Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 PRELIMINARY COURSE

Extension 1 Mathematics

Assessment 2

July 2014

<u>TIME ALLOWED</u>: **75 minutes**

Instructions:

- ***Start each question on a new page.***
- Write your name and class at the top of this page, and on your answer booklet.
- Write in blue or black pen only.
- All necessary working must be shown. <u>Marks may not be awarded for careless or badly arranged work.</u>
- Marks indicated within each question are a guide only and may be varied at the time of marking.
- It is suggested that you spend no more than 7 minutes on Section A.
- Approved calculators may be used.

SECTION A: (5 Marks)

Answers to these multiple choice should be completed on the multiple choice answer sheet supplied with your answer booklet.

All questions are worth 1 mark

1	$\frac{d}{dx}\left(\frac{5}{\sqrt{x}}\right) =$ A. $\frac{5x\sqrt{x}}{2}$ B. $\frac{5}{2x\sqrt{x}}$ C. $\frac{-5x\sqrt{x}}{2}$ D. $\frac{-5}{2x\sqrt{x}}$
2	$\sin(-120^o) =$ A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. $\frac{-\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{2}$
3	The acute angle between the line x=3 and the line $x - \sqrt{3}\,y + 2 = 0$ is: A. 60^o B. 30^o C. 90^o D. 45^o
4	If the endpoints of a diameter of the circle $(x-2)^2 + (y+1)^2 = 25$ are A (-1, -5) and B(k, m) then the values of k and m are: A. $k = 5$ and $m = 3$ B. $k = 3$ and $m = 5$ C. $k = -4$ and $m = -9$ D. $k = -9$ and $m = -4$
5	Given that $\cos A = k$, $k > 0$, and $0^o \leq A \leq 90^o$, then $\sin 2A =$ A. $2\sqrt{1-k^2}$ B. $2\sqrt{1+k^2}$ C $2k\sqrt{1-k^2}$ D. $2k\sqrt{1+k^2}$

SECTION B

(START EACH QUESTION ON A NEW PAGE)

QUESTION 6: (10 Marks)

Marks

(a) Differentiate with respect to x:

(i) $y = (3x^2 - 5)^3$ **1**

(ii) $y = \frac{x^3 - x^2 + 1}{x}$ **1**

(iii) $y = (x+1)\sqrt{x+1}$ **2**

(b) (i) Find the slope of the tangent to the curve $y = x^3 - x^2 - x + 1$ at the point where $x = 1$. **2**

(ii) What does this imply about the x-axis and the curve at the point where $x = 1$? **1**

(c) The lines $3x + 4y - 2 = 0$ and $3x + 4y + k = 0$ are 3 units apart. **3**

Find the two values of k.

QUESTION 7: (9 Marks) ***(Start on a new page)***

Marks

(a) (i) Show that $\dfrac{1}{x+h} - \dfrac{1}{x} = \dfrac{-h}{x(x+h)}$ **1**

(ii) Differentiate $f(x) = \dfrac{1}{x}$ using the method of First Principles. **3**

(b) Show that the equation of the tangent to the curve $y = \dfrac{x+2}{x-1}$ at the point where it crosses the x-axis is $x + 3y + 2 = 0$ **3**

(c) Find the point P which divides the interval joining R(4, 3) to S(2, -1) externally in the ratio 3: 5 **2**

QUESTION 8: (9 Marks) ***(Start on a new page)***

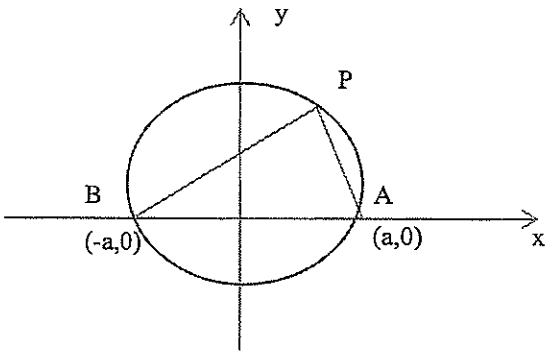
Marks

(a) Find $\lim_{h \to 0}\left\{\frac{(5+h)^2-25}{h}\right\}$ **2**

(b) Give the equation of the perpendicular bisector of the line which joins the points A (3, - 2) and B (5, 2). **3**

Give your answer in general form.

(c) (i) Show that the point P $(a\cos\theta, a\sin\theta)$ lies on the circle $x^2 + y^2 = a^2$ **1**



(ii) Find the gradients of the lines BP and AP **2**

(iii) Deduce that the line AP is at right angles to the line BP. **1**

(Use ONLY the information in parts (i) and (ii). You are NOT to use the circle geometry proof related to the angle in a semi-circle)

QUESTION 9: (9 Marks) ***(Start on a new page)***

Marks

(a) Show that $\tan(x + 45^o) = \frac{sinx+cosx}{cosx-sinx}$ **2**

(b) (i) Find the equation of the normal to $y = x^3 - 2x^2 - 3x + 1$ at P(2, -5). **2**

(ii) Show that there is another point on the curve where the normal to the curve is parallel to the normal at P. **2**
Find the co-ordinates this second point.

(c) (i) Show that $sin\,(A + B) + sin\,(A - B) = 2sinAcosB$ **1**

(ii) Hence, find the value of $sin75^o + \sin 15^o$ **2**

QUESTION 10: (9 Marks) ***(Start on a new page)***

Marks

(a) Find $\lim_{x \to \infty} \frac{2x^2+x}{3x^2-2}$ **1**

(b) (i) Show that the perpendicular distance of the point (4, 5) from the line $y = mx$ is: **1**

$$d = \frac{|4m-5|}{\sqrt{m^2+1}}$$

(ii) If y = mx is a tangent to the circle $(x-4)^2 + (y-5)^2 = 4$, explain why **2**

$$\frac{|4m-5|}{\sqrt{m^2+1}} = 2$$

(c) (i) Show that $\cos 3A = 4\cos^3 A - 3\cos A$ **2**

(ii) Hence solve $4\cos^3 A - 3\cos A = \frac{1}{2}$ for $0^o \leq A \leq 180^o$ **3**

QUESTION 11: (9 Marks) ***(Start on a new page)***

Marks

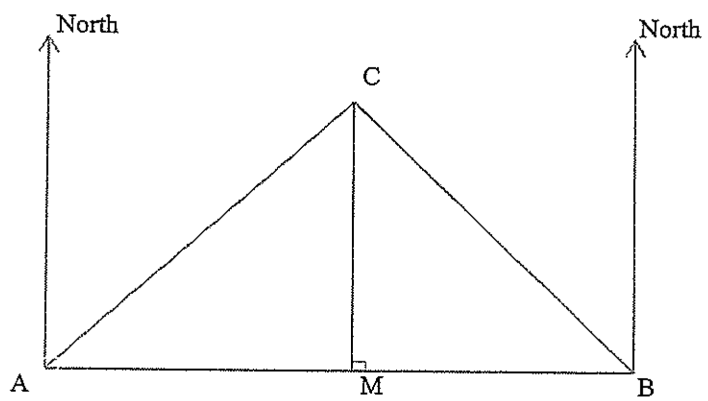
(a) (i) Express $cos\theta + \sqrt{3}\, sin\theta$ in the form $Rcos(\theta - \alpha)$ **2**

(ii) Hence solve the equation **2**

$$cos\theta + \sqrt{3}\, sin\theta = \sqrt{2} \qquad \text{for } 0^o \leq \theta \leq 360^o$$

(b) A surveyor stands at a point A and takes the bearing of a rock C, which he finds to be Nα^oE.
He walks due East 1000m to a second point B where he sees that C has a bearing of Nβ^oW.

He then moves to a third point, M, directly south of C.



(i) Redraw the diagram above, and put on it all of the information contained in the question. **1**

(ii) Prove that from M the distance to the rock C is given by **4**

$$\frac{1000cos\alpha cos\beta}{\sin(\alpha+\beta)}$$

YEAR 11 – SOLUTIONS – EXT1 JUNE 2014

SECT. A. (1) D (2) C (3) A (4) A (5) C

SECTION B QUESTION 6:

(a) (i) $18x(3x^2-5)$ — 1

(ii) $2x - 1 - 1/x^2$ — 1

(iii) $\frac{dy}{dx} = \frac{d}{dx}(x+1)^{3/2}$

$= \begin{cases} \frac{3}{2}(x+1)^{1/2} \\ \frac{3\sqrt{x+1}}{2} \end{cases}$

2. 1 for getting $\frac{d}{dx}(x+1)^{3/2}$

(b) (i) $\frac{dy}{dx} = 3x^2 - 2x - 1$ ← 1

At $x=1$ $\frac{dy}{dx} = 0$ ← 1

(ii) the x-axis is a tangent. 1

(c) point on $3x+4y-2=0$ is $(0, +\frac{1}{2})$ ← 1 for any point

$p = 3 = \left|\frac{0+2+k}{5}\right|$

$\therefore |k+2| = 15 \Rightarrow k = 13$ or $k = -17$ ← 1 each

QUESTION 7:

(a)(i) $\frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{x(x+h)}$ — 1 MARK

$= -h/x(x+h)$

(ii) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{1/(x+h) - 1/x}{h}$ ← 1 for this line.

$= \lim_{h\to 0} \frac{-h}{xh(x+h)}$ ← 1 for using part (i)

$= \lim_{h\to 0} \frac{-1}{x(x+h)}$ ← 1 for answer. Subtract 1 if the word "lim" is used incorrectly in any line.

$= -1/x^2$

(b) $\frac{dy}{dx} = \frac{(x-1)1-(x+2)1}{(x-1)^2}$

$= -3/(x-1)^2$ ← 1 MARK

At $y=0$, $x=-2$

$\therefore m_T = -3/9$

$\therefore m_T = -1/3$ ← 1 MARK

Equation is: $y = -1/3(x+2)$ ← 1 MARK

$x + 3y + 2 = 0$

(c) $k_1 : k_2$
$-3 : 5$

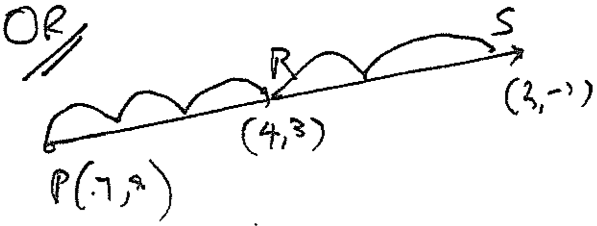
(4,3) (2,−1)

P is $\left(\frac{-3\times 2+5\times 4}{2}, \frac{3+15}{2}\right)$

$= (7,9)$

2 MARKS

OR



QUESTION 8:

(a) $\lim_{h \to 0} \left[\frac{25 + 10h + h^2 - 25}{h}\right]$

$= \lim_{h \to 0} (10 + h)$

$= 10$

2 MARKS – lose 1 for omitting "lim" in line 2

(b) $\text{Slope}_{AB} = 4/2$

$= 2$

Slope (perp) $= -1/2$ ← 1 MARK

midpoint $= (4, 0)$ ← 1 MARK

$y - 0 = -\frac{1}{2}(x - 4)$

$2y = -x + 4$

$x + 2y - 4 = 0$ ← 1 for general form

(c) (i) $x^2 + y^2 = a^2\cos^2\theta + a^2\sin^2\theta$ ← (1)

$= a^2$

(ii) $\text{Slope}_{AP} = \frac{a\sin\theta}{a\cos\theta - a}$

$= \sin\theta / \cos\theta - 1$ (1)

$\text{slope}_{BP} = \frac{a\sin\theta}{a\cos\theta + a}$ (1)

$= \sin\theta / \cos\theta + 1$

(iii) $m_{AP} \cdot m_{BP} = \frac{\sin^2\theta}{\cos^2\theta - 1}$

$= \frac{1 - \cos^2\theta}{\cos^2\theta - 1}$ $\left(\text{or } \frac{\sin^2\theta}{-\sin^2\theta}\right)$ (1)

$= -1$

$\therefore \angle APB = 90°$

QUESTION 9:

(a) $\tan(x+45^\circ) = \dfrac{\tan x + \tan 45}{1-\tan x \tan 45}$

$= \dfrac{\tan x + 1}{1-\tan x}$ — 1 for expansion and $\tan x = 1$

$= \dfrac{\sin x/\cos x + 1}{1 - \sin x/\cos x}$

$= \dfrac{\sin x + \cos x}{\cos x - \sin x}$ ← 1 for simplification

(b) (i) $dy/dx = 3x^2 - 4x - 3$

At $x=2$ $m_T = 1 \Rightarrow m_N = -1$ — 1 MARK

∴ Equation is $y+5 = -1(x-2)$

$x + y + 3 = 0$

(ii) If normals are parallel, so are the tangents — 1 for realising this.

∴ $3x^2 - 4x - 3 = 1$

∴ $3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

∴ $x = 2$ or, $x = -2/3$ ← 1 for all this

↑ found

∴ $y = -8/27 - 8/9 + 2 + 1$

$= 49/27$ — 1 for point

(c)(i) $\sin(A+B) + \sin(A-B)$

$= \sin(A)\cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B$ — 1 MARK

$= 2\sin A \cos B$

(ii) Let $A = 45$ $B = 30$. ← 1 for this

∴ $\sin 75 + \sin 15 = 2\sin 45 \cos 30$

$= 2 \times 1/\sqrt{2} \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}/\sqrt{2}$ or, $\sqrt{6}/2$ ← (1)

QUESTION 10:

(a) $\lim_{x \to \infty} \frac{2x^2 + x}{3x^2 - 2} = \lim_{x \to \infty} \frac{2 + 1/x}{3 - 2/x^2}$

$= 2/3$

1 MARK (answer only)

(b)

(i) $y = mx \Rightarrow mx - y = 0$

$d = \left| \frac{4m + 5(-1) + 0}{\sqrt{m^2+1}} \right|$

$= \frac{|4m - 5|}{\sqrt{m^2+1}}$

} 1 MARK

(ii) $y = mx$ is a tangent to the circle means the distance from the point of intersection to the centre of the circle is a radius, AND is the shortest distance of the line from the origin ie d (above) = 2.

} 2 MARKS for a reasonable explanation.

(b)(i) $\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$

$= (2\cos^2 A - 1)\cos A - 2\sin^2 A \cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$= 4\cos^3 A - 3\cos A$

} 2 MARKS

(ii) $4\cos^3 A - 3\cos A = 1/2$.

means $\cos 3A = 1/2 \quad 0° \le 3A \le 540°$ — 1 for realising this

$\therefore 3A = 60°, 300°, 420°$

$\therefore A = 20°, 100°, 140°$

} 2 MARKS (only 1 for only having 20°)

QUESTION 11:

(a) (i) $R = 2$ $\quad \cos\theta + \sqrt{3}\sin\theta = 2\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)$

$= 2\cos(\theta - \alpha)$ — 1 for R

where $\cos\alpha = \frac{1}{2}$ — 1 for α.

$\Rightarrow \alpha = 60°$

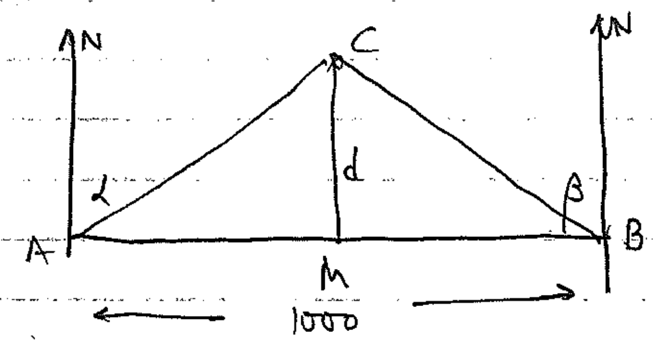
$\therefore$ EXP^N $= 2\cos(\alpha - 60°)$

(ii) $2\cos(\theta - 60°) = \sqrt{2}$ — (2)

$\therefore \cos(\theta - 60°) = 1/\sqrt{2}$. — 1 for each value of θ (none for 375°)

$\therefore \theta - 60° = 45°, 315°$

$\therefore \theta = 105°$, ~~375°~~, $15°$

(b) 

1 MARK

(ii) $\angle CAM = (90 - \alpha)°$ $\quad \angle CBM = (90 - \beta)°$

In $\triangle CBM$, $\quad d/CB = \sin(90 - \beta)$

$= \cos\beta$ — ← (1) for finding CB

In $\triangle ABC$ $\quad \dfrac{CB}{\sin(90 - \alpha)} = \dfrac{1000}{\sin(\alpha + \beta)}$ ← (1)

(1) for $\sin(90 - \beta) = \cos\beta$ and $\sin(90 - \alpha) = \cos\alpha$

$\therefore CB = \dfrac{1000\cos(\alpha)}{\sin(\alpha + \beta)}$

$\therefore d = \dfrac{1000\cos\alpha\cos\beta}{\sin(\alpha + \beta)}$ — 1 MARK