BAULKHAM HILLS HIGH SCHOOL

YEARLY EXAMINATION

2009

YEAR 11

MATHEMATICS ADVANCED

Time Allowed: Three Hours

Directions to Candidates:

- Approved calculators may be used
- Begin each question on a new page
- Write your name and your teacher's name at the top of each page
- Show all necessary working

Question 1 (12 marks) **Start on a SEPARATE page** **Marks**

a) Evaluate correct to 3 significant figures **2**

$$\sqrt{\frac{39.4 - 13.55}{8.63^2 - 1.98}}$$

b) Solve $\frac{x+5}{3} - 2x = 7$ **2**

c) Simplify $2\sqrt{2} + \sqrt{18}$ **2**

d) Express $0.\dot{2}\dot{4}$ as a fraction in its simplest form **2**

e) Find the exact value of

i) $\sin 60^\circ$ **1**

ii) $\cos 225^\circ$ **1**

f) Factorise and simplify **2**

$$\frac{x^2 - 5x - 24}{x^2 + 3x}$$

Question 2 (12 marks) **Start on a SEPARATE page** **Marks**

a) Differentiate

i) $y = 6x^2 - 8x + 7$ **1**

ii) $y = 2\sqrt{x} - \frac{4}{x}$ **2**

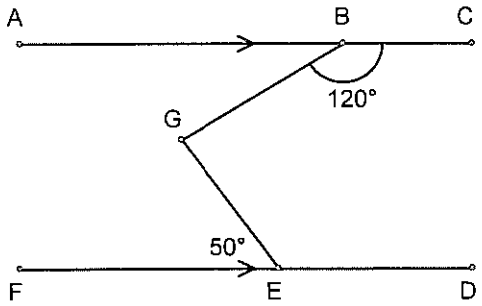
iii) $y = \frac{5x^2}{2x+7}$ **2**

b) Solve simultaneously

$4x - y + 3 = 0$
$xy = 1$ **3**

c) If $\frac{1}{3+2\sqrt{2}} = a + b\sqrt{2}$ find a and b **2**

d) Find the value of the reflex angle $\angle BGE$ **2**
Give reasons.



Question 3 (12 marks) **Start on a SEPARATE page** **Marks**

a) Solve $x^2-6x+5\geq 0$ **2**

b) If $f(x)=\begin{cases} -4 & for \quad x\leq -2 \\ x & for \quad -2<x<1 \\ x^2-1 & for \quad x\geq 1 \end{cases}$

i) Find $f(-1)$ **1**

ii) Find $f(a^2+1)$ **2**
Simplify your answer.

iii) Sketch the graph of $f(x)=y$. **3**

c) Given $\sin\alpha=-\frac{3}{7}$ and $\tan\alpha>0$, find the exact value of $\sec\alpha$ **2**

d) Solve for $0^0\leq\theta\leq 360^0$ **2**

$$\sin\theta+\frac{\sqrt{3}}{2}=0$$

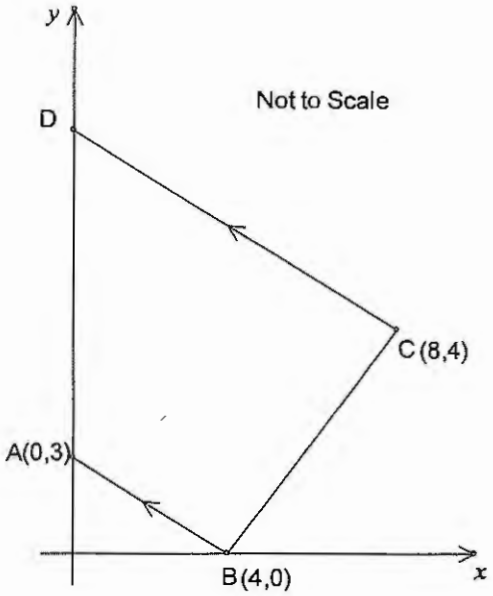
Question 4 (10 marks) **Start on a SEPARATE page** **Marks**

a) i) Show that $\frac{x}{x+2} = 1 - \frac{2}{x+2}$ **1**

ii) Hence or otherwise, sketch the following graph showing all important features

$$y = \frac{x}{x+2}$$ **2**

b)



Given $A(0,3)$, $B(4,0)$, $C(8,4)$ and $AB \| CD$

i) Find the gradient of AB **1**

ii) Show that the equation of AB is $3x+4y-12=0$ **2**

iii) Find the coordinates of D **1**

iv) Find the distance of AB **1**

v) Find the perpendicular distance of C from AB **2**

vi) Find the area of $ABCD$. Answer in exact form. **2**

Question 5 (10 marks) **Start on a SEPARATE page** **Marks**

a) Solve for $0^0 \leq \theta \leq 360^0$

i) $\cos^2\theta - \frac{1}{2} = 0$ **2**

ii) $\sin\theta = \sqrt{3}\cos\theta$ **2**

b) Solve $|4x - 2| \geq 14$. Sketch your solution on the number line **3**

c) The point $P(x, y)$ moves such that PA is perpendicular to PB. **3**

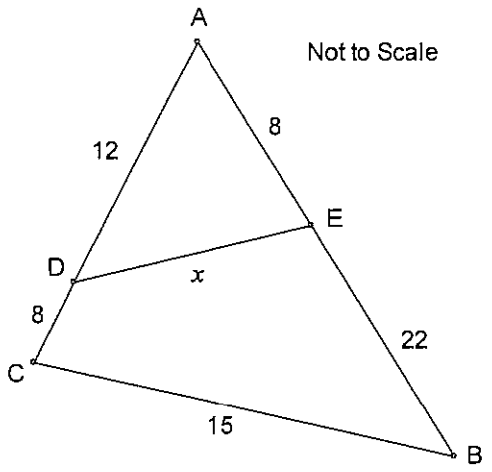
Find the locus of P if A is the point (-3,6) and B is the point (-7,-6)

Describe the locus.

Marks

Question 6 (13 marks) Start on a SEPARATE page

a) Given that $AD = 12$, $AE = 8$, $DC = 8$, $CB = 15$ and $EB = 22$



i) Prove that $\Delta AED \;|||\; \Delta ACB$ **3**

ii) Hence, find the value of x, justifying your working. **2**

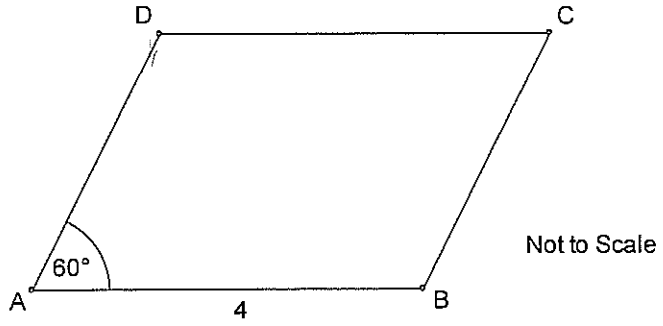
b) Evaluate $\sum_{n=1}^{\infty} 3^{-n}$ **2**

c) For what values of k does the quadratic equation $x^2 - (k+3)x + 4k = 0$

i) have no real roots. **3**

ii) have one real root equal to -3. **1**

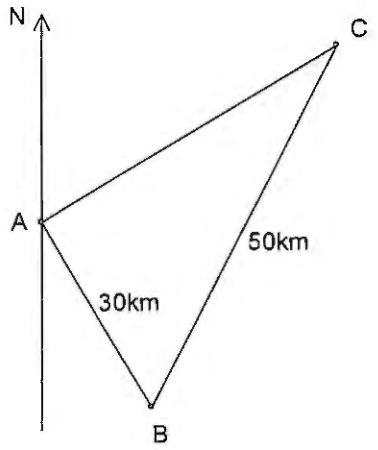
d) Find the area of a rhombus $ABCD$ with sides of 4cm and $\angle DAB = 60^0$ **2**
Answer as an exact value.



Marks

Question 7 (10 marks) Start on a SEPARATE page

a) Two friends drive 30km from A on a bearing of 150°T to point B.
They then drive from B to C, a further 50km on a bearing of 050°T.



i) Show that $\angle ABC = 80^0$ 1

ii) Find the distance from A to C 2

iii) Find the bearing of A from C 2
Show all the working

b) Determine if the function $f(x) = \dfrac{1}{(x-1)(x+1)}$ is an odd or even function, or neither. 2

Justify your answer.

c) Given $kx^2 - (k+1)x - 2k + 3$ 3
find for what value(s) of k, the expression above is negative definite.

Question 8 (12 marks) Start on a SEPARATE page

Marks

a) The first three terms of a geometric series are $2x + 2$, $5x + 1$ and $10x + 2$ **2**
Find the value(s) of x.

b) Simplify $\sec(180^0 - \theta).\tan(90^0 - \theta)$ **3**

c) On a number plane, shade the region satisfying both the inequalities **3**

$y > \sqrt{9-x^2}$ *and* $y \geq x^2 + 3$

d) The quadratic equation $2x^2 - 3x + 6 = 0$ has roots α *and* β

Find the value of

i) $\alpha + \beta$ **1**

ii) $\alpha\,\beta$ **1**

iii) $\alpha^3 + \beta^3$ **3**

Question 9 (12 marks) Start on a SEPARATE page

Marks

a) Given $y = x^3 - 2x^2 + x$

i) Find $\frac{dy}{dx}$ **1**

ii) Find the stationary point(s) and determine their nature **3**

iii) Find the point(s) of inflexion. Justify your answer. **2**

iv) Sketch this curve for $-1 \leq x \leq 2$ showing all relevant features **2**

v) State where this function is increasing **and** concave up. **1**

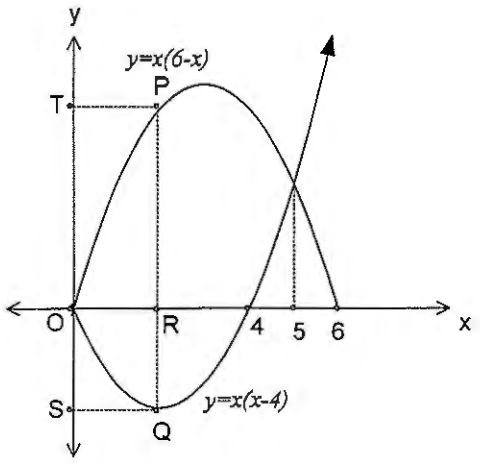
b) Prove $(\cot\theta + \operatorname{cosec}\theta)^2 = \frac{1+\cos\theta}{1-\cos\theta}$ **3**

Marks

Question 10 (10 marks) Start on a SEPARATE page

a) In the diagram below, P is a point on the curve $y = x(6 - x)$ and Q is a point on the curve $y = x(x - 4)$

PQ cuts the x-axis at right angles at R.
S and T are points on y-axis such that $PQST$ is a rectangle.



i) Show that the length of PQ is given by $10x - 2x^2$ **2**

ii) Find an expression for the area $PQST$ as a function of x **1**

iii) Find the value of x which gives the maximum area for $PQST$ $(0 \leq x \leq 5)$ **2**
Justify your answer.

b) Given that the line $lx + my + n = 0$ is a tangent to $x^2 = 4ay$, **3**

Prove that $a = \dfrac{mn}{l^2}$.

End of Paper

Yr. 11 - yearly 2009 - 2unit

Answers

Question 1 (12)

a) $0.597\dot{1}\dot{3}\ldots = 0.5\dot{9}7\dot{1}$ ① ①

b) $\frac{x+5}{3} - 2x = 7$

$x + 5 - 6x = 21$ ✓

$-5x = 16$ ②

$x = -16/5$ ✓

c) $2\sqrt{2} + \sqrt{18} = 2\sqrt{2} + \sqrt{9 \times 2}$

$= 2\sqrt{2} + 3\sqrt{2} \quad = 5\sqrt{2}$ ✓ ✓

d) $x = 0.2424\ldots.$

$100x = 24.2424\ldots.$ ✓

$99x = 24$

$x = \frac{24}{99}$ ✓

e) i) $\sin 60^\circ = \sqrt{3}/2$ ✓

ii) $\cos 225^\circ = -1/\sqrt{2}$ ✓

f) $\frac{x^2 - 5x - 24}{x^2 + 3x} = \frac{(x+3)(x-8)}{x(x+3)}$ ✓

$= \frac{x-8}{x}$ ✓

Question 2 (12)

a) i) $\frac{dy}{dx} = 12x - 8$ ✓

ii) $y = 2x^{\frac{1}{2}} - 4x^{-1}$

$\frac{dy}{dx} = 1x^{-\frac{1}{2}} + 4x^{-2}$ ✓ ✓

iii) $\frac{dy}{dx} = \frac{10x(2x+7) - 2x\,5x^2}{(2x+7)^2}$ ✓

$= \frac{20x^2 + 70x - 10x^2}{(2x+7)^2}$

$= \frac{10x^2 + 70x}{(2x+7)^2}$ ✓

b) (1) $4x - y + 3 = 0$

(2) $xy = 1$

(2) $y = \frac{1}{x}$ ✓

(1) $4x - \frac{1}{x} + 3 = 0$

$4x^2 - 1 + 3x = 0$

$x = \frac{-3 \pm \sqrt{9+16}}{8}$

$(4x-1)(x+1) = 0$

$x = 1/4$, $y = 4$ ✓

$x = -1$, $y = -1$ ✓

Question 2 – cont.

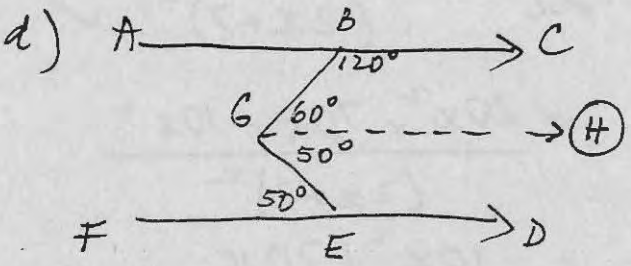
c) $\frac{1}{3+2\sqrt{2}} = a+b\sqrt{2}$

$\frac{3-2\sqrt{2}}{9-4\times 2} = a+b\sqrt{2}$

$3-2\sqrt{2} = a+b\sqrt{2}$

$a = 3$ ✓ $\quad b = -2$ ✓

d)



$\angle BGH = 60°$ co-int. $\angle$'s on AC ∥ GH are ✓ suppl.

$\angle HGE = 50°$ Alternate $\angle$'s on GH ∥ FD

$\therefore \angle BGE = 360° - 60° - 50° =$

$= 250°$ ✓ (angles at point $= 360°$)

Question 3 (12)

a) $x^2 - 6x + 5 \geq 0$

$(x-2)(x-3) \geq 0$ ✓

$x \leq 2$ or $x \geq 3$ ✓

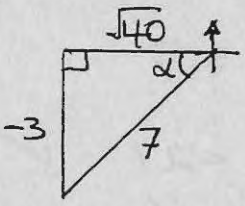
b) i) $f(-1) = -4$ ✓

ii) $f(a^2+1) = (a^2+1)^2 - 1$ ✓

$= a^4 + 2a^2 + 1 - 1 = a^4 + 2a^2$ ✓

(iii) See at end of solns. ✓✓

c)



$\sin\alpha = -\frac{3}{7}$

$\tan\alpha > 0$

$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{-\frac{\sqrt{40}}{7}}$ ✓ $= \pm\frac{7}{\sqrt{40}}$ ✓

d) $\sin\theta + \frac{\sqrt{3}}{2} = 0$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = 60°$ ref acute

$\therefore \theta = 240°, 300°$ ✓ ✓

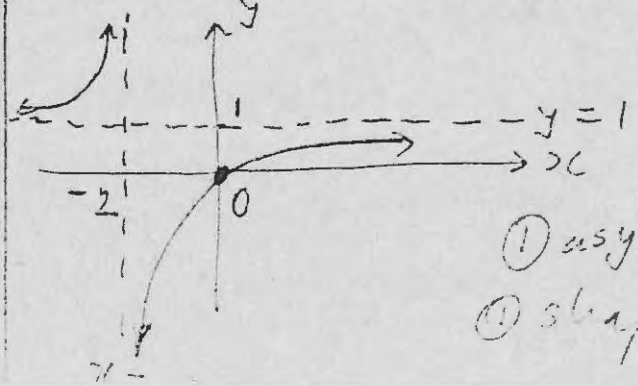
Question 4 (12)

a) i) $\frac{x}{x+2} = 1 - \frac{2}{x+2}$

$RHS = 1 - \frac{2}{x+2} = \frac{x+2-2}{x+2}$ ✓

$= \frac{x}{x+2} = LHS$

ii) $y = \frac{x}{x+2} = 1 - \frac{2}{x+2}$



① asymptotes

① shape

Question 4b

i) $m_{AB} = \frac{-3}{4}$ ✓ $A(0,3)$ $B(4,0)$

ii) $y - 3 = -\frac{3}{4}(x - 0)$ ✓

$y = -\frac{3}{4}x + 3$

$\therefore \frac{3}{4}x + y - 3 = 0$ ✓

$3x + 4y - 12 = 0$

iii) $D(0, 10)$ ✓

iv) $AB = \sqrt{4^2 + 3^2} = 5$ ✓

v) $C(8,4)$ $3x + 4y - 12 = 0$

$d = \frac{|3\times 8 + 4\times 4 - 12|}{5} = \frac{28}{5}$ ✓ ✓

vi) $AB = 5$

$CD = \sqrt{8^2 + 6^2} = 10$

$\therefore$ Area $= \frac{1}{2} \times \frac{28}{5}(5 + 10) = \frac{14}{5} \times 15$ ✓

$= 42$

Question 5 (10)

a) i) $\cos^2\theta - \frac{1}{2} = 0$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$ ✓ ✓

ii) $\sin\theta = \sqrt{3}\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \sqrt{3} = \tan\theta$

$\therefore \theta = 60^\circ, 240^\circ$ ✓ ✓

b) $|4x - 2| \geq 14$ ✓ ✓

$4x - 2 \geq 14$ OR $4x - 2 \leq -14$

$4x \geq 16$ $4x \leq -12$

$\boxed{x \geq 4}$ $\boxed{x \leq -3}$

$x \leq -3$ ✓, $x \geq 4$ ✓

c) $m_{PA} \times m_{PB} = -1$

$A(-3,6)$ $B(-7,-6)$ $P(x,y)$

$\frac{y-6}{x+3} \times \frac{y+6}{x+7} = -1$ ✓

$y^2 - 36 = -(x^2 + 10x + 21)$

$x^2 + 10x + y^2 = -21 + 36$ ✓

$(x+5)^2 + y^2 = -21 + 36 + 25$

$(x+5)^2 + y^2 = 40$

$\therefore$ circle centre $(-5,0)$ ✓

$r = \sqrt{40}$

Question 6 (13)

In ΔAED and ΔACB :-

a) $\angle A$ is in common

$$\frac{AE}{AC} = \frac{8}{20} = \frac{2}{5} \quad \checkmark$$

$$\frac{AD}{AB} = \frac{12}{30} = \frac{2}{5} \quad \checkmark$$

$\therefore \Delta AED \parallel\parallel \Delta ACB$ ✓

(two matching sides are in the same ratio and the included $\angle$'s $=$).

ii) $\frac{x}{15} = \frac{2}{5} \therefore x = 6$ ✓

(since matching sides in similar triangles are in same ratio) ✓

b) $\sum_{n=1}^{\infty} 3^{-n} = \frac{1}{3} + \frac{1}{9} + \ldots$ G.P $a = \frac{1}{3}$ ✓ $r = \frac{1}{3}$

$\therefore$ limiting sum exists $\because |r| < 1$

$$= \frac{1}{2} \quad \checkmark$$

c) $x^2 - (k+3)x + 4k = 0$

$a = 1 \quad b = -(k+3) \quad c = 4k$

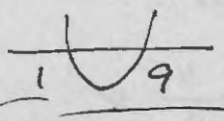
(i) $\Delta < 0$ ✓ no real roots

$b^2 - 4ac = [-(k+3)]^2 - 4 \times 1 \times 4k < 0$

$\therefore k^2 + 6k + 9 - 16k < 0$

$k^2 - 10k + 9 < 0$

$(k-9)(k-1) < 0$ ✓



$1 < k < 9$ ✓

ii) if $\alpha = -3$

$\therefore x^2 - (k+3)x + 4k = 0$

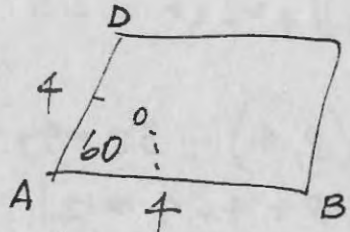
$(-3)^2 - (k+3)x - 3 + 4k = 0$

$9 + 3k + 9 + 4k = 0$

$7k = -18$

$\therefore k = -18/7$ ✓

d)

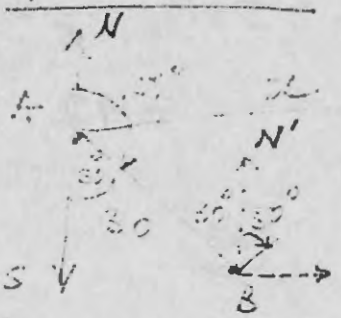


$A = \left(\frac{1}{2} \times 4 \times 4 \times \sin 60^\circ\right) \times 2$ ✓

$= 16 \frac{\sqrt{3}}{2}$ ✓

$\therefore A = 8\sqrt{3} \text{ cm}^2$

Question 7



i) $\angle SAB = 30^\circ$

$\angle ABN' = 30^\circ$ ✓ alt. $\angle$'s on parallel lines

$\therefore \angle ABC = 30^\circ + 50^\circ = 80^\circ$

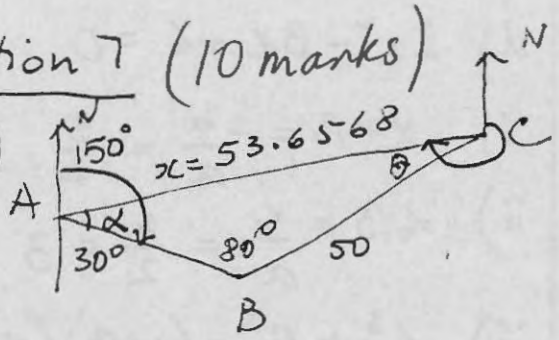
ii) $AC = x$ ✓

$x^2 = 30^2 + 50^2 - 2 \times 30 \times 50 \cos 80^\circ$

$x = 53.6568 \text{ Km}$ ✓

Question 7 (10 marks)

a) iii)



$\frac{\sin\alpha}{50} = \frac{\sin 80^\circ}{53.6568}$ $\therefore \alpha = 66^\circ 35'$ ✓

$\therefore \angle NAC = 150^\circ - \alpha = 83^\circ 25'$

$\therefore \angle NCA = 96^\circ 35'$ (co-int. $\angle$'s on $\parallel$s)

$\therefore$ bearing is $360^\circ - 96^\circ 35'$

$= 263^\circ 25'$ ✓

b) $f(x) = \frac{1}{(x-1)(x+1)} = \frac{1}{x^2-1}$

$f(-x) = \frac{1}{(-x-1)(-x+1)} = \frac{-1}{(x+1)(-x)}$ ✓

$= \frac{-1}{1-x^2} = \frac{-1}{-(x^2-1)} = \frac{1}{x^2-1}$

$\therefore$ even ✓

c) $kx^2 - (k+1)x - 2k + 5$

$\Delta < 0$ and $k < 0$

$b^2 - 4ac < 0$

$[-(k+1)]^2 - 4 \times k \times (-2k+3) < 0$

$k^2 + 2k + 1 + 8k^2 - 12k < 0$

$9k^2 - 10k + 1 < 0$ ✓

$\therefore \frac{1}{9} < k < 1$ ✓

but $k < 0$ $\therefore$ no solution ✓

Question 8

Question 8 (13 marks)

a) since G.P.: $\frac{5x+1}{2x+2} = \frac{10x+2}{5x+1}$ ✓

$\therefore (5x+1)^2 = (10x+2)(2x+2)$

$25x^2 + 10x + 1 = 20x^2 + 24x + 4$

$5x^2 - 14x - 3 = 0$

$(5x+1)(x-3) = 0$

$x = -\frac{1}{5}$ or $x = 3$ ✓

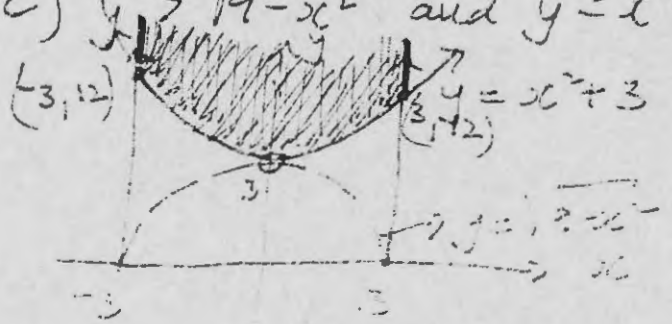
b) $\sec(180° - \theta) \cdot \tan(90° - \theta)$

$= \frac{1}{\cos(180-\theta)} \cdot \cot\theta$ ✓

$= \frac{1}{-\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$= -\frac{1}{\sin\theta}$ or $-\operatorname{cosec}\theta$ ✓

c) $y > \sqrt{9-x^2}$ and $y = x^2 + 3$



① semicircle & parabola

① dotted, full boundaries

① region

d) $2x^2 - 3x + 6 = 0$

i) $\alpha + \beta = -\frac{b}{a} = \frac{3}{2}$ ✓

ii) $\alpha\beta = \frac{c}{a} = \frac{6}{2} = 3$ ✓

iii) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)$ ✓

$= (\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta)$ ✓

$= \frac{3}{2} \times \left[\left(\frac{3}{2}\right)^2 - 3 \times 3\right]$

$= -\frac{81}{8}$ ✓

Question 9 (12 marks)

a) i) $y = x^3 - 2x^2 + x$

$\frac{dy}{dx} = 3x^2 - 4x + 1$ ✓

ii) $\frac{dy}{dx} = 0 = 3x^2 - 4x + 1$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}$ $x = 1$

$y = \frac{4}{27}$ $y = 0$ ✓

nature:

$\frac{d^2y}{dx^2} = 6x - 4$ ✓

at $x = 1$ $\therefore \frac{d^2y}{dx^2} = 2 > 0 \therefore (1, 0)$ ✓ min.

at $x = 1/3$ $\therefore \frac{d^2y}{dx^2} = -2 \therefore \left(\frac{1}{3}, \frac{4}{27}\right)$ max.

Question 9

a) iii) $\frac{d^2y}{dx^2} = 6x - 4$

$0 = 6x - 4$

$\frac{4}{6} = x = \frac{2}{3}$ ✓ $y = \frac{4}{27}$

- Test Concavity:-

Since at $x = \frac{1}{3}$ $\frac{d^2y}{dx^2} = -2$ ...

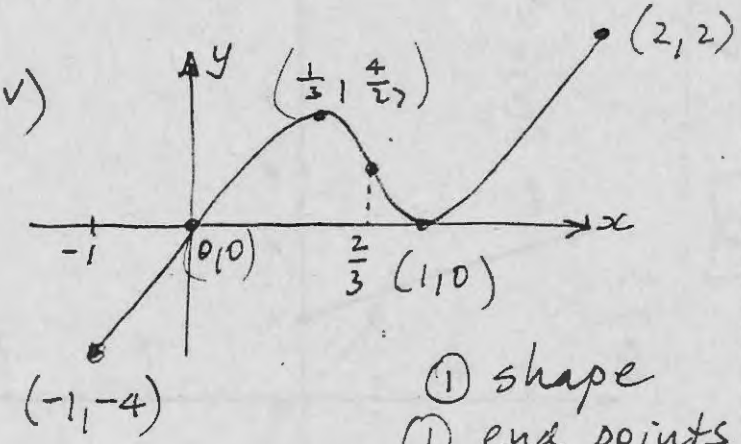
and at $x = \frac{2}{3}$ ✓ $\frac{d^2y}{dx^2} > 0$ ∴ concave up

∴ $\left(\frac{2}{3}, \frac{4}{27}\right)$ pt. of infl.

or

x	$\frac{1}{3}$	$\frac{2}{3}$	1
$\frac{d^2y}{dx^2}$	-2	0	$+2$

iv)



① shape
① end points

v) ✓ $\frac{2}{3} < x \leq 2$ (on domain from (iv)

or $x > \frac{2}{3}$ if all real are taken into domain

9b) LHS $= (\cot\theta + \operatorname{cosec}\theta)^2 =$

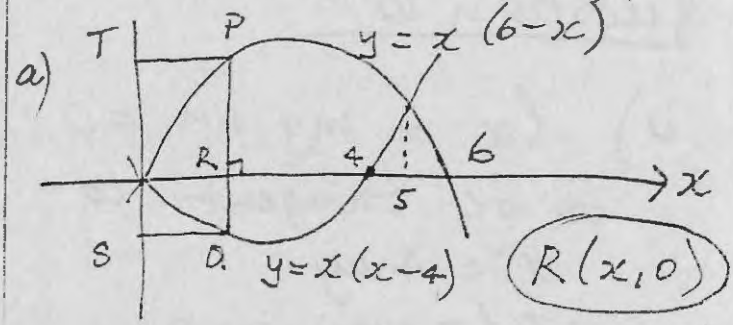
$\cot^2\theta + 2\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta} + \frac{1}{\sin^2\theta} = \frac{\cos^2\theta + 2\cos\theta + 1}{\sin^2\theta}$ ✓

R.H.S $= \frac{1+\cos\theta}{1-\cos\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$ ✓ $= \frac{1+2\cos\theta+\cos^2\theta}{\sin^2\theta} =$ LHS

or different method

Question 10 (8 marks)

a)



R(x, 0)

i) P(x, x(6-x)) ✓

Q (x, x(x-4))

∴ $PQ = \sqrt{0 + (y_P - y_Q)^2}$

or $PQ = y_P - y_Q = x(6-x) - x(x-4)$ ✓

$= -x^2 + 4x + 6x - x^2$

$= -2x^2 + 10x$

ii) Area $= x \times PQ$

$= x(-2x^2 + 10x)$ ✓

$A = -2x^3 + 10x^2$

iii) $\frac{dA}{dx} = 20x - 6x^2$

$0 = 2x(10 - 3x)$

$x = 0$ $\quad x = \frac{10}{3}$ ✓

$\frac{d^2A}{dx^2} = 20 - 12x$

at $x = 0$ $\frac{d^2A}{dx^2} = 20$ ∴ Min.

at $x = \frac{10}{3}$ $\frac{d^2A}{dx^2} = -20$ ∴ Max ✓

∴ Max. A is at $x = \frac{10}{3}$

Question 10

b) $lx + my + n = 0$ to be tangent to $x^2 = 4ay$

$\therefore$ (1) $lx + my + n = 0$ } one solution
(2) $x^2 = 4ay$

$\therefore$ simultaneously solve

(2) $y = \frac{x^2}{4a}$

(1) $lx + m \cdot \frac{x^2}{4a} + n = 0$

$x^2 \cdot \frac{m}{4a} + l \cdot x + n = 0$ ✓

for one solution $\boxed{\Delta = 0}$

$\Delta = b^2 - 4ac$

$0 = l^2 - 4 \times \frac{m}{4a} \times n$ ✓

$\frac{mn}{l^2} = a$ ✓

$\therefore$

3b) iii)

(1) for each part with correct endpoints.