 3) and D (5, 6) **3**

Give your answer in general form.

(b) (i) Prove that $\sin(x + y) + \sin(x - y) = 2\sin x\cos y$ **1**

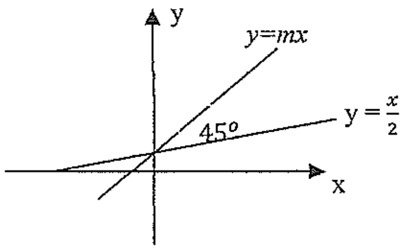
(ii) Hence find the exact value of $\sin 75^o + \sin 15^o$ **2**

(c) Find $\lim_{h \to 0} \left[\frac{(3+h)^2 - 9}{h}\right]$ **2**

QUESTION 9: (9 Marks) Start a New Page

Marks

(a) The angle between the 2 lines shown below is 45^o. **2**



Find the value of m.

(b) Find the co-ordinates of the point which divides the line joining the points (-2, 7) to (1, 5) externally in the ratio 2:1. **2**

(c) Shade the region which solves both of the following inequalities simultaneously: **4**

$$(x-2)^2 + (y+2)^2 \leq 4 \quad \text{and} \quad x - y - 4 \leq 0$$

(d) Differentiate with respect to x: **1**

$$\frac{4x^3 - x}{x^2}$$

QUESTION 10: (9 Marks) Start a New Page

Marks

(a) (i) Express $\sqrt{3}cosx + sinx$ in the form $Rcos(x - \alpha)$ where $0^o \leq \alpha \leq 90^o$ **2**

(ii) Hence, or otherwise, solve $\sqrt{3}cosx + sinx = 1$ for $0^o \leq x \leq 360^o$ **2**

(b) The circle $(x-2)^2 + y^2 = 1$ has its centre at Q

(i) Give the coordinates of Q **1**

(ii) Sketch the circle **1**

(iii) P is a point on the circle in the 1st quadrant so that the angle POQ is a maximum. (O is the origin).
What type of line is OP with respect to the circle? **1**

(iv) Find the size of the angle POQ. **2**

QUESTION 11: (9 Marks) Start a new page.

Marks

(a) Differentiate $f(x) = x^2 + x$ using the method of First Principles. **3**

(b) Find the exact value of $cos\frac{\pi}{12}$ using the expansion for cos $2x$ **2**

DO NOT EVALUATE THIS.

(c) (i) By finding their perpendicular distances to the origin, or otherwise, show that the lines $x + y = 1$ and $x + 7y - 5 = 0$ are both tangents to the same circle. **3**

(ii) Give the equation of this circle. **1**

YEAR 11 PRELIMINARY COURSE 2013

SOLUTIONS — ASSESSMENT 2 – JULY.

EXTENSION 1

PART A

1/ A   2/ A   3/ B   4/ D   5/ C

PART B

QUESTION 6:

(a) $\sqrt{3}/2$ — 1 MARK

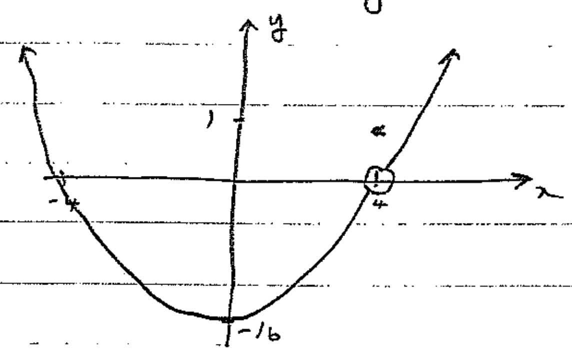
(b) $2\sin x = \cos x$ — 1 for converting $\sin(x+90^\circ)$

$\therefore \tan x = \frac{1}{2}$ — 1 for answer.

(c) (i) $x^2(x-4) - 16(x-4)$

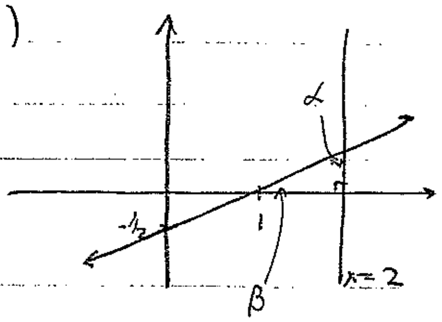
$= (x-4)^2(x+4)$ — 1 MARK or $(x-4)(x-4)(x+4)$

(ii) Line becomes $y = (x-4)(x+4)$ except at $x = 4$



1 for sketch. (everything must be correct!)

(d)



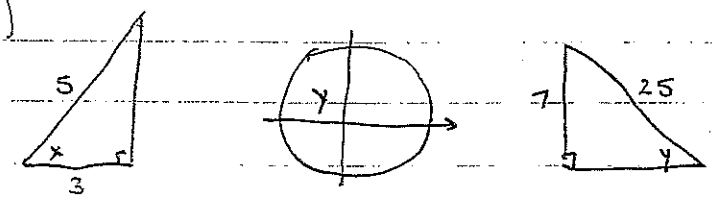
$\tan\beta = \frac{1}{2}$

$\beta = 26^\circ 34'$ — 1 for β

$\alpha = 90 - \beta$

$= 63^\circ 26'$ — 1 for α.

(e)



$\cos(x-y) = \cos x\cos y + \sin x \sin y$ — 1 for $\cos y$ $\left(-\frac{24}{25}\right)$

$= \frac{3}{5}\left(-\frac{24}{25}\right) + \frac{4}{5}\left(\frac{7}{25}\right)$ — 1 for $\sin x$ $\left(\frac{4}{5}\right)$

$= \frac{-72+28}{125}$

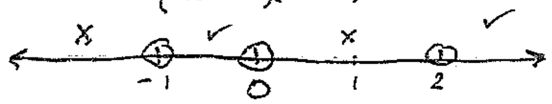
$= -\frac{44}{125}$ — 1 for answer.

QUESTION 7:

(a) C.V. $x \neq 0$

The equation is: $x^2 - x = 2$

$(x-2)(x+1) = 0$



$-1 < x < 0$ or $x > 2$

1 for each region = 2 MARKS.

(b) A point on $2x - y + 2 = 0$ is $(0, +2)$.

$$p = \left| \frac{2.0 - 1(+2) - 5}{\sqrt{5}} \right|$$

$= 7/\sqrt{5}$ units

1 for method

1 for answer.

(c) (i)

$$AC/100 = \frac{1}{\tan 20^\circ}$$ ← ①

$AC = 274.75\text{m}$ ← ①

(ii) $AB^2 = AC^2 + 600^2 - 2 \times 600 \times AC \cos 60^\circ$ ← ①

$\therefore AB^2 = (274.75)^2 + 36000 - 1200(274.75)(1/2)$

$\therefore AB = 520.28\text{ m}$ ← ①

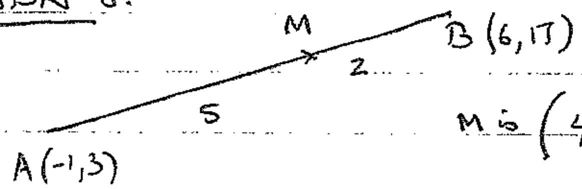
(d) $\frac{d}{dx} 3\sqrt{x} = 3(1/2)x^{-1/2}$

$$= \frac{3}{2\sqrt{x}}$$

1 for either but no penalty for getting the simplification wrong if the first answer is correct.

QUESTION 8:

(a)(i)



M is (4,13)

OR

$$M = \left(\frac{6.5 + (-1)2}{7}, \frac{17.5 + 3.2}{7} \right)$$

$$= (4,13)$$

1 MARK for answer, by any method.

(ii) $m_{OB} = 3/6 = 1/2$

$\therefore m_{perpendicular} = -2.$ ← 1 MARK

$\therefore$ Line is:

$$y - 13 = -2(x-4)$$

$$2x + y - 21 = 0$$

(2) MARKS (less 1 if not in general form)

(b) (i) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$

1 MARK.

(ii) In the above, let $x = 45°, y = 30°$ — 1 MARK

$$\therefore \sin 75° + \sin 15° = 2 \sin 45 \cos 30°$$

$$= 2(1/\sqrt{2})(\sqrt{3}/2)$$

$$= \sqrt{3}/\sqrt{2} \text{ OR } \sqrt{6}/2$$

1 MARK for either

(c) $\lim_{h \to 0} \left[\frac{(3+h)^2 - 9}{h} \right]$

$$= \lim_{h \to 0} \left[\frac{9 + 6h + h^2 - 9}{h} \right]$$

$$= \lim_{h \to 0} (6 + h)$$

$$= 6$$ ← (1)

(1) MARK for keeping the "$\lim_{h \to 0}$" until the end and in the correct spot (ie not before the =)

QUESTION 9:

(a). $m_1 = m \quad m_2 = 1/2$ — ① MARK for ½.

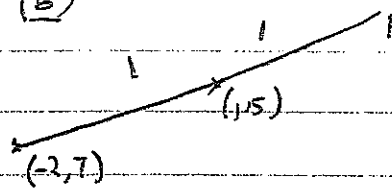
$$\tan 45^\circ = \left|\frac{m - 1/2}{1 + m/2}\right|$$

$$\therefore 1 = \left|\frac{2m-1}{2+m}\right|$$

$\therefore 2 + m = 2m - 1$

$m = 3$ ← 1 for answer

(b)

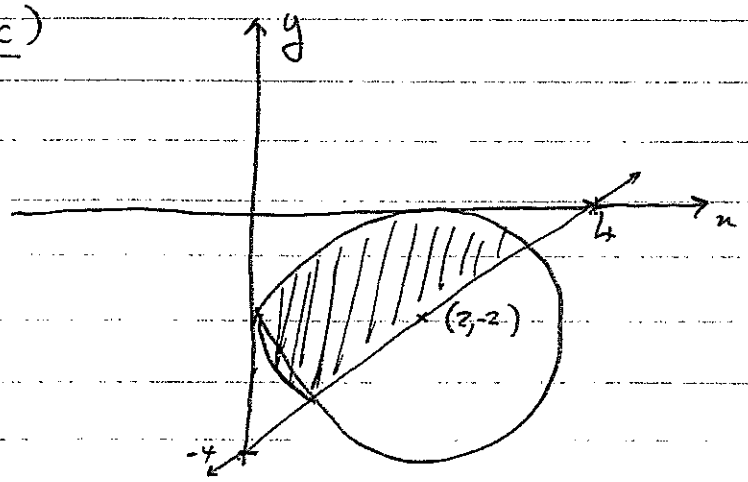


$k_1 = 2,\ k_2 = -1$ ← ② for either method

OR

$$P = \left(\frac{-1.-2+2.1}{1}, \frac{-1.7+10}{1}\right) = (4, 3)$$

(c)



1 MARK for circle in correct spot

1 MARK for correct straight line.

2 for correct shading. (No need to show that line goes through the centre)

(d) $\frac{d}{dx}\left(4x - \frac{1}{x}\right)$

$$= 4 + \frac{1}{x^2}$$

OR $\frac{4x^2+1}{x^2}$

① for either answer

QUESTION 10:

(a) (i) $R = 2$

$\sqrt{3}\cos x + \sin x = 2\left[\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right]$

$= 2\cos(x - \alpha)$

where $\cos\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 30°$ (or $\frac{\pi}{6}$) — ① for α

① for R

$\therefore \sqrt{3}\cos x + \sin x = 2\cos(x - 30°)$

(ii) $\sqrt{3}\cos x + \sin x = 1$

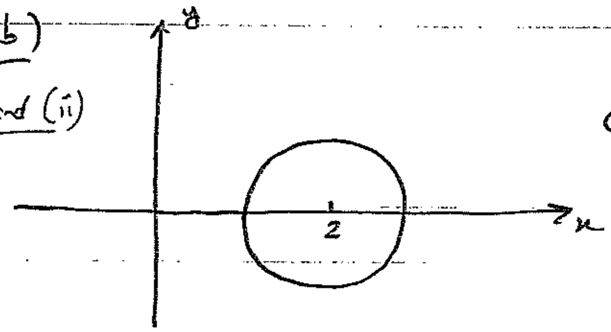
$\therefore 2\cos(x - 30°) = 1$

$\therefore \cos(x - 30°) = \frac{1}{2}$

$\therefore x - 30° = 60°$ or $300°$

$\therefore x = 90°$ or $x = 330°$ — 1 for each answer = ②

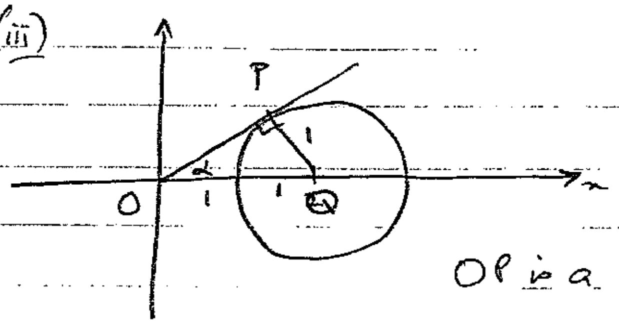
(b) (i) and (ii)



Q is (2,0) — 1 MARK

1 for sketch.

(iii)



OP is a tangent — 1 MARK.

Since $\angle OPQ = 90°$, and $PQ = 1$, $OQ = 2$

$\sin\alpha = \frac{1}{2}$ } ① for relating all these.

$\therefore \alpha = 30°$ ← ① MARK

QUESTION 11:

(a) $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2+(x+h)-(x^2+x)}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2+h}{h}$

$= \lim_{h\to 0} 2x+h+1$

$= 2x+1$

3 MARKS

Look for:

- correct expansions
- keeping $\lim_{h\to 0}$ to the right of = signs.
- keeping the $\lim_{h\to 0}$ until they let $h=0$.

[the answer is immaterial]

(b) (i) $\cos 2x = 2\cos^2 x - 1$

$\therefore \cos \pi/6 = 2\cos^2(\pi/12) - 1$

$\therefore \sqrt{3}/2 = 2\cos^2(\pi/12) - 1$

$\therefore \cos^2 \pi/12 = \frac{\sqrt{3}/2 + 1}{2}$

$= \frac{\sqrt{3}+2}{4}$

$\therefore \cos \pi/12 = \sqrt{\frac{\sqrt{3}+2}{4}}$

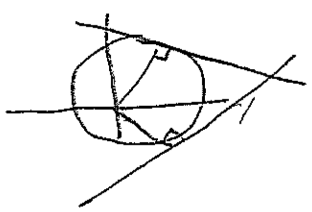
} 1 for realising this

1 for answer. Don't worry if they put $\pm$.

(c) (i) $p_1 = \left|-1/\sqrt{2}\right|$ $\quad p_2 = \left|-5/\sqrt{50}\right|$

$= 1/\sqrt{2}$ $\quad = 1/\sqrt{2}$

$\therefore$ The perpendicular distances to the origin are the same. So they are both lines on the circumference of a circle and tangents to it



3 MARKS for anything reasonably convincing.

(ii) $r = 1/\sqrt{2}$, $C:(0,0)$

$x^2+y^2 = 1/2$

1 MARK