Baulkham Hills High School

YEAR 11

YEARLY EXAMINATION

2008

MATHEMATICS
ADVANCED

Time allowed: Three hours

DIRECTIONS TO CANDIDATES:

- Approved calculators may be used
- Begin each question on a new page.
- Show all necessary working.

Marks

Question 1

a) Factorise completely

i) $a^2 - 25$ 1

ii) $x^2 - 3x + 2$ 1

iii) $4m^3 - 32n^3$ 2

b) Solve for x.

i) $6x^2 - x - 2 \geq 0$ 2

ii) $|x + 2| = 4$ 2

iii) $\dfrac{2x+3}{3} - \dfrac{3x-1}{4} = 3$ 2

c) Evaluate correct to 2 sig. figures.

$\sqrt[3]{\dfrac{86.54 \times 3.16}{2.6^3}}$ 2

Question 2

a) Convert $0.\dot{2}\dot{3}$ to a fraction in its simplest form. 2

b) Simplify $\dfrac{3^3}{(3^x)^3 \times 3^{-3x}}$ 2

c) Express $\dfrac{3\sqrt{2}+1}{2\sqrt{3}-1}$ with a rational denominator. 2

d) If $(2\sqrt{3} - 2)(\sqrt{3} - 1) = a - \sqrt{b}$, find a and b. 2

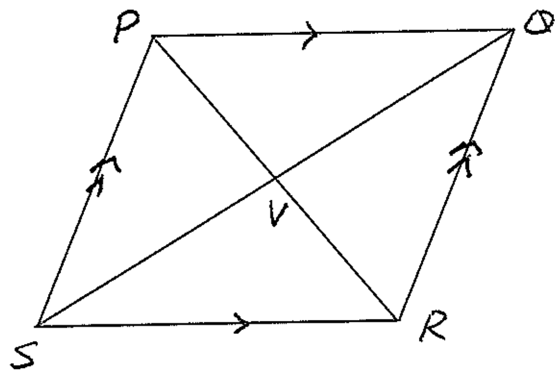
e) At a shop all prices increased by 35%. One item now costs \$81.20. What is the old price of this item? 2

f) Express $(a^{\frac{1}{2}} - b^{\frac{-1}{2}})^2$ without indices. 2

Question 3

Marks

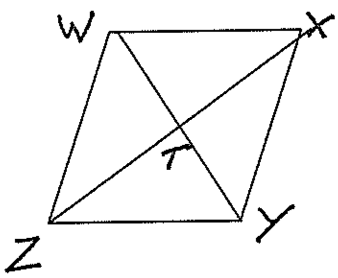
a)



NOT TO SCALE

PQRS is a parallelogram whose diagonals intersect at V. Given that $\angle QPR = 68^\circ$ and $\angle QSR = 47^\circ$, find the size of $\angle SVR$. Give reasons for your answer. 2

b) In the diagram below, WXYZ is a rhombus

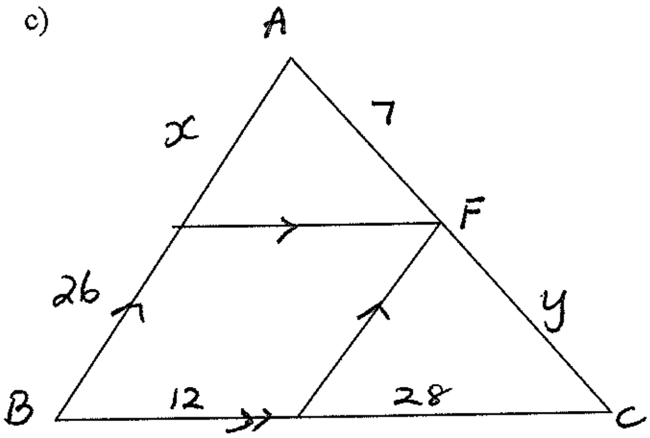


NOT TO SCALE

(i) Prove that $\Delta WXT \equiv \Delta YZT$ 3

(ii) State why $\angle WTX = 90^\circ$. 1

c) 4



NOT TO SCALE

In the diagram, BA is parallel to DF and EF parallel to BC. Find x and y (correct to 1decimal place). Give reasons.

d) Calculate the size of each internal angle of a regular 20 sided polygon. 2

Question 4

a) Differentiate 7

$i)\ x^3 - 2x^2 + 1$

$ii)\ \sqrt{x^2 - 6}$

$iii)\ (2x+1)^8$

$iv)\ \dfrac{5x^3 - x}{x^2}$

b) Simplify 6

$i)\ \dfrac{m^6 \times m^4}{m^8}$

$ii)\ \dfrac{m+4}{m^2 + 2m - 8}$

$iii)\ \dfrac{3}{m^2 - 4} - \dfrac{5}{2m-4}$

c) Consider the arithmetic sequence: a, 3a -1, 5a -2

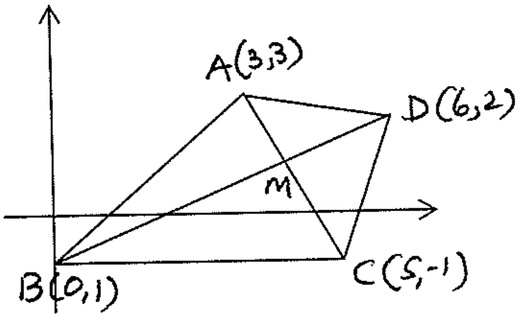
i) Find the common difference. 1

ii) Find the sum of the first twenty terms. 2

Marks

Question 5

In a quadrilateral ABCD, the points A, B, C and D are (3,3), (0-1) and (5, -1)and (6,2) respectively. The line BD bisects the line AC at right angles at the point M



NOT TO SCALE

a) Find the distance of BD. 1
b) Show that the gradient of BD is ½. 1
c) Show that the equation of the line BD is x- 2y – 2 = 0. 2
d) Show that the equation of the line AC is 2x + y – 9 = 0. 3
e) What are the coordinates of M? 2
f) Find the area of ABCD. 3

Question 6

a) If $f(x) = 2x^2 + 5x + 1$ calculate f(-3). 2

b) Consider the function $y = \sqrt{16 - x^2}$

   i) State the domain. 1
   ii) Show that it is an even function. 1

c) The equation $x^2 - 6x + y^2 - 12y + 20 = 0$ represents a circle.

   i) Show that the centre is (3,6) and the radius is 5. 2
   ii) Show that the line x -12 y + 1 = 0 does not intersect or touch the circle. 2

d) Show the region represented by $y < 4 - x^2$ and $y \geq 3^x - 3$. 4

Marks

Question7

a) Sketch each of the following showing all important features: 9

i) $y = 3^x$

ii) $(x - 2)^2 + y^2 = 4$

iii) $y = \frac{-1}{x}$

iv) $y = |2x - 4|$

b) What is the range of $f(x) = \frac{1}{\sqrt{8 - 2x}}$ 2

c) Find the exact value of 3

i) $\sin 330^0$

ii) $\cos 240^0$

iii) $\sec 225^0$

d) Find the equation of the tangent to the curve $y = 4 - x^2$ at $x = 1$. 2

QUESTION 8

a) If $90^0 \leq \theta \leq 180^0$ and $\sin \theta = \frac{2}{3}$, find the exact value of $\cos \theta$. 2

b) Solve for θ where $0^0 \leq \theta \leq 360^0$.

i) $\sin \theta = \frac{1}{\sqrt{2}}$ 2

ii) $3 \tan^3 \theta = \tan \theta$ 3

c) A person walks 10 km East from A to B and then 12km South East to C.

i) Draw a neat diagram and mark the given information 2

ii) Find the distance to the nearest km between A and C. 2

iii) What is the bearing of A from C? 2

Marks

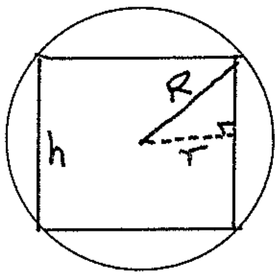
Question 9

a) The population of kangaroos in a nature reserve is given by

$$P = 100 + 6t^2 - 2t^3 \text{ (where t is in years)}$$

i) find the stationary points of the function and determine their nature. 4
ii) Find any point of inflexion. 2
iii) Sketch the function in the domain $0 \le t \le 5$ 2
iv) When was the population of the kangaroos increasing most rapidly. 1

b) A metal worker is required to cut a circular cylinder with radius r cm , height h cm, from a solid sphere of metal radius R =4cm. The diagram below shows a cross-section of the sphere and the cylinder.



(i) Show that the volume V cm^3 of the cylinder is given by 2

$$V = \frac{1}{4}\pi h(64 - h^2)$$

(ii) Find the value of h such that the volume of the cylinder is maximum. 3

Question 10

i) For what values of m does the quadratic equation $x^2 + mx + (m+1)^2 = 0$ have two real roots. 3

ii) If a:b:c is 1:2:3 and $a^2 + b^2 + c^2 = 504$ find the value of a, where a is a positive number 2

iii) If α and β are the roots of equation $x^2 + 5x - 8 = 0$ find the value of $(\alpha + 2)(\beta + 2)$. 2

iv) A series is defined by $s_n = 3n^2 - 11n$. Prove that the series s_n is an arithmetic progression. 2

v) Consider the series $\cos^2 x + \cos^4 x + \cos^6 x + ...$ $\quad 0° < x < 90°$ 3
   i) show that a limiting sum exists.
   ii) Find the limiting sum expressing the answer in its simplest form.

END OF EXAMINATION

2008 2U yearly exam, year 11.

Question 1

a) (i) $(a-5)(a+5)$ ✓

(ii) $(x-2)(x-1)$ ✓

(iii) $4(m^3-8n^3)$ ✓

$= 4(m-2n)(m^2+2mn+4n^2)$ ✓

b) (i) $6x^2-4x+3x-2 \geq 0$

$2x(3x-2)+1(3x-2) \geq 0$

$(2x+1)(3x-2) \geq 0$ ✓

$x \leq -\frac{1}{2}$ or $x \geq \frac{2}{3}$ ✓

(ii) $x+2=4 \quad \therefore x=2$ ✓

$-x-2=4 \quad x=-6$ ✓

(iii) $4(2x+3)-3(3x-1)=36$ ✓

$-x+15=36$

$x=-21$ ✓

c) $= 0. \ldots$

$\doteqdot \ldots$ ✓

$= 2.496479$ ✓

$\doteqdot 2.5$ ✓

(12)

Question 2

a) $x = 0.\dot{2}\dot{3}$

$100x = 23.2323\ldots$ ✓

$99x = 23$

$x = \frac{23}{99}$ ✓

b) $\frac{3^3}{3^0} = 27$ ✓ ✓

(c) $\frac{3\sqrt{2}+1}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1}$ ✓

$= \frac{6\sqrt{6}+2\sqrt{3}+3\sqrt{2}+1}{12-1}$

$= \frac{6\sqrt{6}+2\sqrt{3}+3\sqrt{2}+1}{11}$ ✓

(d) $(2\sqrt{3}-2)(\sqrt{3}-1)$

$= 6-2\sqrt{3}-2\sqrt{3}+2$

$= 8-4\sqrt{3}$ ✓

$= 8-\sqrt{48}$

$a=8 \quad b=48$ ✓

e) $\frac{81.20}{135} \times 100$ ✓ $= \$60.15$ ✓

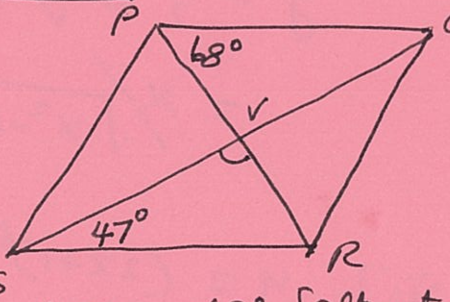
f) $(a^{\frac{1}{2}}-b^{-\frac{1}{2}})^2 = a-2a^{\frac{1}{2}}b^{-\frac{1}{2}}+b$ ✓

$= a+b-2\sqrt{\frac{a}{b}}$ ✓

(12)

Question 3

a)



$\angle PRS = 68°$ [alternate ∡ in || lines]

$\therefore S\hat{V}R = 180-(47+68)$ [∡ sum of Δ is 180°]

$= 65°$

(b) In ΔWXT and ZTY

$\angle WTX = \angle ZTY$ (vert-opp ∡) ✓

$WT = TY$ ✓ } diagonals bisect each other

~~TX~~ $= TS$

$\Delta WTX \equiv \Delta ZTY$. [SAS] ✓

(c) Diagonals of a rhombus meet at 90° ✓

Question 3. (ctd)

$$\frac{y}{7} = \frac{28}{12} \checkmark \Rightarrow y = 16.3$$ ✓

$$\frac{x}{26} = \frac{7}{y} \checkmark \Rightarrow x = 11.1$$

~~sides of~~ intercepts of the same set of ∥ lines are in proportion! ✓

d) $\text{exterior } \angle = \frac{360}{20} = 18$ ✓

$\therefore \text{interior } \angle = 162$ ✓

12

Question 4

i) $y = x^3 - 2x^2 + 1$ ✓

$$\frac{dy}{dx} = 3x^2 - 4x + 0.$$

ii) $y = \sqrt{x^2 - 6}$

$$y' = \frac{1}{2}(x^2-6)^{-1/2} \times 2x. \checkmark \checkmark$$

$$= x / \sqrt{x^2 - 6}$$

iii) $y = (2x+1)^8$

$$y' = 8 \checkmark (2x+1)^7 \times 2 \checkmark$$

$$= 16(2x+1)^7$$

(iv) $y = \frac{5x^3 - x}{x^2} = 5x - \frac{1}{x}.$

$$y' = 5 \checkmark + \frac{1}{x^2}. \checkmark$$

Question 4 (ctd)

b)

i) m^2 ✓

ii) $\frac{\cancel{m+4}}{\cancel{(m+4)}(m-2)} = \frac{1}{m-2}$ ✓ ✓

iii) $\frac{3}{(m-2)(m+2)} - \frac{5}{2(m-2)}$ ✓ ✓

$$= \frac{1}{(m-2)}\left[\frac{3}{m+2} - \frac{5}{2}\right]$$

$$= \frac{-4-5m}{2(m-2)(m+2)}. \checkmark$$

c) Common difference

i) $= 3a - 1 - a = 2a - 1$ ✓

$n = 20.$

(ii) $S_{20} = \frac{\cancel{20}^{10}}{\cancel{2}}\left\{2a + 19(2a-1)\right\}$ ✓

$$= 20a + 380a - 190$$

$$= \underline{\underline{400a - 190}}. \checkmark$$

16

Question 5

a) $d_{BD} = \sqrt{(6-0)^2 + (2-1)^2}$

$= \sqrt{36+9}$

$= \sqrt{45}$ ✓

b) $m_{BD} = \frac{2-1}{6-0} = \frac{1}{2}$ ✓

c) $y - 2 = \frac{1}{2}(x-6)$ ✓

$2y - 4 = x - 6$

$x - 2y - 2 = 0$.

d) $y + 1 = \frac{3+1}{3-5}(x-5)$

$y + 1 = \frac{4}{-2}(x-5)$

$y + 1 = -2x + 10$

$2x + y - 9 = 0$.

e) $M(4,1)$ ✓✓

f) $d_{AM} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$ ✓

$\text{Area} = \frac{1}{2} BD \times AM \times 2$.

$= \sqrt{45} \times \sqrt{5}$

$= 15 \text{ unit}^2$ ✓

Question 6.

a) $f(-3) = 2 \times -3^2 + 5 \times -3 + 1$ ✓

$= 18 - 15 + 1$

$= 4$ ✓

b) (i) $-4 \le x \le 4$ ✓

(ii) $f(-x) = \sqrt{16 - (-x)^2}$

$= 16 - x^2$

$= f(x)$ ✓

Hence it is an even function.

c) $x^2 - 6x + y^2 - 12y + 20 = 0$.

(i) $x^2 - 6x + 9 + y^2 - 12y + 36 + 20 = 45$

$(x-3)^2 + (y-6)^2 = 25$ ✓

$= 5^2$

∴ Centre $(3,6)$ radius 5.

$x - 12y + 1 = 0$.

$x = 12y - 1$

$(12y - 1 - 3)^2 + (y-6)^2 = 25$

$16(3y-1)^2 + (y-6)^2 = 25$

$16[9y^2 - 6y + 1] + y^2 - 12y + 36 = 25$

$145y^2 - 96y + 16 - 12y + 36 = 25$

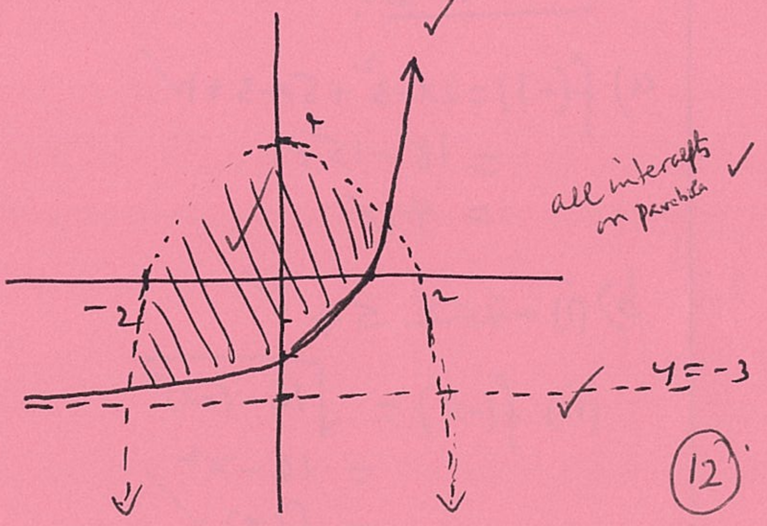
$145y^2 - 108y + 27 = 0$ ✓

$\Delta = 108^2 - 4 \times 27 \times 145 < 0$. ✓

∴ There are no real roots.

Hence $x - 12y + 1 = 0$ does not touch the circle.

6 d.

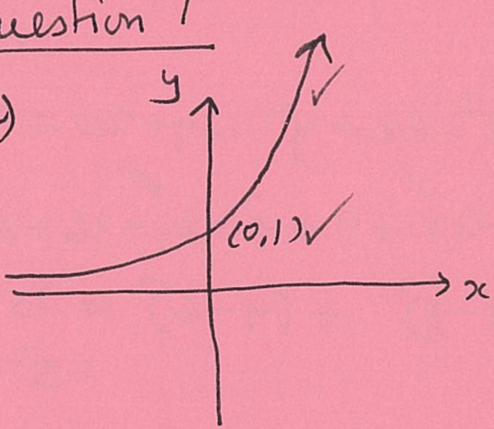


all intercepts on parabola ✓

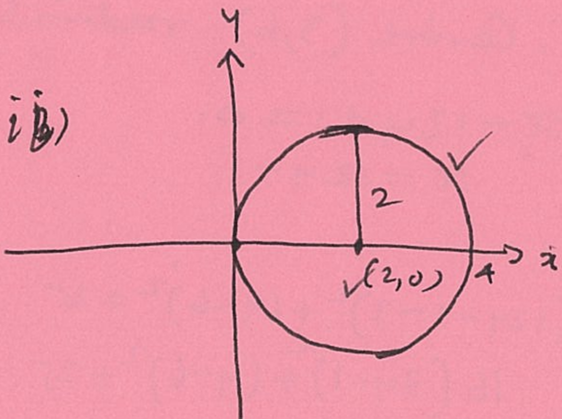
(12)

Question 7

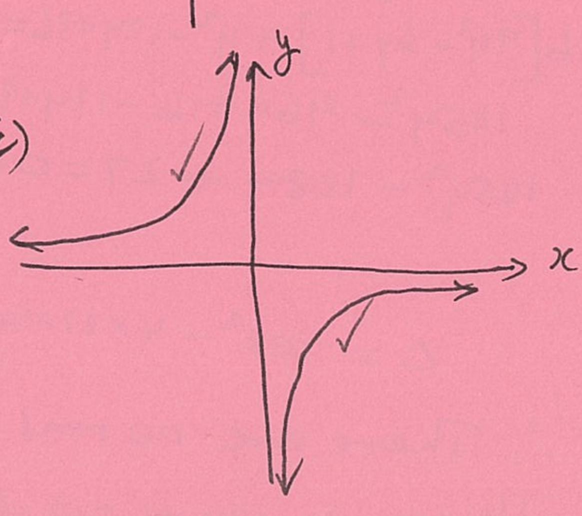
a) i)



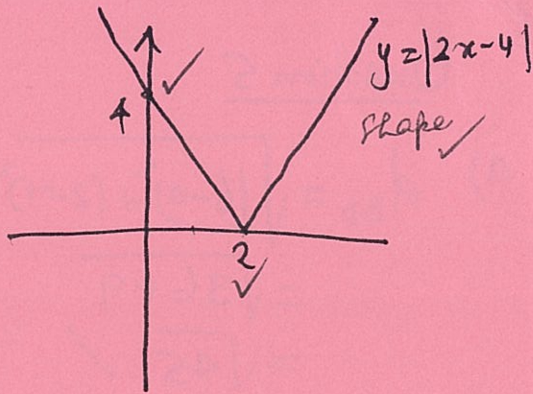
ii)



iii)



(iv)



b) all real f(x) s.t f(x)>

c) (i) $\sin(360 - 30^\circ) = -\frac{1}{2}$ ✓

(ii) $\cos(180 + 60^\circ) = -\frac{1}{2}$ ✓

(iii) $\sec 225^\circ = \frac{1}{\cos(180+45)} = -\sqrt{2}$ ✓

d) $y' = -2x.$

$y'_{x=1} = -2$ ✓

$x = 1 \Rightarrow y = 4 - 1 = 3.$

$y - 3 = -2(x - 1).$

$y + 2x - 5 = 0.$

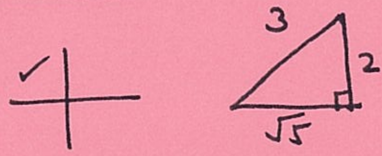
(15)

Toman.

Ursino

Nada J.

Question 8

a) 

$\cos\theta = -\sqrt{5}/3$ ✓

b)(i)  $\sin\theta = \frac{1}{\sqrt{2}}$

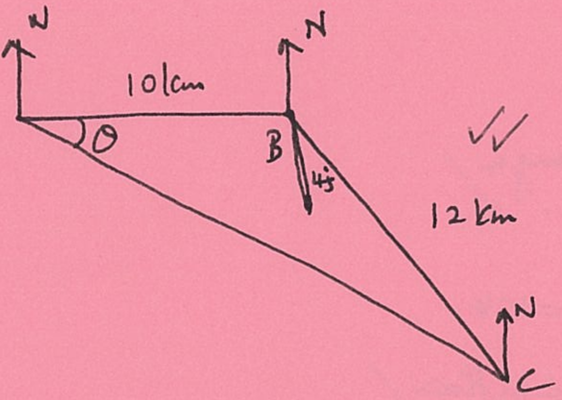
$\theta = 45^\circ, 135^\circ$ ✓✓

(ii) $\tan\theta(3\tan^2\theta - 1) = 0$

$\therefore \tan\theta = 0 \quad \tan\theta = \pm\frac{1}{\sqrt{3}}$ ✓

$\theta = 0^\circ, 180^\circ, 360^\circ$ ✓

$\theta = 30^\circ, 210^\circ, 150^\circ, 330^\circ$ ✓

c) (i) 

✓✓

(ii) $AC^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos 135^\circ$ ✓

$AC \doteqdot 20$ km. ✓

(iii) $\frac{12}{\sin\theta} = \frac{20}{\sin 135^\circ}$ ✓

$\sin\theta = \frac{12 \times \sin 135^\circ}{20}$

$= 0.41717$

$\theta = 24^\circ 39'$

13

Bearing N $65^\circ 21'$ W

$294^\circ 39'$ T ✓

Question 9.

i) $P = 100 + 6t^2 - 2t^3$

$P' = -6t^2 + 12t \quad P'' = -12t + 12.$

Stationary pts occur when $P' = 0$.

$-6t^2 + 12t = 0 \implies t = 0, t = 2$

$t = 0 \implies P = 100. \quad A(0,100)$ ✓

$t = 2 \implies P = 100 + 24 - 16 \quad B(2,108)$ ✓

at $(0,100)$ $P' = 0$ and $P'' > 0$

$\therefore$ $(0,100)$ minimum turning pt ✓

at $(2,108)$ $P' = 0$ and $P'' < 0$.

$\therefore$ $(2,108)$ is a maximum turning pt. ✓

(ii) when $P'' = 0 \implies t = 1$

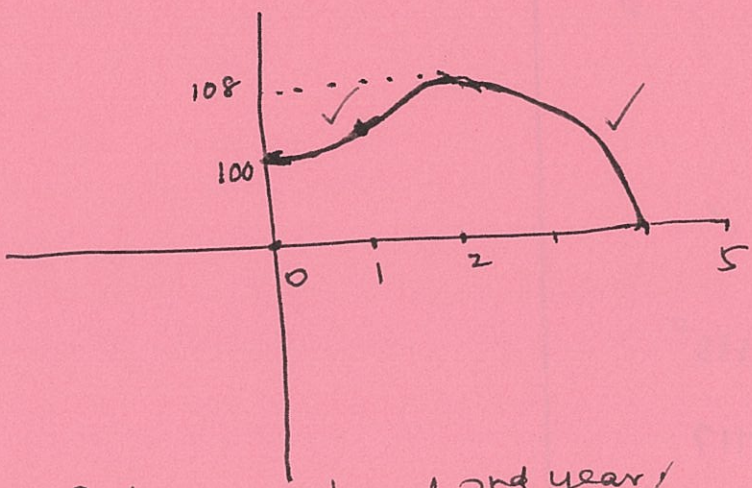
	1^-	1	1^+
P''	+ve	0	-ve

$\therefore$ There is a change in concavity. ✓

$t = 1 \implies P = 100 + 6 - 2 = 104.$

$\therefore$ $(1,104)$ is a point of inflexion ✓

(iii)



(iv) Between 1st and 2nd year ✓

Question 9

b).

(i) $V = \pi r^2 \cdot h$. — ②

Also $R^2 - r^2 = \left(\frac{h}{2}\right)^2$ ✓

$16 - \frac{h^2}{4} = r^2$. — ①

Sub ① in ②.

$V = \pi h \cdot \left(16 - \frac{h^2}{4}\right)$. ✓

$= \frac{\pi h}{4}(64 - h^2)$.

(ii) $V = 16\pi h - \frac{\pi h^3}{4}$.

$\frac{dV}{dh} = 16\pi - \frac{3}{4}\pi h^2$ ✓

For a max volume

$\frac{dV}{dh} = 0$ and $\frac{d^2V}{dh^2} < 0$. ✓

$64\pi - 3\pi h^2 = 0$

$h^2 = \frac{64}{3}$.

$h = 8/\sqrt{3}$ ✓

$\frac{d^2V}{dh^2} = 0 - \frac{6\pi}{4} \cdot h$. this is always < 0. for all h.

$\therefore$ Value for h for max Volume

is $8/\sqrt{3}$ or $\frac{8\sqrt{3}}{3}$ cm //.

Question 10

i) $a = 1 \quad b = m \quad c = (m+1)^2$

For real roots $\Delta \geq 0$ ✓

$m^2 - 4 \times 1 (m+1)^2 \geq 0.$

$m^2 - 2^2(m+1)^2 \geq 0$

$[m - 2(m+1)][m + 2m + 2] \geq 0.$

$(-m-2)(3m+2) \geq 0.$

$(m+2)(3m+2) \leq 0$ ✓

$-2 \leq m \leq -2/3$ ✓

(ii) $1a : 2a : 3a.$

$a^2 + (2a)^2 + (3a)^2 = 504$ ✓

$14a^2 = 504$

$a^2 = 36$

$a = 6$ ✓

(iii) $\alpha + \beta = -5, \quad \alpha\beta = -8$

$(\alpha+2)(\beta+2) = \alpha\beta + 2(\alpha+\beta) + 4.$ ✓

$= -8 + 2(-5) + 4$

$= -14$ ✓

(iv) $S_1 = -8, \quad S_2 = -10, \quad S_3 = 27 - 33 = -6$

✓ $T_1 = -8$, $T_2 = -2$, $T_3 = 4.$ (+6, +6.)

There exists a common diff. ✓

$\therefore$ S_n defines an AP.

~~(iv) $\sin^2 x + \sin^4 x + \sin^6 x - - - -$~~

~~$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \sin^2 x.$~~

~~$\therefore$ there exist a common ratio $\sin^2 x$.~~

~~$\forall\ 0 < x < 90 \quad 0 < \sin x < 1$~~

~~ie $0 < \sin^2 x < 1$~~

~~$\therefore S_\infty = \frac{a}{1-r} = \frac{\sin^2 x}{1-\sin^2 x} = \frac{\sin^2 x}{\cos^2 x}$~~

~~$= \tan^2 x.$~~

~~$\forall\ 0 < x < 90$ $\tan^2 x$ is defined hence exist.~~

(v) (ii) $\cos^2 x + \cos^4 x + \cos^6 x + - - - - -$

common ratio $\frac{\cos^4 x}{\cos^2 x} = \cos^2 x$ ✓

$\forall\ 0 < x < 90 \quad -1 < \cos x < 1$

$\therefore \quad \cos^2 x < 1$ ✓

Hence the common ratio < 1

$\therefore S_\infty = \frac{a}{1-r}$

$= \frac{\cos^2 x}{1 - \cos^2 x} = \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$ ✓

12