2017 SYDNEY BOYS HIGH SCHOOL
YEAR 11 YEARLY

Mathematics Accelerated

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using **black** pen
- Board-approved calculators may be used
- A reference sheet is provided with this paper
- Leave your answers in the simplest exact form, unless otherwise stated
- All necessary working should be shown in every question if full marks are to be awarded
- Marks may **NOT** be awarded for messy or badly arranged work
- In Questions 7 – 13, show relevant mathematical reasoning and/or calculations
- Start each **NEW** question in a separate answer booklet

Total marks – 57

Section I Pages 2 – 4

8 marks

- Attempt Questions 1 – 8
- Allow about 10 minutes for this section

Section II Pages 5– 8

49 marks

- Attempt Questions 9 – 12
- Allow about 1 hour and 20 minutes for this section

Examiner: *B.Kilmore.*

Section I – Multiple Choice

5 Marks
Attempt question 1 − 5
Allow approximately 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 − 10

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1. The expression $1-\sin^2\left(\frac{\pi}{2}-\theta\right)$ is equal to

   (A) $\cos\left(\frac{\pi}{2}-\theta\right)$

   (B) $\sin\left(\frac{\pi}{2}-\theta\right)$

   (C) $\cos^2\theta$

   (D) $\sin^2\theta$

2. If the period is π , the range is $-2\le y\le 4$, and the horizontal translation is $\frac{\pi}{4}$, the equation for the trigonometric function in the form $y=A\cos(Bx+C)+D$ is

   (A) $y=3-2\cos\left(\pi x+\frac{\pi}{4}\right)$

   (B) $y=3\cos\left(2x-\frac{\pi}{2}\right)+1$

   (C) $y=3\cos\left(2x+\frac{\pi}{2}\right)-1$

   (D) $y=3\cos\left(3x-\frac{3\pi}{4}\right)-1$

3. If $f'(x)=e^{2x}+k$ and $f(x)$ has a stationary point at (0, 2), where k is a constant, then $f(1)$ is equal to

   (A) $\frac{1}{2}e^2+\frac{1}{2}$

   (B) $e^2+\frac{1}{2}$

   (C) e^2-1

   (D) e^4

4. The derivative of $f(x)=\dfrac{\sin 4x}{4x+1}$ is

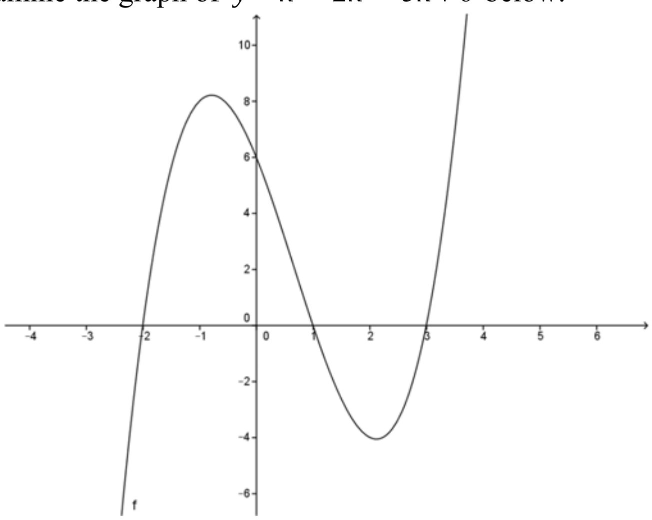
(A) $\dfrac{4(4x+1)\cos x-4\sin 4x}{(4x+1)^2}$

(B) $\dfrac{(4x+1)\cos 4x-4\sin 4x}{(4x+1)^2}$

(C) $\dfrac{4(4x+1)\cos 4x-4\sin 4x}{(4x+1)^2}$

(D) $\dfrac{4\sin(4x)-4(4x+1)\cos(4x)}{(4x+1)^2}$

5. Examine the graph of $y=x^3-2x^2-5x+6$ below.



The area between the curve and the X-axis from $x=-2$ and $x=3$ is equal to

(A) $10\dfrac{5}{12}$ *sq units*

(B) $22\dfrac{5}{12}$ *sq units*

(C) $21\dfrac{1}{12}$ *sq units*

(D) $11\dfrac{3}{4}$ *sq units*

6. The indefinite integral $\int\left(\cos\frac{x}{3}-3\sin 3x\right)dx$ is equal to

(A) $\sin\frac{x}{3}+\cos 3x+C$

(B) $\frac{1}{3}\sin\frac{x}{3}+\cos 3x+C$

(C) $\frac{1}{3}\sin\frac{x}{3}-\cos 3x+C$

(D) $3\sin\frac{x}{3}+\cos 3x+C$

7. A trigonometric function is $f(x)=3\cos 2(x-\pi)+1$. Its amplitude, period and range are given by

	Amplitude	Period	Range
(A)	3	π	R
(B)	2	$\frac{2\pi}{3}$	$-4\le y\le 4$
(C)	3	π	$-2\le y\le 4$
(D)	π	3	$-2\le y\le 4$

8. The sum of the solutions of the equation $\sin(4x)=0.5$ for $0\le x\le\frac{\pi}{2}$ is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{24}$

(C) $\frac{3\pi}{4}$

(D) π

Section II

50 marks
Attempt Questions 6 – 9
Allow about 1 hour and 20 minutes for this section

Answer each question in a NEW writing booklet. Extra pages are available

In Questions 11 – 14, your responses **should include** relevant mathematical reasoning and/or calculations.

Question 9 (13 Marks) Start a NEW Writing Booklet

(a) Differentiate

(i) $y = \dfrac{\cos 3x}{x^2}$ **2**

(ii) $y = e^x \sin 5x$ **2**

(b) If $y = x^2 \ln(2x)$

(i) find $\dfrac{dy}{dx}$ **2**

(ii) hence find $\int x \ln(2x) dx$ **2**

(c) After heavy rain, the flood gate of a dam was opened. Water was released from the dam at a rate of $31.8t$ litres / second where t is measured in seconds after the gate is opened.

(i) Find an expression for the total volume of water, V, released from the dam in k seconds. **1**

(ii) Calculate the time it took for 10^9 *litres* of water to be released from the dam. Express your answer to the nearest second. **2**

(d) Consider the function $f(x) = 2^{-|x|}$.

(i) Show algebraically that $f(x)$ is an even function. **1**

(ii) Sketch the graph of $y = f(x)$. **1**

Question 10 (12 Marks) Start a NEW Writing Booklet

(a) Find

(i) $\int \tan x \, dx$ **1**

(ii) $\int \frac{e}{\pi x - 1} dx$ **2**

(b) A particle is moving on the X-axis and is initially at the origin. Its velocity, $v \; m/s$,

at time t seconds is given by $v = \frac{4}{t+1} - 2t$.

(i) What is the initial velocity of the particle? **1**

(ii) Find the time when the particle changes direction. **2**

(iii) Find the distance travelled by the particle in the first 3 seconds. **3**

(c) As soon as an organism dies, the rate at which Carbon-14 (C-14) will decay is given as

$\frac{dx}{dt} = -kx$ where $k > 0$ and x is the amount of C-14 present in the organism at time t.

Given that the half-life of C-14 is 5730 years, if the organism is found to contain 0.2% of its original C-14, for how long has the organism been left to decay? **3**

Question 11 (12 Marks) Start a NEW Writing Booklet

(a) Consider the two functions $f(x)$ and $g(x)$ where $f(x)=\dfrac{e^x-e^{-x}}{2}$ and $g(x)=\dfrac{e^x+e^{-x}}{2}$

(i) Show that the graph $y=f(x)$ is increasing for all values of x and that there is a point of inflection at $x=0$. **2**

(ii) Show that the graph of $y=g(x)$ has a minimum at $x=0$. **2**

(iii) Let $y=f(x)$. Show that this equation can be written in the form $e^{2x}-2ye^x-1=0$. Hence make x the subject of the formula. **3**

(c) At the start of a month, Henry deposited M dollars into a new bank account and kept depositing M dollars at the start of each of the following months.

The money in the account was earning interest at the rate of 0.4% per month. compounding monthly.

At the same time, his son started putting \$10 in a jar at the start of that month, and each month after that he put into the jar \$5 more than the previous month.

i) Show that the son has saved \$3510 after 3 years. **2**

ii) If the total money saved by Henry and his son is \$18035 after 3 years, find the value of M to the nearest dollar. **3**

Question 12 (12 Marks) Start a NEW Writing Booklet

(a) The rate at which substances are removed from the blood by the kidneys is proportional to the amount of the substance left in the blood. A patient is administered $100mg$ of a drug intravenously at 11am. At 1pm, 67mg of the drug remains in the blood.

(i) Find the equation relating the amount of drug in the blood , A , to the time elapsed, t. **2**

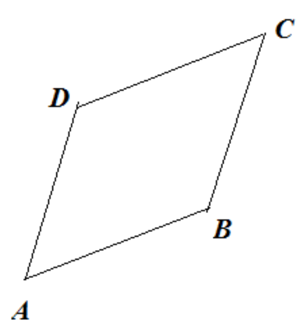
(ii) At 3pm, the patient is accidentally given another full dose of $100mg$. When will the amount of blood in the patient's blood be back down to 100mg? **2**

(b) For the two curves $y = x^2 - x + 1$ and $y = 2x^2 - 4x + 3$, find

(i) the area between them. **2**

(ii) the volume obtained when this area is rotated about the X-axis. **2**

(c) In the diagram ABCD is a rhombus with $AC = 6\sqrt{3}\ cm$ and $BD = 6\ cm$.



(i) Show that the rhombus has side length $6\ cm$. **2**

(ii) If $A(0,0)$ and $B(3\sqrt{3},3)$, find the coordinates of C. **2**

End of Exam

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2017 SYDNEY BOYS HIGH SCHOOL
Year 11 Yearly

Mathematics Accelerated

Sample Solutions

Question	Teacher
Q1 – 8	–
Q9	EC
Q10	AF
Q11	PSP
Q12	PB

MC Answers

1. D
2. B
3. A
4. C
5. C
6. D
7. C
8. A

2U Y11 Yearly 2017 Multiple choice solutions

Mean (out of 8): 6.93

1. $1-\sin^3\left(\frac{\pi}{2}-\theta\right)$

$= 1-\cos^2\theta$

$= \sin^2\theta$ (D)

A	2
B	1
C	10
D	92

2. A – Range $1 \le y \le 5$ ✗

B – Period $= \frac{2\pi}{2} = \pi$

Range $-3+1 \le y \le 3+1$

$\therefore -2 \le y \le 4$

$2x - \frac{\pi}{2} = 2\left(x - \frac{\pi}{4}\right)$

$\therefore$ Horizontal translation of $\frac{\pi}{4}$ ✓

C – Range $-4 \le y \le 2$ ✗

D – Range $-4 \le y \le 2$ ✗ (B)

A	1
B	96
C	4
D	3

3. When $x=0$, $f'(x)=0$

$\therefore e^0 + k = 0$

$\therefore k = -1$

$\therefore f'(x) = e^{2x} - 1$

$\therefore f(x) = \frac{1}{2}e^{2x} - x + c$

$f(0) = \frac{1}{2} - 0 + c = 2$

$\therefore c = 1\frac{1}{2}$

$\therefore f(x) = \frac{1}{2}e^{2x} - x + 1\frac{1}{2}$

$\therefore f(1) = \frac{1}{2}e^2 - 1 + 1\frac{1}{2}$

$= \frac{1}{2}e^2 + \frac{1}{2}$ (A)

A	93
B	8
C	4
D	0

4. $f'(x) = \frac{(4x+1).\cos 4x.4 - \sin 4x.4}{(4x+1)^2}$

$= \frac{4(4x+1)\cos 4x - 4\sin 4x}{(4x+1)^2}$ (C)

A	5
B	4
C	95
D	1

5. Area $= \int_{-2}^{1} (x^3 - 2x^2 - 5x + 6)\,dx$

$- \int_{1}^{3} (x^3 - 2x^2 - 5x + 6)\,dx$

$= \left[\frac{x^4}{4} - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x\right]_{-2}^{1}$

$- \left[\frac{x^4}{4} - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x\right]_{1}^{3}$

$= \left[\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6\right] - \left[\frac{16}{4} + \frac{16}{3} - 10 - 12\right]$

$- \left\{ \left[\frac{81}{4} - 18 - \frac{45}{2} + 18\right] - \left[\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6\right] \right\}$

$= \frac{253}{12}$

$= 21\frac{1}{12}$ sq units.

(C)

A	4
B	12
C	88
D	1

6. $\int \left(\cos\frac{x}{3} - 3\sin 3x \right) dx$

$= 3\sin\frac{x}{3} + \cos 3x + c$ (D)

A	1
B	4
C	0
D	100

7. $f(x) = 3 \cos 2(x - \pi) + 1$

↑ amplitude; $\frac{2\pi}{2} = \pi$ period

Range $-3+1 \le f(x) \le 3+1$

$-2 \le f(x) \le 4$

(C)

A	4
B	0
C	101
D	0

8. $\sin(4x) = 0.5 \quad 0 \le x \le \frac{\pi}{2}$

$\therefore 4x = \frac{\pi}{6}$ or $\frac{5\pi}{6} \quad \therefore 0 \le 4x \le 2\pi$

$\therefore x = \frac{\pi}{24}$ or $\frac{5\pi}{24}$

Sum of roots $= \frac{\pi}{24} + \frac{5\pi}{24}$

$= \frac{6\pi}{24}$

$= \frac{\pi}{4}$ (A)

A	76
B	17
C	2
D	10

Question (9) (a)

(a)

(i) $y = \frac{\cos 3x}{x^2}$

$\frac{dy}{dx} = \frac{-x^{2}(3\sin 3x) - 2x(\cos 3x)}{x^{4}}$

$= \frac{-(3x\sin 3x + 2\cos 3x)}{x^3}$

(ii) $\frac{dy}{dx} = e^{x}\sin 5x + 5\cos 5x\, e^{x}$

$= e^{x}(\sin 5x + 5\cos 5x)$

(b)

(i) $\ln 2x(2x) + x^2 \times \frac{1}{x}$

$\frac{dy}{dx} = x(2\ln 2x + 1)$.

(ii) $\therefore 2\int x\ln(2x)\,dx + \frac{x^2}{2} = x^2\ln(2x) + c$

$\Longrightarrow \int x\ln(2x)\,dx = \frac{x^2}{2}\ln 2x + \frac{x^2}{4} + c$.

(c) $\frac{dv}{dt} = 31.8t$

(i) $\therefore v = \int_0^k (31.8t)\,dt$

$= [15.9t^2]_0^k$

$= 15.9k^2$

(ii) $15.9k^2 = 10^9$

$k^2 = \frac{10^9}{15.9}$

$k = 10^4 \times 0.793$

$= 7931$

(d)

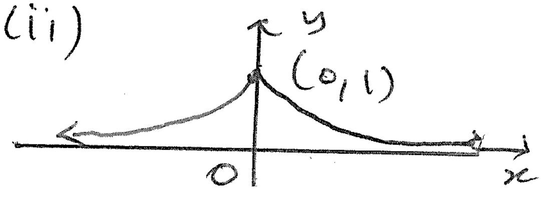
(i) $f(-x) = 2^{-|-x|}$

$= 2^{-|x|}$

$= f(x)$

$\therefore f$ is even

(ii)



Comment: Question

(a)
(i) & (ii)

Answered very well by most students.
Some students have not simplify the derivative by "cancelling" the x.

(b) A basic "differentiate and then integrate" question. Students who recognised that the final answer should be halved to get $\int x \ln(2x)\,dx$ are most successful.

(c) Students made the careless mistake of selecting
(i) $C = k$ as the answer rather then $\frac{k^2}{2}$.

(ii) A mistake was taken off by error carried was given (solution was simplied) even if the continued working was correct.

10) a) i) $\int \tan x \, dx$

$= -\int \frac{-\sin x}{\cos x} \, dx$

$= -\ln|\cos x| + C$

ii) $\int \frac{e}{\pi x - 1} \, dx$

$= \frac{e}{\pi} \int \frac{\pi}{\pi x - 1} \, dx$

$= \frac{e}{\pi} \ln|\pi x - 1| + C$

COMMENT: There is no reason why students should not know the integral of $\tan x$.
Given its not on the reference sheet we must be able to use a trig. identity to put in a better form.

b) i) $v = \frac{4}{t+1} - 2t$

when $t = 0$

$v = \frac{4}{(0)+1} - 2(0)$

$v = 4$ m/s.

ii) when $v = 0$

$\frac{4}{t+1} - 2t = 0$

$4 - 2t^2 - 2t = 0$

$-2(t^2 + t - 2) = 0$

$-2(t+2)(t-1) = 0$

$t = 1, -2$

particle changes direction when $t = 1$ s.

iii) $v = \frac{4}{t+1} - 2t$

$x = 4\ln|t+1| - t^2 + c$

when $t=0$, $x=0$

$0 = 4\ln|(0)+1| - (0)^2 + C$

$c = 0$

$x = 4\ln|t+1| - t^2$

when $t=1$

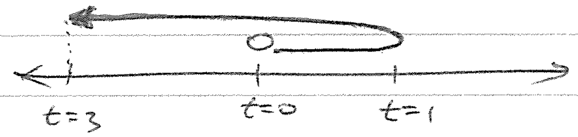
$x = 4\ln|(1)+1| - (1)^2$

$= 4\ln 2 - 1$

when $t=3$

$x = 4\ln|(3)+1| - (3)^2$

$= 4\ln 4 - 9$



Distance travelled is $2(4\ln 2 - 1) - (4\ln 4 - 9)$

$= 8\ln 2 - 2 - 8\ln 2 + 9$

$= 7$ m

COMMENT:

Part (i) & (ii) were generally done well. Many mistakes were made in part (iii).

Some students found the displacement when $t=3$.

Many students made errors in simplifying, particularly if they used absolute value.

ALTERNATIVELY

Distance Travelled $= \int_0^1 v\,dt + \left|\int_1^3 v\,dt\right|$

$= \int_0^1 v\,dt - \int_1^3 v\,dt$.

c) $x = x_0 e^{-kt}$

when $t = 5730$

$x = \frac{x_0}{2}$

$\frac{x_0}{2} = x_0 e^{-k(5730)}$

$e^{-5730k} = \frac{1}{2}$

$-5730k = \ln(\frac{1}{2})$

$k = \frac{-\ln(\frac{1}{2})}{5730}$

≈ 0.00012

$x = x_0 e^{-0.00012t}$

when $x = 0.2\% x_0$

$= 0.002 x_0$

$0.002 x_0 = x_0 e^{-0.00012t}$

$-0.00012t = \ln(0.002)$

$t = \frac{-\ln(0.002)}{0.00012}$

$t = 51373.94$ years

The organism has been left to decay approximately 51374 years.

COMMENT:

- Students need to recognise that $x = x_0 e^{-kt}$ is a solution to $\frac{dx}{dt} = -kx$.
- We are looking for the time it takes for $x = 0.002x_0$ not $x = 0.2x_0$.
- Even if $k = 0.00012$ is used in working $k = -\frac{\ln(\frac{1}{2})}{5730}$ should be used in calculations

Question 11 SOLUTIONS **(12 Marks)**

(a) Consider the two functions $f(x)$ and $g(x)$ where

$$f(x)=\frac{e^x-e^{-x}}{2} \text{ and } g(x)=\frac{e^x+e^{-x}}{2}$$

(i) Show that the graph $y=f(x)$ is increasing for all values of x and that there is a point of inflection at $x=0$. **2**

$$f'(x)=\frac{e^x+e^{-x}}{2}=g(x)$$

Given that $e^{\pm x}>0$ for all x, then $f'(x)>0$ i.e. increasing for all x.

$$f''(x)=\frac{e^x-e^{-x}}{2}=f(x)$$

At $x=0$, $f''(0)=\dfrac{e^0-e^0}{2}=0$, but is there a change in concavity?

x	-1	0	1
$f''(x)$	$\frac{1}{2}(e^{-1}-e)\doteqdot -1.18$	0	$\frac{1}{2}(e-e^{-1})\doteqdot 1.18$

As there is a change in concavity, then there is a point of inflexion at $x=0$.

Comment

This is a "Show that" question, which means that students must show more detail.

Too many students are using the quotient rule to differentiate.

The argument that as $x\to\infty$, $y\to\infty$ (and similarly $x\to-\infty$, $y\to-\infty$) does not show that the graph is increasing e.g. $y=x^4+x$

Students were penalised if they didn't show a change of concavity.

(ii) Show that the graph of $y=g(x)$ has a minimum at $x=0$. **2**

From (i) above $g'(x)=f(x)=\dfrac{e^x-e^{-x}}{2}$.

Solving $g'(x)=0 \Leftrightarrow \dfrac{e^x-e^{-x}}{2}=0$

$\therefore e^x=e^{-x}\Leftrightarrow x=0$ (considering y-intercepts)

$g''(x)=f'(x)=\dfrac{e^x+e^{-x}}{2}>0$ from (i).

So there is an (absolute) minimum at (0, 1).

Comment

Students who recognised the connection between $f(x)$ and $g(x)$ were more able to be efficient with this part i.e. speed and accuracy.

Students need to remind themselves that they need to show numbers when justifying max and min, even more so when it is a "show that question."

Question 11 SOLUTIONS (continued)

(a) (iii) Let $y = f(x)$. Show that this equation can be written in the form **3**

$$e^{2x} - 2ye^x - 1 = 0 .$$

Hence make x the subject of the formula.

$$y = \frac{e^x - e^{-x}}{2} \Rightarrow 2y = e^x - \frac{1}{e^x}$$
$$\therefore 2e^x y = e^{2x} - 1 \Rightarrow e^{2x} - (2y)e^x - 1 = 0$$

Note that $e^{2x} = (e^x)^2$.

So either by using a substitution $u = e^x$ or going straight to the quadratic formula with $(e^x)^2 - (2y)e^x - 1 = 0$

$$\therefore e^x = \frac{2y \pm \sqrt{4y^2 + 4}}{2}$$
$$= y \pm \sqrt{y^2 + 1}$$

As $y - \sqrt{y^2 + 1} < 0$ and $e^x > 0$ then $e^x = y + \sqrt{y^2 + 1}$.

$$\therefore x = \log_e \left(y + \sqrt{y^2 + 1} \right)$$

Comment

Many students either didn't see the last part of this question or were unable to do it.

Question 11 SOLUTIONS (continued)

(b) At the start of a month, Henry deposited M dollars into a new bank account and kept depositing M dollars at the start of each of the following months. The money in the account was earning interest at the rate of 0.4% per month. compounding monthly. At the same time, his son started putting \$10 in a jar at the start of that month, and each month after that he put into the jar \$5 more than the previous month.

(i) Show that the son has saved \$3510 after 3 years. **2**

The amounts that Henry's son puts in the jar forms an arithmetic series with $a = 10$ and $d = 5$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 10 + 5(n-1) \\ &= 5n + 5 \end{aligned}$$

So $T_{36} = 185$

The total amount put in the jar is $S_{36} = \dfrac{36}{2}(10 + 185) = 3510$.

Comment

Generally well done.

Students were penalised for not showing enough detail as it is a ”Show that” question.

(b) (ii) If the total money saved by Henry and his son is \$18 035 after 3 years, find the value of M to the nearest whole number. **3**

How much did Henry save?

Method 1:

The first \$$M$ accrues $M(1.004)^{36}$.

The first \$$M$ accrues $M(1.004)^{35}$.

$\vdots$

The last \$$M$ accrues $M(1.004)$.

So the total is $M(1.004)+M(1.004)^2+...+M(1.004)^{36}$

$$\text{Total} = M\left[\underbrace{(1.004)+(1.004)^2+...+(1.004)^{36}}_{\text{GP: } a=1.004,\ r=1.004,\ n=36}\right]$$

$$= M\times\frac{1.004(1.004^{36}-1)}{1.004-1}$$

$$= 251M(1.004^{36}-1)$$

Method 2:

Let A_n be the amount accrued after n months

$$A_1 = M(1.004)$$

$$A_2 = (A_1 + M)(1.004)$$

$$= M(1.004)^2 + M(1.004)$$

$$A_3 = (A_2 + M)(1.004)$$

$$= M(1.004)^3 + M(1.004)^3 + M(1.004)$$

$$\therefore A_{36} = M(1.004)^{36} + M(1.004)^{35} + ... + M(1.004)$$

This now follows the same as Method 1.

From (i): $251M(1.004^{36}-1)+3510=18\,035$

$\therefore 251M(1.004^{36}-1)=14\,525$

$$\therefore M = \frac{14\,525}{251(1.004^{36}-1)}$$

$$\doteqdot 374$$

Comment

Students were penalised for not showing enough detail.

This part was not a difficult example of a “superannuation” problem, yet too many students were either using r = 0.004, 0.04, 0.4, 1.4 or 1.04

Students who used method 2 and made an initial logical error could only get a maximum mark of 2.

QUESTION 12.

(a)(i) Given that $\frac{dA}{dt} \propto A$ where A = AMOUNT of DRUG IN BLOOD in mg
t = time (hours)

$$\therefore \frac{dA}{dt} = +kA.$$

$$\frac{dt}{dA} = \frac{+1}{kA}.$$

$$t = \frac{1}{k}\ln A + c.$$

$$\ln A = kt - kc.$$

$$A = e^{kt-kc}$$

$$A = A_0 e^{kt}.$$

now $A_0 = 100$.

& $A = 67$ when $t = 2$.

$$\therefore 67 = 100e^{2k}.$$

$$0.67 = e^{2k}.$$

$$\ln 0.67 = 2k.$$

$$k = \frac{1}{2}\ln 0.67$$

$$\doteqdot -0.2002$$

$$\therefore \boxed{A = 100e^{-0.2002t}}$$

COMMENT: most were awarded 2 marks for this part.

(ii) at 3pm $t = 4$.

$$\therefore A = 100 e^{-0.2002 \times 4}$$

$$\doteqdot 44.90.$$

Let $A = 144.90 \, e^{-0.2002t}$.

To find when (ie at what time)

$A = 100$.

Let $100 = 144.9 e^{-0.2002t}$.

$$\ln\left(\frac{100}{144.9}\right) = -0.2002t$$

$$t = \frac{-1}{0.2002} \ln \frac{100}{144.9}$$

$$\doteqdot 1.8525.$$

this converts into 1 hour 51 mins

$\therefore$ at **4.51 PM.**

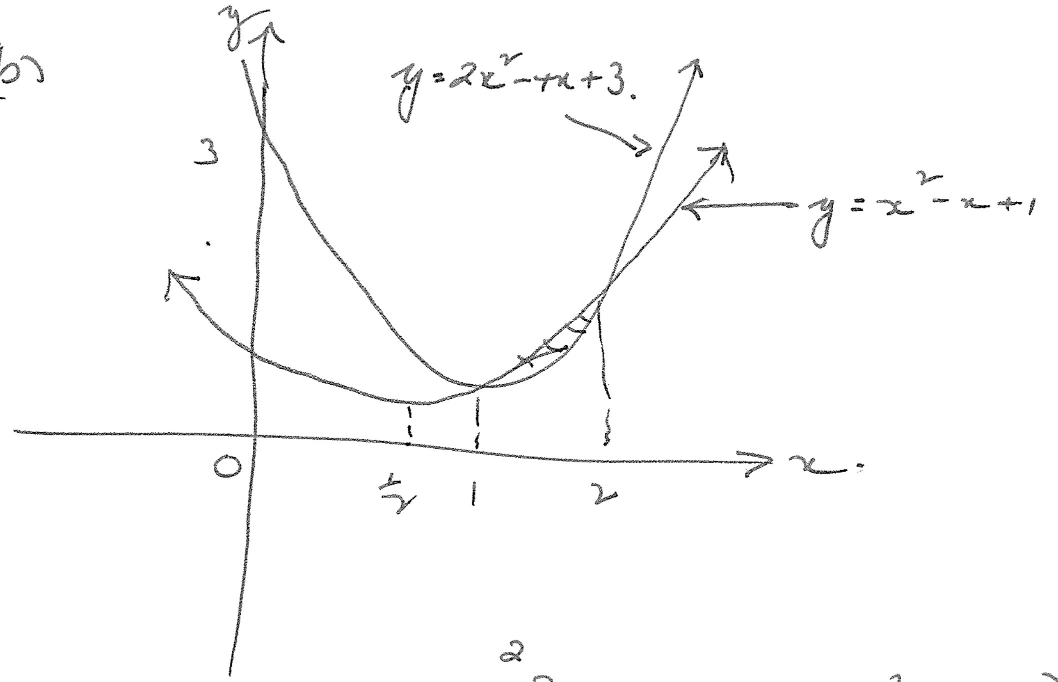
COMMENT The question asked for 'when'

$\therefore$ 1.8525 hours was not the required answer.

nor was the question asked for the nearest hour.

ie. 4.51 PM was required.

(b)



(i) $$\text{Area} = \int_1^2 \left[(x^2 - x + 1) - (2x^2 - 4x + 3)\right] dx$$

$$= \int_1^2 (-x^2 + 3x - 2)\, dx$$

$$= \left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x\right]_1^2$$

$$= \left(\frac{-8}{3} + 6 - 4\right) - \left(-\frac{1}{3} + \frac{3}{2} - 2\right)$$

$$= -\frac{2}{3} + \frac{5}{6}.$$

$$= \frac{1}{6}\ u^2$$

COMMENT The most common mistake was to have the curves in the wrong order and getting $-\frac{1}{6}\ u^2$.

(11) Volume $= \pi \int_1^2 \left[(x^2-x+1)^2 - (2x^2-4x+3)^2 \right] dx$

$= \pi \int_1^2 (3x^2-5x+4)(-x^2+3x-2)\, dx$

$= \pi \int_1^2 (-3x^4+14x^3-25x^2+22x-8)\, dx$

$= \pi \cdot \left[\frac{-3x^5}{5} + \frac{14}{4}x^4 - \frac{25x^3}{3} + \frac{22x^2}{2} - 8x \right]_1^2$

$= \pi \cdot \left[\frac{-3x^5}{5} + \frac{7x^4}{2} - \frac{25}{3}x^3 + 11x^2 - 8x \right]_1^2$

$= \pi \left[\left(-\frac{96}{5} + 15 - \frac{200}{3} + 44 - 16 \right) - \left(-\frac{3}{5} + \frac{7}{2} - \frac{25}{3} + 11 - 8 \right) \right]_1^2$

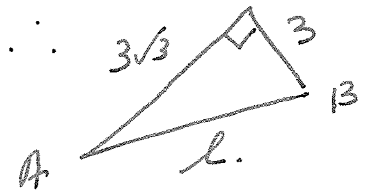
$= \pi \left(\frac{312}{5} - \frac{371}{6} \right)$

$= \pi \, \frac{1872 - 1855}{30}$

$= \frac{17\pi}{30} \ u^3$.

COMMENT: The algebra proved too difficult for most. Using the difference of two squares was probably the best approach.

(C) (i) now the diagonals of a rhombus bisect at right-angles



$\therefore l^2 = 9 + 27$

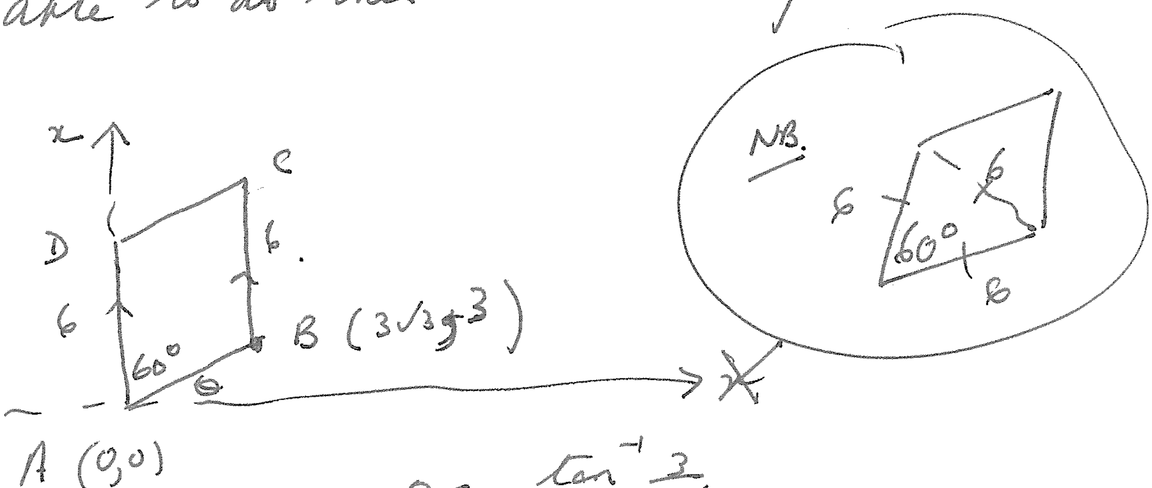
$= 36.$

$l = 6$

$\therefore$ side length is 6 cm.

COMMENT most students were able to do this successfully.

(ii)



$\theta = \tan^{-1}\frac{3}{3\sqrt{3}}$

$= \tan^{-1}\frac{1}{\sqrt{3}}$

$= 30°$

$\therefore D\hat{A}X = 90°$ $(60° + 30°)$

$\therefore AD \parallel CB.$

$\therefore C$ is $(3\sqrt{3}, 9)$

COMMENT Very few were able to do this question.