SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2012
Year 11 ACCELERANTS
YEARLY EXAMINATION

Mathematics

General Instructions:

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
  Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided on the back of the Multiple Choice answer sheet
- All necessary working should be shown in every question if full marks are to be awarded
- Marks may NOT be awarded for messy or badly arranged work
- Answer in simplest exact form unless otherwise stated

Total marks – 70 Marks

Section I Pages 2–5
10 marks

- Attempt Questions 1–10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section

Section II Pages 6–9
60 marks

- Attempt Questions 11–14
- Allow about 75 minutes for this section
- For Questions 11–14, start a new answer booklet per question

Examiner: Mr D. Hespe

Section I — 10 marks

Select the alternative A, B, C, or D that best answers the question.
Fill in the response oval on your multiple choice answer sheet.

Marks

1. The graph of $x^3 + 2x^2 + x - 2$ has: 1

   (A) 2 points of inflexion

   (B) 1 turning point and 1 point of inflexion

   (C) 3 turning points

   (D) 2 turning points

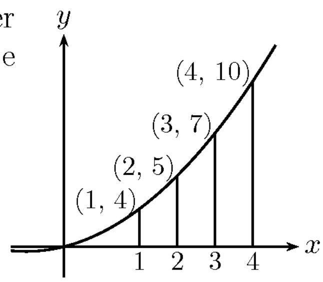
2. A student is using the trapezoidal rule to find the area under the curve at right from $x = 1$ to $x = 4$. His answer should be approximately equal to: 1

   (A) 26 sq. units

   (B) 22 sq. units

   (C) 19 sq. units

   (D) 16 sq. units



3. $\sin y + \sin(x - y) = \sin x$ for all y provided x is 1

   (A) $\dfrac{\pi}{3}$

   (B) $\dfrac{\pi}{2}$

   (C) π

   (D) 2π

4. If $y = 5e^{-6x}$ then $\dfrac{dy}{dx}$ is equal to 1

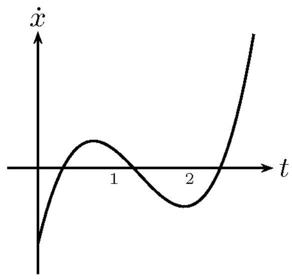
   (A) $-6e^{-6x}$

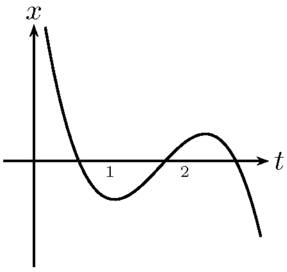
   (B) $5e^{-6x-1}$

   (C) $-30e^{-6x}$

   (D) $5e^{-7x}$

5. The graph to the right shows how the velocity of a particle varies with time. Its displacement-time graph is best given by 1



(A)


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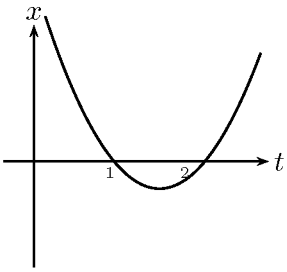
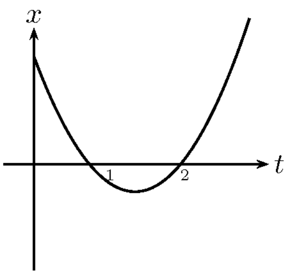
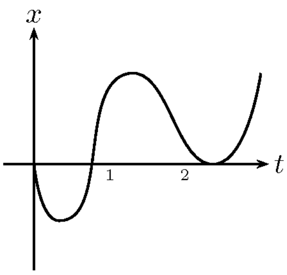


(C)


(D)


6. The area bounded by the graph of $f(x) = e^x - 1$, the x-axis, and the line $x = 2$ is equal to: 1

(A) $e^2 - 1$

(B) $e^2 - 3$

(C) $e^2 + 1$

(D) $e^2 + 3$

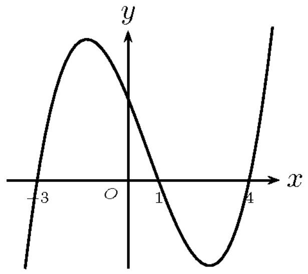
7. The graph of $g(x)$ has the following properties: $\boxed{1}$

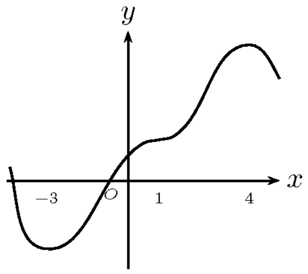
i $g'(x) = 0$ if $x = -3$, 1 and 4

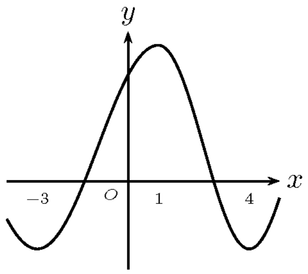
ii $g'(x) < 0$ if $x < -3$ and $1 < x < 4$

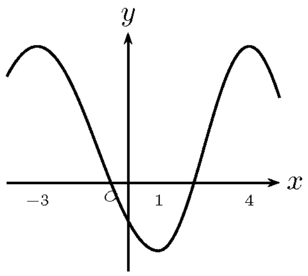
iii $g'(x) > 0$ for all other x

then the graph of $g(x)$ could be:

(A) 

(B) 

(C) 

(D) 

8. The indefinite integral $\displaystyle\int \left(\cos\frac{x}{3} - 3\sin 3x\right)\,dx$ is equal to: $\boxed{1}$

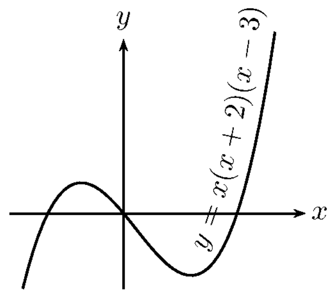
(A) $\frac{1}{3}\sin\dfrac{x}{3} + 9\cos 3x + c$

(B) $3\sin\dfrac{x}{3} - \cos 3x + c$

(C) $-3\sin\dfrac{x}{3} + \cos 3x + c$

(D) $3\sin\dfrac{x}{3} + \cos 3x + c$

9. The area bounded by the curve on the graph at right and the x-axis is equal to: [1]



(A) $20\frac{5}{12}$ sq. units

(B) $21\frac{1}{12}$ sq. units

(C) $10\frac{5}{12}$ sq. units

(D) $-10\frac{5}{12}$ sq. units

10. If $\dfrac{dR}{dt} = kR$ then which of the following is *not* possible? [1]

(A) $R = ke^{10t}$

(B) $R = 10ke^{kt}$

(C) $R = 20e^{kt}$

(D) $R = -20e^{kt}$

End Multiple Choice Questions

Section II— 60 marks

Marks

Question 11 (15 marks) (use a separate answer booklet)

(a) Find $\dfrac{dy}{dx}$ if $y = e^{\cos x}$. [1]

(b) Find the indefinite integrals:

(i) $\displaystyle\int \frac{dx}{3x-2}$ [1]

(ii) $\displaystyle\int \frac{6}{e^{2x}}\,dx$ [1]

(c) The gradient function of a curve is $7-4x$ and the curve passes through the point $(1,\ 10)$. Find its equation. [2]

(d) Find the area enclosed by the curve $y = 6x - x^2$ and the x-axis. [2]

(e) A minor segment of a circle has an area of $50\,\text{cm}^2$ and subtends a central angle of $\frac{\pi}{5}$ radians. Find the radius of the circle correct to one decimal place. [2]

(f) A particle moves in a straight line and its displacement from the origin is given by $s = 5 - 6t + t^2$, find:

(i) the distance of the particle from the origin after 2 seconds, [1]

(ii) the times when the particle is at the origin, [2]

(iii) at what instant the velocity is zero, [2]

(iv) the acceleration of the particle. [1]

Marks

Question 12 (15 marks) (use a separate answer booklet)

(a) Differentiate and simplify:

(i) $2xe^{2x}$, **1**

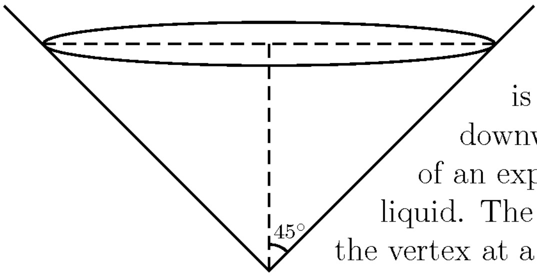
(ii) $\dfrac{\cos x}{1-\sin x}$. **2**

(b) Find correct to four significant figures $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x}\,dx$. **2**

(c) Evaluate $\displaystyle\int_0^2 |x^2+2x-3|\,dx$ (HINT: sketch the curve first). **2**

(d) Find a primitive of the function $\tan^2\theta$. **2**

(e) **6**



A hollow cone of semi-vertical angle 45° is held with its axis vertical and vertex downwards (see diagram). At the beginning of an experiment, it is filled with 390 cm^3 of liquid. The liquid runs out through a small hole at the vertex at a constant rate of 2 cm^{3}s^{-1}. Find the rate at which the depth of the liquid is decreasing 3 minutes after the start of the experiment. Give your answer correct to 3 significant figures.

Marks

Question 13 (15 marks) (use a separate answer booklet)

(a) Find where the tangent to $x^3 + 2x + 1$ at the point where $x = -1$ meets the curve again. **4**

(b) Find the volume of the solid formed when the region enclosed by the graph of $y = \ln x$, the x-axis, the y-axis and the line $y = \ln 3$ is rotated about the y-axis. Give your answer in exact form. **4**

(c) The equation of a curve C is given as $y = \dfrac{x^2}{x + \lambda}$ where λ is a non-zero constant.

(i) Write down the equations of the asymptotes of C. **3**

(ii) Draw, on separate diagrams, a sketch of C for the cases where **4**

(α) $\lambda > 0$

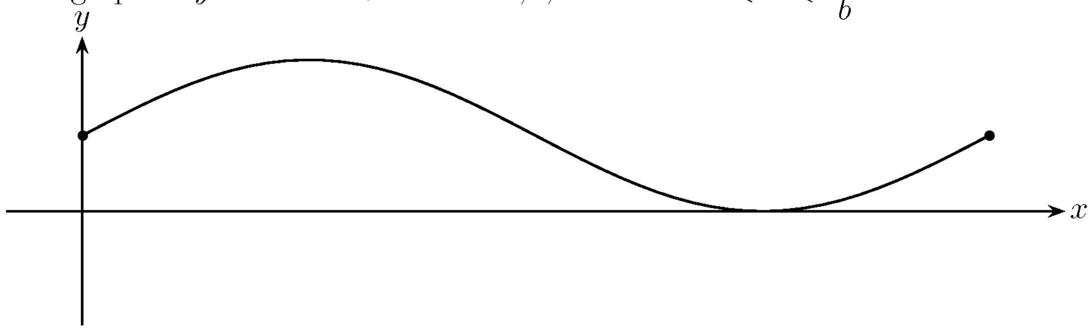
(β) $\lambda < 0$.

Marks

Question 14 (15 marks) (use a separate answer booklet)

(a) It is given that $\ln a = x$ and $\ln b = y$. Express $\ln\left(\dfrac{a^2 b}{e}\right)$ in terms of x and y. **2**

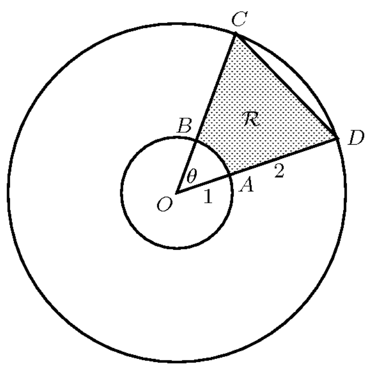
(b) The graph of $y = a\sin bx + c$ where $a, b, c \in \mathbb{R}$ and $0 \leqslant x \leqslant \dfrac{2\pi}{b}$ is drawn below. **3**



Copy the diagram to your answer booklet. On the same set of axes, sketch the graph of $y = -2a\sin\left(\frac{b}{2}x\right)$.

(c) To historically date objects less than 50 000 years old, carbon 14 dating is used. Carbon 14 has a half-life of 5370 years. Before death animals and plants have a reading for carbon 14 of 12.5 counts per minute on a radiation counter. Show that the decay rate for carbon 14 is -1.29×10^{-4}. If a piece of wood from an excavation site has a reading of 7 counts per minute, show that the wood's age is approximately 4500 years. **4**

(d)



The diagram shows two circles, of radii 1 and 3, each with centre O. The angle between the lines OAD and OBC is θ radians. The shaded region $\mathcal{R}$ is bounded by the minor arc AB and the lines BC, CD, and DA.

(i) Find the area of $\mathcal{R}$. **2**

(ii) Find the value of θ for which the area of $\mathcal{R}$ is greatest. **2**

(iii) Find the greatest value of θ which ensures that the whole of the line segment CD lies between the two circles. **2**

End of Paper

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Student Number:________________________

Mathematics

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Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

Section I: **Multiple choice answer sheet.**

Completely colour the cell representing your answer. Use black pen.

1. A B C D ● (D)
2. A B C ● D (C)
3. A B C D ● (D)
4. A B C ● D (C)
5. A B C D ● (D)
6. A B ● C D (B)
7. A B C ● D (C)
8. A B C D ● (D)
9. A B ● C D (B)
10. A ● B C D (A)

Year 11 Accelerated Mathematics Exam Soln:

Question 11:

a)

$$y = e^{\cos x}$$
$$\frac{dy}{dx} = -\sin x \, e^{\cos x}$$

b)

(i)

$$\int \frac{dx}{3x-2}$$
$$= \frac{1}{3}\int \frac{3.dx}{3x-2}$$
$$= \frac{1}{3}\ln(3x-2) + C$$

(ii)

$$\int \frac{6}{e^{2x}}.dx$$
$$= \int 6\,e^{-2x}.dx$$
$$= \frac{6e^{-2x}}{-2} + C$$
$$= -3e^{-2x} + C$$

c) $y' = 7 - 4x$

$y = 7x - 2x^2 + C$

At $(1, 10)$

$10 = 7 - 2 + C$

$\therefore C = 5$

$y = 7x - 2x^2 + 5$

d)

$$A = \int_0^6 6x - x^2 .dx$$
$$= \left[3x^2 - \frac{x^3}{3}\right]_0^6$$
$$A = 36\,u^2$$

e)

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

$$50 = \frac{1}{2}r^2\left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right)$$

$$r^2 = \frac{100}{\frac{\pi}{5} - \sin\frac{\pi}{5}}$$

$r = 49.7\ cm$ (nearest 1 d.p.)

f) $s = 5 - 6t + t^2$

(i) When $t = 2$

$s = 5 - 6(2) + 2^2$

$s = -3$

Distance = 3 units

(ii) When $s = 0$

$5 - 6t + t^2 = 0$

$(t - 5)(t - 1) = 0$

$t = 1, \quad 5$

(iii) $\frac{ds}{dt} = -6 + 2t$

When $\frac{ds}{dt} = 0$

$-6 + 2t = 0$

$2t = 6$

$t = 3$

(iv) $\frac{d^2s}{dt^2} = 2$

QUESTION 12.

(a) (i)

$$y = 2x e^{2x}$$

$$y' = 4x e^{2x} + 2e^{2x}$$

$$= 2e^{2x}(2x+1)$$

(ii)

$$y = \frac{\cos x}{1-\sin x}$$

$$y' = \frac{(1-\sin x)(-\sin x) - \cos x \times -\cos x}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$$

$$= \frac{1-\sin x}{(1-\sin x)^2}$$

$$= \frac{1}{1-\sin x}.$$

(b).

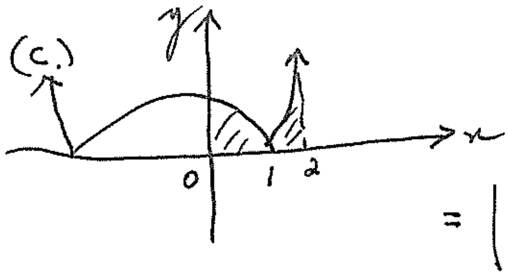
$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x} \, dx = \Big[\ln(2+\sin x)\Big]_0^{\frac{\pi}{2}}$$

$$= \ln(2+1) - \ln(2+0)$$

$$= \ln 3/2.$$

$$\doteqdot 0.4055.$$

(c).



$$\int_0^2 |x^2+2x-3|\, dx$$

$$= \left|\int_0^1 (x^2+2x-3)\, dx\right| + \int_1^2 (x^2+2x-3)\, dx.$$

$$= \left|\left[\frac{x^3}{3}+x^2-3x\right]_0^1\right| + \left[\frac{x^3}{3}+x^2-3x\right]_1^2$$

$$= \left|\tfrac{1}{3}+1-3\right| + \tfrac{8}{3}+4-6-\left(\tfrac{1}{3}+1-3\right)$$

$$= 1\tfrac{2}{3} + \tfrac{2}{3} + 1\tfrac{2}{3}$$

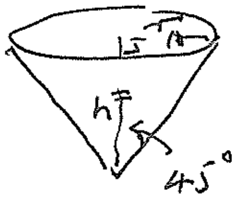
$$= 4.$$

(d) Let $f'(\theta) = \tan^2\theta$

$= \sec^2\theta - 1$

$\therefore f(\theta) = \tan\theta - \theta + c.$

(e).



NB $h = r$.

$\therefore V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{1}{3}\pi h^3$

$\therefore \frac{dV}{dh} = \pi h^2.$

[Now after 3 minutes there is 30 cm³ remaining at which time.

$30 = \frac{1}{3}\pi h^3.$

$\therefore \frac{90}{\pi} = h^3$

$\therefore h = \left(\frac{90}{\pi}\right)^{\frac{1}{3}}$ — (A)]

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{1}{\pi h^2} \times -2.$

$= \frac{-2}{\pi \left(\frac{90}{\pi}\right)^{2/3}}$

$\doteqdot -0{\cdot}0680$ cm/s.

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Question 13

(a) $y = x^3 + 2x + 1$

$y' = 3x^2 + 2$

$y'(-1) = 3 + 2 = 5$ $y(-1) = -1 - 2 + 1 = -2$

$\therefore$ Tangent

$y + 2 = 5(x+1)$

$= 5x + 5$

$y = 5x + 3$

To find intersections, substitute for y:

$x^3 + 2x + 1 = 5x + 3$

$x^3 - 3x - 2 = 0$

Two solutions are $x = -1$

By division

$$\begin{array}{r} x^2 - x - 2 \\ x+1 \overline{) x^3 \quad\quad - 3x - 2} \\ \underline{x^3 + x^2} \quad\quad\quad\quad \\ -x^2 - 3x \quad\quad \\ \underline{-x^2 - x} \quad\quad \\ -2x - 2 \end{array}$$

$x^2 - x - 2 = 0$

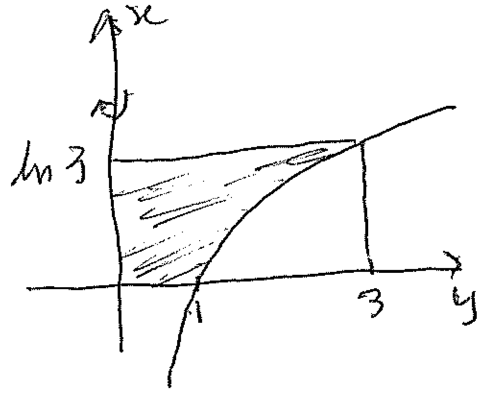
$(x-2)(x+1) = 0$

$x = 2, -1$

$\therefore$ Next meets curve when $x = 2$, $y = 13$

i.e. $(2, 13)$ [4]

(b)



$y = \ln x$

$x = e^y$

$V = \pi \int_c^d x^2 \, dy$

$= \pi \int_0^{\ln 3} e^{2y} \, dy$

$= \pi \left[\frac{1}{2} e^{2y} \right]_0^{\ln 3}$

$= \frac{\pi}{2} \left[(e^{\ln 3})^2 - e^0 \right]$

$= \frac{\pi}{2} [9 - 1]$

$= 4\pi$ [4]

(c) $y = \dfrac{x^2}{x + \lambda}$

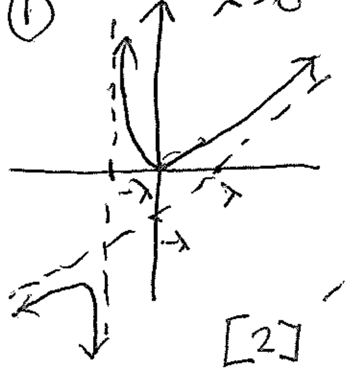
Asymptotes: $x = -\lambda$ Vertical   [3]

Oblique: $y = \dfrac{(x^2 - \lambda^2) + \lambda^2}{x + \lambda}$

$= (x - \lambda) + \dfrac{\lambda^2}{x + \lambda}$

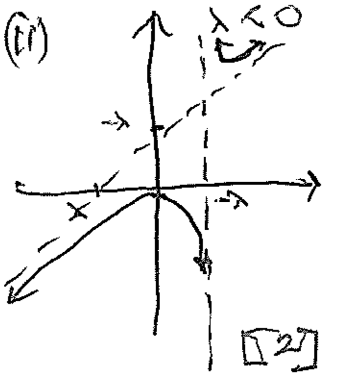
As $x \to \pm\infty$, $y \to x - \lambda$

(i) $\lambda > 0$



[2]

(ii) $\lambda < 0$



[2]

Question 14

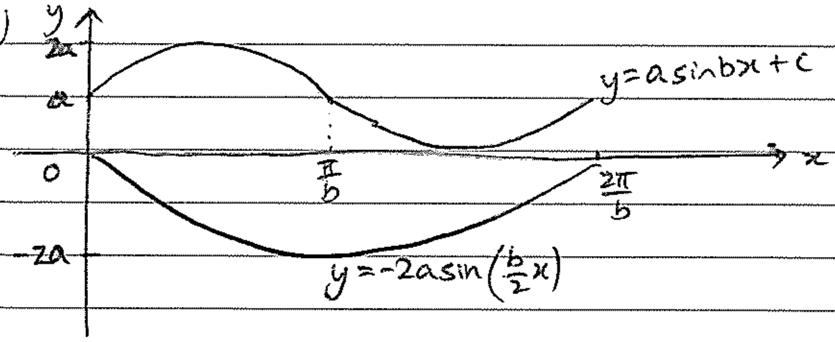
(a) $\ln\left(\frac{a^2b}{e}\right) = \ln(a^2b) - \ln e$

$= \ln a^2 + \ln b - \ln e$

$= 2\ln a + \ln b - \ln e$

$= 2x + y - 1$

(b)



Note: $c = a$

(c) $P = Ae^{kt}$

when $t = 5370$

$P = \frac{A}{2}$

$\frac{A}{2} = Ae^{5370k}$

$\frac{1}{2} = e^{5370k}$

$5370k = \ln(\frac{1}{2})$

$k = \frac{1}{5370} \cdot \ln(\frac{1}{2})$

$k \doteqdot -1.29 \times 10^{-4}$ which is the growth rate.

$P = Ae^{kt}$

when $t=0$

$P = 12.5$

$12.5 = Ae^0$

$A = 12.5$

$P = 12.5e^{kt}$

when $P = 7$

$7 = 12.5e^{kt}$

$e^{kt} = 0.56$

$kt = \ln(0.56)$

$t = \frac{\ln(0.56)}{k}$, where $k = -1.29 \times 10^{-4}$

$t \doteqdot 4500$ years

d) i) $A = \frac{1}{2}(3)(3)\sin\theta - \frac{1}{2}(1)^2\theta$

$= \frac{9}{2}\sin\theta - \frac{\theta}{2}$

ii) $\frac{dA}{d\theta} = \frac{9}{2}\cos\theta - \frac{1}{2}$

let $\frac{dA}{d\theta} = 0$ for stat. points

$\frac{9}{2}\cos\theta - \frac{1}{2} = 0$

$\frac{9}{2}\cos\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{9}$

$\theta = \cos^{-1}\left(\frac{1}{9}\right)$

≈ 1.46

$\frac{d^2A}{d\theta^2} = -\frac{9}{2}\sin\theta$

when $\theta = 1.46$

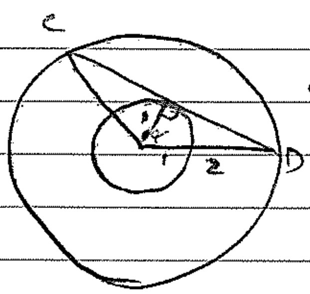
$\frac{d^2A}{d\theta^2} = -4.47\ldots$

< 0 ✓

$\therefore$ Max. Area when $\theta = \cos^{-1}\left(\frac{1}{9}\right)$

$\approx 1.46.$

iii)



The greatest value of θ will involve considering when CD is a tangent to smaller circle.

$\cos\alpha = \frac{1}{3}$

$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

$2\alpha = 2\cos^{-1}\left(\frac{1}{3}\right)$

≈ 2.46

For CD to lie between the two circles

$0 < \theta < \underline{2.46}.$