SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2015

Year 11 Yearly Examination

Mathematics Accelerated

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Leave your answers in the simplest exact form, unless otherwise stated.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Start each **NEW** question in a separate answer booklet.

Total Marks – 70

Section I Pages 2–4

10 Marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II Pages 5–10

60 marks

- Attempt Questions 11–16
- Allow about 1 hour and 45 minutes for this section

Examiner: *R Dowdell*

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

1 A straight line has a gradient of -2 and passes through the point (4, 1). What is its equation?

A $y + 2x = 6$

B $y = 2x - 6$

C $y + 2x - 9 = 0$

D $2y = x - 2$

2 What is the sum of the series

$$1 + \frac{1}{3} + \frac{1}{3^2} + \ \ldots. \ \ ?$$

A 2

B $\frac{3}{2}$

C $\frac{4}{3}$

D $\frac{10}{9}$

3 What is the expression

$$\frac{\tan\theta}{\sec\theta}$$

equivalent to?

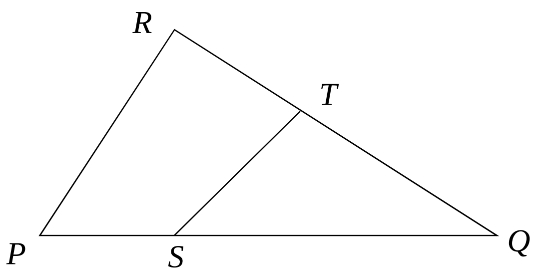
A $\frac{\cos^2\theta}{\sin\theta}$

B $\frac{\sin\theta}{\cos^2\theta}$

C $\cos\theta$

D $\sin\theta$

4



In the triangle PQR, $\angle PST$ is equal to 146°, $TS = TQ$ and $PQ = QR$. What is the size of $\angle PRQ$?

A 54°

B 68°

C 73°

D 75°

5

The perimeter of a rhombus is 60 cm. If the length of the longer diagonal is 24 cm, what is the length of the shorter diagonal?

A 20 cm

B 18 cm

C 15 cm

D 9 cm

6

What is the value of

$$\int_0^1 \sqrt{x^2 - 2x + 1}\, dx?$$

A -1

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 1

7 If $\sin(x+20)^\circ = \cos x^\circ$ and x° is acute,
what is the value of x?

A 35

B 45

C 55

D 70

8 The probability of having a particular disease is 5%. The test to determine if a person has this disease or not gives the correct result 83% of the time. What is the probability that a randomly selected person tests positive?

A 0.0415

B 0.203

C 0.246

D 0.83

9 Which equation is equivalent to $\log_{10} y = 3\log_{10} x + 2$?

A $y = 3x + 2$

B $y = 100x^3$

C $y = x^3 + 2$

D $y = x^3 + 100$

10 What is

$$\lim_{h\to 0}\frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}?$$

A 0

B $\frac{1}{2}$

C $\frac{1}{32}$

D It cannot be determined from the information given

Section II

60 marks
Attempt Questions 11-16
Allow about 1 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (10 marks) Start a NEW Writing Booklet

(a) Differentiate the following functions:

(i) $(2x^3 - 1)^4$ **2**

(ii) $x^2 \sin x$ **2**

(iii) $\log_e\left(\frac{x-1}{x+2}\right)$ **2**

(b) Evaluate the following integrals:

(i) $\int_0^1 (3x^4 - 1)\,dx$ **2**

(ii) $\int_0^{\frac{\pi}{4}} \sin 2x\,dx$ **2**

Question 12 (10 marks) Start a NEW Writing Booklet

(a) The size of a colony of bugs is given by $P = 400e^{kt}$ where P is the population after t days.

(i) If there are 560 bugs after 1 day, find the value of k, correct to 2 decimal places. **2**

(ii) When will the colony triple in size? (Answer to the nearest day) **1**

(iii) What is the growth rate of the population after 3 days? **2**

(b) The rate of inflation measures the rate of change of prices. Between January 2014 and December 2014, prices were rising but the rate of inflation was falling. Draw a graph of prices as a function of time that fits this description. **2**

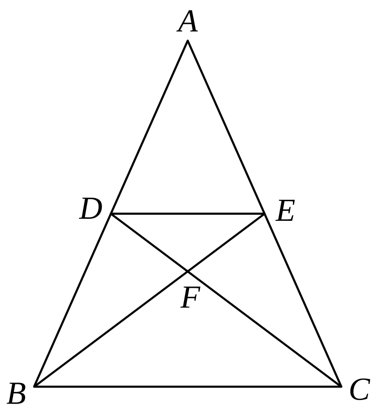
(c) A radioactive substance has a half-life of 100 years.

(i) What amount of the radioactive substance would remain after a period of 500 years? **1**

(ii) How long would it take for a mass of 10 kg to reduce to 8 kg? **2**

Question 13 (10 marks) Start a NEW Writing Booklet

(a) In the diagram below, <ADE = <AED and DF = EF.



NOT TO SCALE

(i) Copy the diagram into your answer booklet. **2**

(ii) Prove that $\Delta\, ABE \equiv \Delta\, ACD$, giving reasons. **3**

(iii) Hence prove that $\Delta\, BFC$ is isosceles.

(b) The sum of the first 9 terms of the series **3**

$$\log_3\left(\frac{1}{x}\right)+\log_3\left(\frac{1}{x^2}\right)+\log_3\left(\frac{1}{x^3}\right)+\ldots\ldots \text{ is } -270.$$

Find the value of x.

(c) Find the equation of the line (in general form) perpendicular to $3x-4y+12=0$ and intersecting it on the x-axis. **2**

Question 14 (10 marks) Start a NEW Writing Booklet

(a) A particle moves in a straight line such that its displacement, x cm, after t seconds is given by $x = t^3 - 9t^2 + 24t - 7$.

(i) When and where does the particle first change direction? **2**

(ii) Calculate the total distance travelled in the first 3 seconds. **2**

(iii) What is the average speed in the first second? **1**

(b) The points A (3, -5) and B (5, 9) are at opposite ends of the diameter of a circle.

Find

(i) the centre of the circle; **1**

(ii) the radius of the circle; **1**

(iii) the two values of k, such that the point (9, k) lies on the circle. **1**

(c) Estimate $\int_0^2 \sin(1 + x^2)\, dx$ by using Simpson's Rule with three function values. **2**

Question 15 (10 marks) Start a NEW Writing Booklet

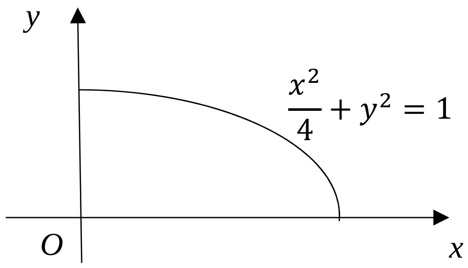
(a) The equation of a parabola is $y^2 = 12(x - 1)$.

(i) Find the coordinates of the vertex of the parabola. **1**

(ii) Find the coordinates of the focus of the parabola. **1**

(iii) Find the equation of the directrix of the parabola. **1**

(b) **2**



The part of the curve $\frac{x^2}{4} + y^2 = 1$ that lies in the first quadrant is rotated about the *x* axis.

Find the volume of the solid of revolution.

(c) A container, initially empty, is filled so that the volume *V*, in litres, at the time *t* minutes is given by $V = 32t - 4t^2, \ 0 \leq t \leq 4$.

(i) How much water is in the container when it is full? **2**

(ii) After how many minutes was the container three quarters full? **2**

(iii) At what rate was the water flowing into the container when the container was three quarters full? **1**

Question 16 (10 marks) Start a NEW Writing Booklet

(a) Consider the function $y = 108x - x^4$. **5**

(i) Find any stationary points on the curve.

(ii) Does the function have any points of inflexion?

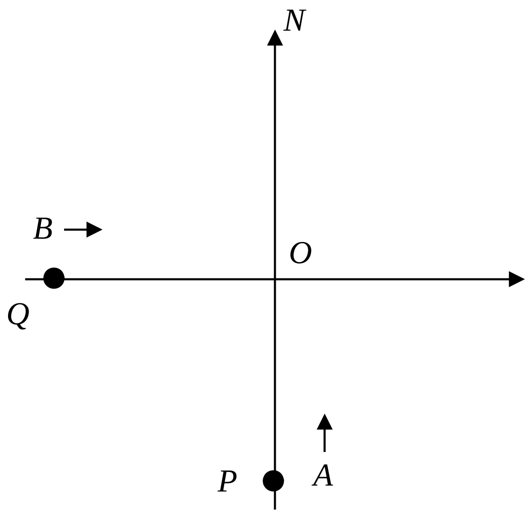
(iii) Sketch the function.

(b) Two walkers *A* and *B* start walking at noon towards a point *O*. **5**
A starts at P, 9 km due south of *O* and *B* starts at *Q*, 13 km west of *O*.

A walks at 4 km/hr and B at 3 km/hr.

Show that if there distance apart after t hours is d km, then $d^2 = 25t^2 - 150t + 250$.

Hence, find when their distance apart is least and what this distance is.



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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Year 11 Yearly

Accelerated Mathematics

Sample Solutions

Question	Teacher
Q11	**JM**
Q12	**AMG**
Q13	**PP**
Q14	**BK**
Q15	**DH**
Q16	**PB**

MC Answers

1. C
2. B
3. D
4. C
5. B
6. C
7. A
8. B
9. B
10. B

Section I 10 marks

1 A straight line has a gradient of -2 and passes through the point (4, 1). What is its equation?

A $y + 2x = 6$

B $y = 2x - 6$

C $y + 2x - 9 = 0$

D $2y = x - 2$

$$y = mx + b$$
$$(4, 1)\ \&\ m = -2$$
$$1 = (-2)(4) + b$$
$$b = 9$$
$$\therefore y = -2x + 9$$
$$2x + y - 9 = 0$$

2 What is the sum of the series

$$1 + \frac{1}{3} + \frac{1}{3^2} + \ \ldots \ ?$$

A 2

B $\frac{3}{2}$

C $\frac{4}{3}$

D $\frac{10}{9}$

Sum to infinity

$$S_\infty = \frac{a}{1-r}$$
$$a = 1\ \&\ r = \frac{1}{3}$$
$$S_\infty = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

3 What is the expression

$$\frac{\tan\theta}{\sec\theta}$$

equivalent to?

A $\frac{\cos^2\theta}{\sin\theta}$

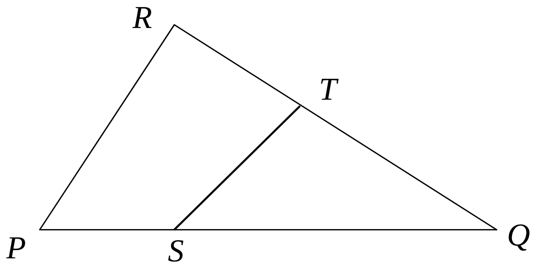
B $\frac{\sin\theta}{\cos^2\theta}$

C $\cos\theta$

D $\sin\theta$

$$\begin{aligned}\frac{\tan\theta}{\sec\theta} &= \frac{\sin\theta}{\cos\theta} \div \frac{1}{\cos\theta} \\ &= \frac{\sin\theta}{\cos\theta} \times \cos\theta \\ &= \sin\theta\end{aligned}$$

4



In the triangle PQR, $\angle PST$ is equal to 146°, $TS = TQ$ and $PQ = QR$. What is the size of $\angle PRQ$?

A 54°

B 68°

C 73°

D 75°

In ΔTSQ

$\angle TSQ = 180° - 146°$

$= 34°$ (Straight line PQ)

$\angle SQT = 34°$ (Isosceles $\Delta\ SQT$)

In ΔPQR

$\angle PRQ = \dfrac{180 - 34}{2} = 73°$ (Isosceles ΔPRQ)

5 The perimeter of a rhombus is 60 cm. If the length of the longer diagonal is 24 cm, what is the length of the shorter diagonal?

A 20 cm

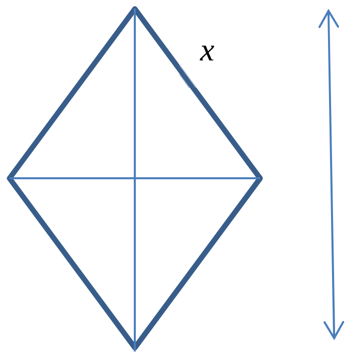
B 18 cm

C 15 cm

D 9 cm

$P = 60 = 4x$

$x = 15cm$



$\sqrt{15^2 - 12^2} = 9$

Length of shorter side = 2(9)=18cm

6 What is the value of

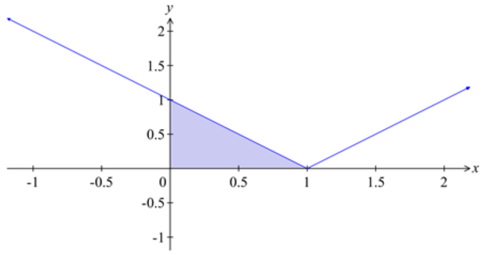
$$\int_0^1 \sqrt{x^2 - 2x + 1}\, dx?$$

A -1

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 1



$$\int_0^1 \left(x^2 - 2x + 1\right)^{\frac{1}{2}} dx = \int_0^1 \sqrt{\left(x-1\right)^2}\, dx$$

$$= \int_0^1 \left|x - 1\right| dx \quad \left[\text{area of shaded triangle}\right]$$

$$= \frac{1}{2}$$

7 If $\sin(x + 20)^\circ = \cos x^\circ$, what is the value of x?

A 35

B 45

C 55

D 70

7

$\sin(x+20)^\circ = \cos x^\circ$

$90 - (x+20) = x$

$70 = 2x$

$x = 35^\circ$

8 The probability of having a particular disease is 5%. The test to determine if a person has this disease or not gives the correct result 83% of the time. What is the probability that a randomly selected person tests positive?

A 0.0415

B 0.203

C 0.246

D 0.83

Required probability is made up of those that are positive and get a correct result and those that are negative and get a false result.

$$P = (0.05)(0.83) + (0.95)(0.17)$$
$$= 0.203$$

9 Which equation is equivalent to $\log_{10} y = 3\log_{10} x + 2$?

A $y = 3x + 2$

B $y = 100x^3$

C $y = x^3 + 2$

D $y = x^3 + 100$

$$\log_{10} y = 3\log_{10} x + 2$$
$$\therefore \log_{10} y = \log_{10} x^3 + 2$$
$$\therefore \log_{10} y - \log_{10} x^3 = 2$$
$$\therefore \log_{10} \frac{y}{x^3} = 2$$
$$\therefore \frac{y}{x^3} = 10^2$$
$$\therefore y = 100x^3$$

10 What is

$$\lim_{h \to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}?$$

A 0

B $\frac{1}{2}$

C $\frac{1}{32}$

D It cannot be determined from the information given

Let $f(x) = 8x^8$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(\tfrac{1}{2}) = \lim_{h \to 0} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h}$$

$$= \lim_{h \to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$$

$$f'(x) = 64x^7$$

$$\therefore f'(\tfrac{1}{2}) = 64(\tfrac{1}{2})^7$$

$$= \tfrac{1}{2}$$

$$\therefore \lim_{h \to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h} = \tfrac{1}{2}$$

Section II

Question 11

(a) i) $\frac{d}{dx}(2x^3-1)^4 = 4(2x^3-1)^3 . 6x^2$

$= 24x^2(2x^3-1)^3$

ii) $\frac{d}{dx}(x^2 \sin x)$ Let $u = x^2$, $v = \sin x$; $u' = 2x$, $v' = \cos x$

$= vu' + uv'$

$= \sin x . 2x + x^2 . \cos x$

$= x(2\sin x + x\cos x)$

iii) $\frac{d}{dx}\left(\log_e\left(\frac{x-1}{x+2}\right)\right) = \frac{d}{dx}\left(\log_e(x-1) - \log_e(x+2)\right)$

$= \frac{1}{x-1} - \frac{1}{x+2}$

$= \frac{x+2-(x-1)}{(x-1)(x+2)}$

$= \frac{3}{(x-1)(x+2)}$

Comments:
These questions were generally done well.
With part iii) many students didn't simplify the logarithm first.

(b) i) $\int_0^1 (3x^4-1)\,dx = \left[\frac{3x^5}{5} - x\right]_0^1$

$= \left(\frac{3(1)^5}{5} - 1\right) - \left(\frac{3(0)^5}{5} - 0\right)$

$= -\frac{2}{5}$

ii) $\int_0^{\pi/4} \sin 2x \, dx$

$$= \left[-\frac{1}{2} \cos 2x \right]_0^{\pi/4}$$

$$= \left(-\frac{1}{2} \cos 2\left(\frac{\pi}{4}\right)\right) - \left(-\frac{1}{2} \cos 2(0)\right)$$

$$= \left(-\frac{1}{2}(0)\right) - \left(-\frac{1}{2}(1)\right)$$

$$= \frac{1}{2}$$

Comments:

These questions were generally done well.

Question 12

(a) $P = 400e^{kt}$

(i) When $t = 1$, $P = 560$

$$\therefore \frac{560}{400} = e^k$$
$$k = \ln\left(\frac{560}{400}\right)$$
$$= 3364.....$$
$$\simeq 0.34 \quad \text{to 2 d.p.}$$

Comment: Universally well-answered.

(ii) The colony has tripled when P = 1200.

$$1200 = 400e^{kt}$$
$$e^{kt} = 3$$
$$kt = \ln 3$$
$$t = \frac{\ln 3}{k}$$
$$= 3.2650....$$
$$\simeq 3 \text{ days, to the nearest day.}$$

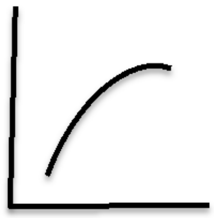
Comment: Generally well-answered, although some thought it would be better to round up – they were given full marks. Some others tripled the first day population, and received no marks ("correct substitution into a correct formula").

(iii) Growth rate after 3 days:

$$\frac{dP}{dt} = 400ke^{3k}$$
$$= 369.3 \text{ bugs/day}$$

Comment: Many rounded k to 0.34 before calculation, to get 377. They were given full marks.

(b) $\frac{dP}{dt} > 0, \quad \frac{d^2P}{dt^2} < 0$



Comment: Almost every candidate got this right.

(c) (i) 500 years is 5 half-lives, thus

$$M_0 \to M_0 \times \left(\tfrac{1}{2}\right)^5 = \frac{M_0}{32}$$

Hence $\frac{1}{32}$ part remains.

Comment: Some did not understand the question. Almost all got it right.

(ii) $M = M_0 e^{-kt}$

When t = 100, the mass is halved.

$$\frac{1}{2} = e^{-100k}$$
$$-100k = \ln\left(\tfrac{1}{2}\right)$$
$$k = \frac{\ln 2}{100}$$
$$= 0.00693...$$

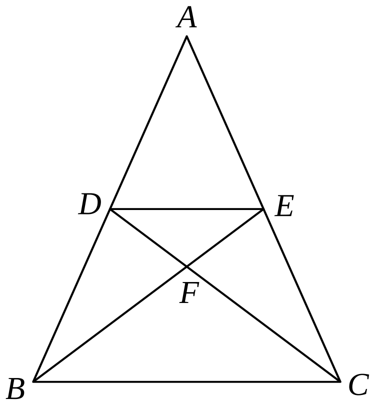
Now let the original mass be 10, and find time to decay to 8.

$$8 = 10e^{-kt}$$
$$-kt = \ln(0.8)$$
$$t = \frac{\ln(0.8)}{-k}$$
$$\simeq 32.19 \quad \text{years.}$$

Comment: Generally very well answered. No particular error stood out.

Question 13 (10 marks)

(a) In the diagram below, $\angle ADE = \angle AED$ and $DF = EF$.



NOT TO SCALE

(i) Copy the diagram into your answer booklet.

(ii) Prove that $\Delta ABE \equiv \Delta ACD$, giving reasons. **2**

$\angle ADE = \angle AED$ (given)
$\angle EDF = \angle DEF$ (equal angles opposite equal sides, DF and EF)

$\angle ADC = \angle ADE + \angle EDF$
$= \angle AED + \angle DEF$ (from above)
$= \angle AEB$

This or it's equivalent was needed.

In ΔABE and ΔACD
$AE = AD$ (given)
$\angle AEB = \angle ADC$ (proven above)
$\angle BAE = \angle CAD$ (common angle)
$\therefore \Delta ABE \equiv \Delta ACD$ (AAS)

(iii) Hence prove that ΔBFC is isosceles. **3**

Method 1:
$EB = DC$ (matching sides of congruent triangles)
$FB = EB - EF$
$= DC - DF$ (from above)
$\therefore FB = FC$
$\therefore \Delta FBC$ is isosceles.

Method 2:
$AB = AC$ (matching sides of congruent triangles)
$\therefore \angle ABC = \angle ACB$ (equal angles opposite equal sides)
$\angle ABE = \angle ACD$ (matching angles of congruent triangles)
$\therefore \angle FBC = \angle FCB$ (subtraction of equal angles from equal angles)
$\therefore FB = FC$ (equal sides opposite equal angles)
$\therefore \Delta BFC$ is isosceles

Comment

Students that made simple mistakes with reasons or inappropriate use of symbols lost a $\frac{1}{2}$ mark for each mistake. Students that made a logical error lost 1 mark for each error.
In part (iii), students who made the question easier could only score 1 mark.

Students are encouraged to plan their answer first before starting to write their solution down.

(b) The sum of the first 9 terms of the series **3**

$$\log_3\left(\frac{1}{x}\right)+\log_3\left(\frac{1}{x^2}\right)+\log_3\left(\frac{1}{x^3}\right)+\ldots \text{ is } -270.$$

Find the value of x.

..

Arithmetic series with $a=-\log_3 x$ and $d=-\log_3 x$

$$S_n=\frac{n}{2}(a+l)$$

$$S_9=\frac{9}{2}(-\log_3 x-9\log_3 x)$$

$$=-45\log_3 x$$

$$\therefore -45\log_3 x=-270$$

$$\therefore \log_3 x=6$$

$$\therefore x=3^6=729$$

Comment

Generally when students started from the basis of an arithmetic series they were successful. Students that made the mistake of assuming that the series was geometric could only score 1 mark, but then for only some substantive work e.g. showing $r = 2$ or equivalent.

(c) Find the equation of the line (in general form) perpendicular to $3x-4y+12=0$ and intersecting it on the x-axis. **2**

x-intercept of $3x-4y+12=0$: $x=-4$

Method 1:

The line perpendicular to $3x-4y+12=0$ is $4x+3y+k=0$

Substitute (–4, 0): $4\times(-4)+3\times 0+k=0\Rightarrow k=16$

$$\therefore 4x+3y+16=0$$

Method 2:

$$3x-4y+12=0\Rightarrow 4y=3x+12$$

$$\therefore y=\frac{3}{4}x+3$$

$$\therefore m_{\text{perp}}=-\frac{4}{3}$$

$$\therefore y-0=-\frac{4}{3}(x+4)$$

$$\therefore 3y=-4x-16$$

$$\therefore 4x+3y+16=0$$

Comment

A surprising number of students could not work out the x-intercept.

Q14.

(a)(i) $x = t^3 - 9t^2 + 24t - 7$

$V = 3t^2 - 18t + 24$

For stat pts $V = 0 \Rightarrow t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

$t = 2$ or $t = 4$

$\therefore$ particle 1st changes direction when $t = 2$ sec

(2)

$x(2) = 8 - 36 + 48 - 7$

$x = 13$ cm

ie particle 1st changes direction when $t = 2$ sec at $x = 13$ cm

(ii) Distance travelled in first 3 sec

$= |x(3) - x(2)| + |x(2) - x(0)|$

$= |11 - 13| + |13 - (-7)|$

$= 2 + 20$

$= 22$ cm

(2)

(iii) Av. speed in 1st second $= \dfrac{x(1) - x(0)}{1 - 0}$

$= \dfrac{9 - -7}{1}$

$= 16$ cm/sec

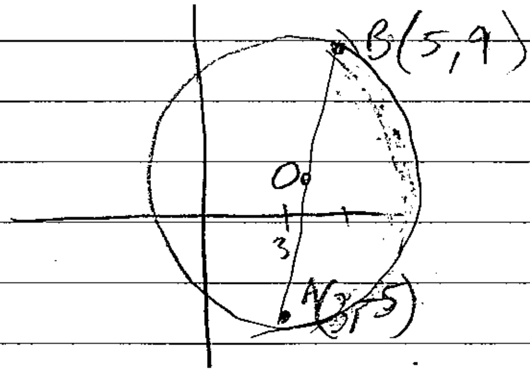
(1)

(a) was done well.

(b) Common mistake was to find displacement rather than distance. 1 mark awarded if students realized they had to split time into 0-2 sec and 2-3 seconds.

(c) Common error was to find the average of the velocities.

(b)



(i) Centre = midpoint AB = $(4, 2)$ ✓

(ii) Radius = $\sqrt{(5-4)^2+(9-2)^2}$

$= \sqrt{1+49}$

$= \sqrt{50} = 5\sqrt{2}$ ✓

(iii) $(9, k)$ lies on circle. Eqn of circle is

$(x-4)^2+(y-2)^2=50$.

3

Sub in $(9,k)$ $\Rightarrow$ $25+(k-2)^2=50$

$(k-2)^2 = 25$

$k-2 = \pm 5$

$k = 2 \pm 5$

$k = -3$ or 7 ✓

This section was generally done well.

(c) $\int_0^2 \sin(1+x^2)\,dx$

x	0	1	2
y	$\sin 1$	$\sin 2$	$\sin 5$

$A = \frac{1}{3}\left[\text{ends} + 4(\text{odds})\right]$

$= \frac{1}{3}\left[\sin 1 + \sin 5 + 4\sin 2\right]$

$= \frac{1}{3}\left[0.8414 + -0.9589 + 3.637\right]$

$= 1.1732$

$A = 1.17$ to 2dp.

2

Many students made the mistake of using degrees instead of radians.

2015 Accelerated Mathematics Yearly:
Solutions— Question 15

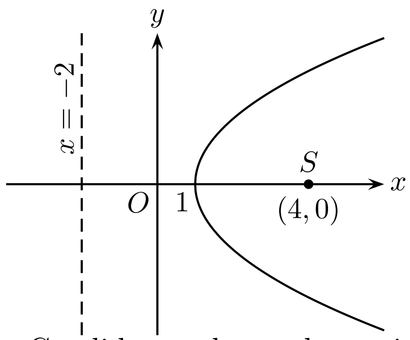
15. (a) The equation of a parabola is $y^2 = 12(x-1)$.

(i) Find the coordinates of the vertex of the parabola. 1

Solution: Vertex $(1, 0)$.
Comment: Generally well done.
Students at this level should be aware that ordered pairs are written in parentheses, although no marks were deducted for failure to observe this convention.

(ii) Find the coordinates of the focus of the parabola. 1

Solution:

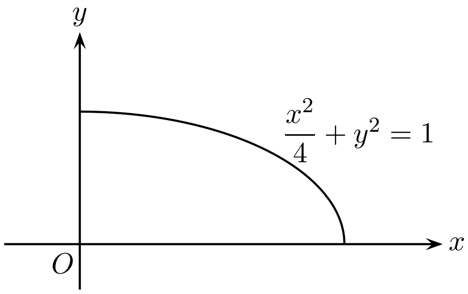


$y^2 = 4 \times 3(x-1)$.
Focal length is 3.
Focus $(4, 0)$.

Comment: Candidates who made a quick sketch had fewer errors than those who did not.

(iii) Find the equation of the directrix of the parabola. 1

Solution: Directrix $x = -2$.
Comment: Too many candidates do not realise that the directrix is a line, not a point (the word "equation" in the question should have been sufficient reminder).

(b)  **2**

The part of the curve $\frac{x^2}{4} + y^2 = 1$ that lies in the first quadrant is rotated about the x-axis.
Find the volume of the solid of revolution.

Solution:

$$\begin{aligned} \text{When } x &= 0,\ y = 1, \\ \text{when } y &= 0,\ x = 2. \\ \text{Vol.} &= \pi \int_0^2 y^2\,dx, \\ &= \pi \int_0^2 \left(1 - \frac{x^2}{4}\right) dx, \\ &= \pi \left[x - \frac{x^3}{12}\right]_0^2, \\ &= \pi\big(2 - \tfrac{2}{3} - 0 + 0\big), \\ &= \frac{4\pi}{3}. \end{aligned}$$

Comment: This was generally well done. Candidates who gave an approximate answer *after* the exact solution were not penalised.

(c) A container, initially empty, is filled so that the volume V, in litres, at the time t minutes is given by $V = 32t - 4t^2,\ 0 \leqslant t \leqslant 4$.

(i) How much water is in the container when it is full? **2**

Solution:

$$\begin{aligned} \frac{dV}{dt} &= 32 - 8t, \\ &= 0 \text{ when } t = 4. \\ \frac{d^2V}{dt^2} &= -8,\ \textit{i.e.}\text{ maximum}, \\ \therefore V_{\text{full}} &= 32 \times 4 - 4 \times 4^2, \\ &= 64\,\text{L}. \end{aligned}$$

Comment: To gain both marks, candidates had to show why they chose $t = 4$ by either differentiation to establish a maximum, or by a clearly drawn sketch, or by the year 10 method of finding the vertex.

(ii) After how many minutes was the container three quarters full? 2

Solution:

$$\begin{aligned} \tfrac{3}{4} \text{ of } 64 &= 48, \\ 48 &= 32t - 4t^2, \\ t^2 - 8t + 12 &= 0, \\ (t-6)(t-2) &= 0, \\ t &= 6, 2. \\ \text{But } t &\leqslant 4, \\ \text{so } t &= 2 \text{ minutes.} \end{aligned}$$

Comment: The first mark was for getting the correct factorisation. The second for a clear explanation of why $t = 2$ is the only correct solution.

(iii) At what rate was the water flowing into the container when the container was three quarters full? 1

Solution:

$$\begin{aligned} \frac{dV}{dt} &= 32 - 8 \times 2, \\ &= 16. \end{aligned}$$

i.e. 16 L/min.

Comment: This was well done by the majority of candidates, although some were confused about what the units are.

QUESTION 16.

(a).(i) $y = 108x - x^4$

$y' = 108 - 4x^3$

$y'' = -12x^2$.

For stationary points

$y' = 0$

ie. $108 - 4x^3 = 0$.

$x^3 = 27$

$x = 3$

$\therefore (3, 243)$ is stationary

when $x = 3$ $y'' = -12 \times 9$

$= -108$

< 0 $\therefore$ MAX. TURNING POINT.

COMMENT. The vast majority of students gained full marks.

(ii). For points of inflexion

let $y'' = 0$

$-12x^2 = 0$

$x = 0$

Test

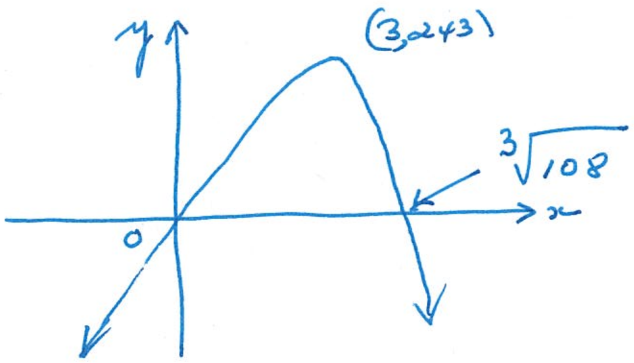
x	-1	0	1
y''	-12	0	-12

$\therefore$ NO CHANGE IN CONCAVITY

$\therefore$ NO. PT of INFLEXION.

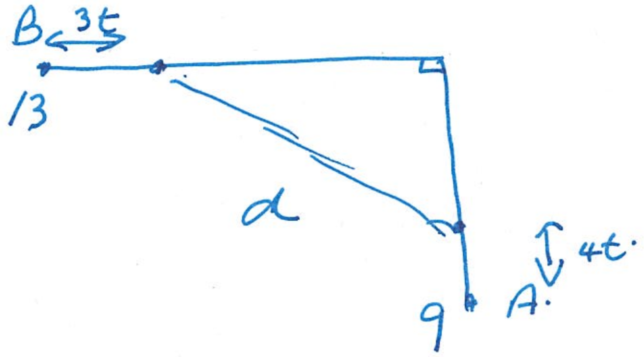
COMMENT Important to test for change in concavity USING NUMBERS (VALUES) for y''.

(iii)



COMMENT: Well done.

(b)



$\therefore \quad d^2 = (13-3t)^2 + (9-4t)^2$

$= 169 - 78t + 9t^2 + 81 - 72t + 16t^2$

$= 25t^2 - 150t + 250.$

$(d^2)' = 50t - 150.$

$(d^2)'' = 50.$

$\therefore$ Min. at $t = 3$ NB $(d^2)'' > 0$

at $t = 3$

$d^2 = 25 \times 9 - 150 \times 3 + 250$

$= 225 + 250 - 450$

$= 25$

$\therefore d = 5$ $\therefore$ AFTER 3 HOURS the DISTANCE APART is 5 km

COMMENT
A FEW DID NOT SHOW A MINIMUM.
OTHERS DIFFERENTIATED d rather than d^2.
MOST SCORED FULL MARKS