BAULKHAM HILLS HIGH SCHOOL

YEARLY EXAMINATION

YEAR 11

MATHEMATICS ADVANCED 2010

General Instructions

- Exam time – 3 hours
- Reading time – 5 minutes
- Start each question on a new page
- All necessary working should be shown
- Write your student number at the top of each page of your answer booklet
- Board approved calculators may be used
- Write, using black or blue pen

Total Marks: 120

Attempt ALL questions

Question 1 (12 marks) **Marks**

a) Evaluate, correct to 2 decimal places. **2**

$19^{-0.5}$

b) Express $0.\dot{4}\dot{7}$ as a fraction in its simplest form. **2**

c) Solve $4 - 5x < 11$ **2**

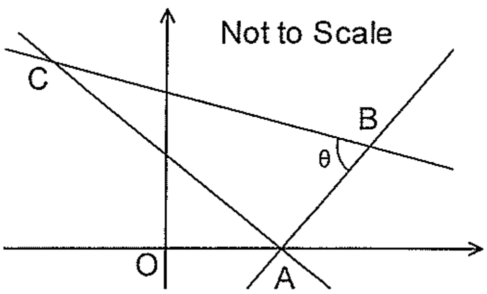
d) Factorise $3x^2 - 5x - 2$ **2**

e) By showing full working, rationalise the denominator for the algebraic expression **2**

$$\frac{3}{\sqrt{x}+2}$$

f) Simplify $\frac{2}{3} - \frac{x-1}{4}$ **2**

Question 2 (12 marks) - Start a new page



Copy the diagram onto your writing pad.

The diagram shows the points $A(1,0)$, $B(4,1)$ and $C(-1,6)$ in the cartesian plane.
Angle ABC is θ

a) Show that A and C lie on the line $3x + y = 3$ **2**

b) Show that the gradient of AB is $\frac{1}{3}$ **1**

c) Show that the length of AB is $\sqrt{10}$ units **1**

d) Show that AB and AC are perpendicular **1**

e) Find $\tan\theta$ **2**

f) Find the equation of the circle with centre A that passes through B **2**

g) Point D lies on the interval AC such that $AD = AB$. Find the co-ordinates of D **2**

h) On your diagram, shade the region satisfying the inequality $3x + y \leq 3$ **1**

Question 3 **(12 marks) - Start a new page** **Marks**

a) Differentiate the following

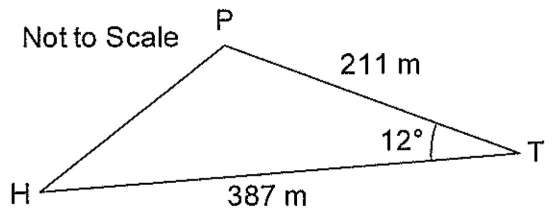
i) $y = (x^2 - 5)(2x + 1)$ **2**

ii) $y = \left(3x^3 - 7\right)^4$ **2**

iii) $y = \dfrac{x^2 - 7}{2x + 3}$ **2**

b) Find the equation of the tangent to the curve $y = x^3 - 3x^2 + 3x - 1$ at the point where $x = 2$ **3**

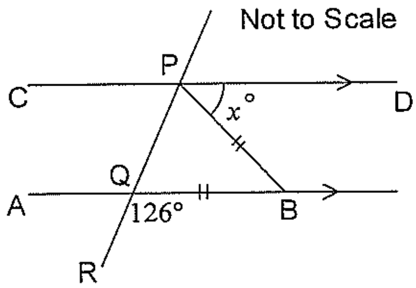
c) **3**



On a golf course, the distance from a tee T to the hole H is 387 metres.
A golfer's ball comes to rest at P, 211 metres from T.
Given that $\angle PTH = 12°$ how far is it from P to H.

Question 4 **(12 marks) - Start a new page**

a)



In the diagram CD is parallel to AB and PB = QB.
Copy the diagram onto your pad and find the value of $x°$ giving complete reasons. **3**

b) The equation of a parabola is $x^2 = 12(y - 2)$ **3**

i) Find the co-ordinates of the vertex of the parabola.

ii) Find the equation of the directrix of the parabola.

c) For what value of k does $3x^2 + 2kx + 3 = 0$ have equal roots. **3**

d) Find the equation of the normal to the curve $y = \sqrt{x + 2}$ at the point where $x = 7$ **3**

Question 5 **(12 marks) - Start a new page** **Marks**

a) Given $\sin\theta = \frac{3}{7}$ and that $\tan\theta < 0$ find the exact value of $\sec\theta$ for $0 \le \theta \le 360°$ without finding θ and showing all working. **3**

b) If $x = \sqrt{a} - 1$ find $x^2 - \dfrac{1}{x}$ **2**

c) State the domain and range for each function

i) $y = \sqrt{4 - x^2}$ **2**

ii) $y = \dfrac{1}{(x^2 - 1)}$ **2**

d) Find the exact solution of

$$x = \frac{x+4}{x-1}$$ **3**

Question 6 **(12 marks) - Start a new page**

a) Solve the simultaneous equations **3**

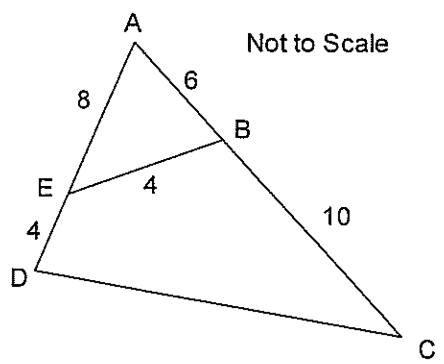
$3x - 2y = 6$

$4x + 5y = 31$

b) Find the perpendicular distance from $P(7, -1)$ to the straight line $3x + 4y - 7 = 0$ **3**

c) Express $\dfrac{6^n + 3^n}{2^{n+1} + 2}$ as a fraction in its simplest form. **2**

d)



In the diagram $AB = 6, BC = 10,\ AE = 8,\ ED = 4$

i) Prove ΔABE and ΔADC are similar **2**

ii) Find the length of CD **2**

Question 7 (12 marks) - **Start a new page** **Marks**

a) Without solving the equation $5x^2 + 3x + 6 = 0$
Find the value of

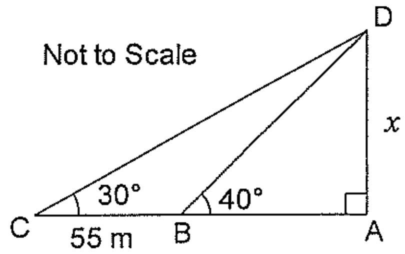
i) $\alpha + \beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **2**

iv) $\alpha^2 + \beta^2$ **2**

b)



In the diagram $\angle DAC = 90°, \angle DBA = 40°,$
$\angle DCA = 30°$, $CB = 55\,m$ and $AD = x\,m$

i) Find DB **2**

ii) Hence find x **2**

c) Find the values of x for which $|3x - 2| \leq 6$ **2**

Question 8 (12 marks) - **Start a new page**

a) Solve $4x^2 + 4x - 3 = 0$ by the method of completing the square. **3**

b) Prove that $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \sec\theta\,\text{cosec}\,\theta$ **3**

c) Solve the following equation **3**

$$9^x - 12(3^x) + 27 = 0$$

d) The point $P(x, y)$ moves so that its distance AP from the point $A(5,1)$ is always twice its distance BP from the point $B(-1,4)$. **3**
Show that the equation of the locus is

$$x^2 + y^2 + 6x - 10y + 14 = 0$$

Question 9 **(12 marks) - Start a new page** **Marks**

a) Consider the function $f(x) = 3x^4 - 4x^3$

i) Find the co-ordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **4**

ii) Find the co-ordinates of any points of inflexion **2**

iii) Find the greatest and the least values of $y = f(x)$ in the domain $-1 \leq x \leq 2$ **2**

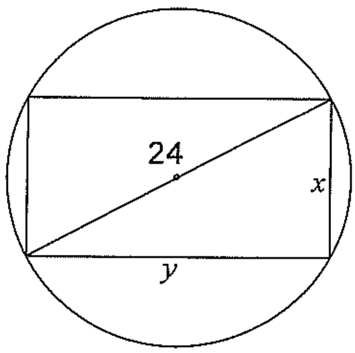
iv) Draw a neat sketch of the curve $y = f(x)$ for $-1 \leq x \leq 2$ **2**

b) Express $\dfrac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta$ as a single trigonometric ratio **2**

and hence solve $\dfrac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = 1$ for $0 \leq \theta \leq 360°$

Question 10 **(12 marks) - Start a new page**

a)



A rectangle of sides $x\ cm$ and $y\ cm$ is contained in a circle of diameter $24cm$ as shown

i) Show that the area A of the rectangle is given by $A = x\sqrt{576 - x^2}$ **2**

ii) Find $\frac{dA}{dx}$ **2**

iii) Find the area of the largest rectangle that can be drawn in this circle. **3**

b) Prove that $\dfrac{1}{\sec x + \tan x} = \sec x - \tan x$ **3**

c) Make y the subject if $2\sqrt{x}\sqrt{y} - y + x^2 = x$ **2**

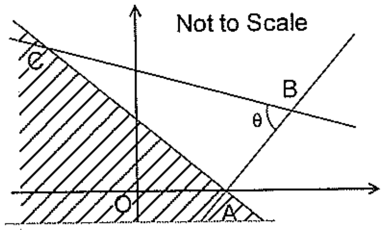
End of Paper

YR 11 ADVANCED SOLUTIONS
YEARLY 2010

Question 1 (12 marks)

a)	$= 0.229415\ldots$ $= 0.23$	2
b)	$n = 0.4747\ldots$ $100n = 47.4747\ldots$ $99n = 47$ $n = \frac{47}{99}$	2
c)	$-5x < 7$ $x > -\frac{7}{5}$	2
d)	$(3x+1)(x-2)$	2
e)	$\frac{3}{\sqrt{x}+2} \times \frac{\sqrt{x}-2}{\sqrt{x}-2} = \frac{3\sqrt{x}-6}{x-4}$	2
f)	$\frac{2\times 4 - 3(x-1)}{12} = \frac{8-3x+3}{12}$ $= \frac{11-3x}{12}$	2

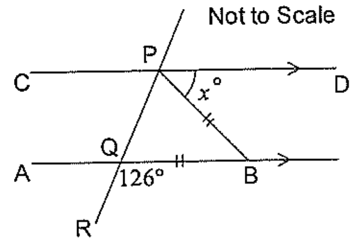
Question 2 (12 marks)

a)	$3x + y = 3$ subst $x = 1, y = 0$ $LHS = 3 + 0 = 3 = RHS$ subst $x = -1,\ y = 6$ $\therefore 3 \times -1 + 6 = -3 + 6$ $= 3 = RHS$ $\therefore A$ and C lie in $3x + y = 3$	2
b)	$m_{AB} = \frac{1-0}{4-1} = \frac{1}{3}$	1
c)	$AB = \sqrt{(4-1)^2 + (1-0)^2}$ $= \sqrt{10}$	1
d)	$m_{AC} = \frac{6-0}{-1-1} = -3$ $m_{AC} \times m_{AB} = -3 \times \frac{1}{3} = -1$ $\therefore AB$ and AC are perpendicular	1
e)	$\tan\theta = \frac{AC}{AB} = \frac{\sqrt{2^2+6^2}}{\sqrt{10}} = 2$	2
f)	$(x-1)^2 + y^2 = \left(\sqrt{10}\right)^2$ $x^2 + y^2 - 2x - 9 = 0$	2
g)	$\tan\theta = 2 \quad AC = 2AB$ $\therefore D$ is midpoint of AC $\therefore D\ (0,3)$ OTHER METHODS	2
h)		1

Question 3 (12 marks)

a)	i) $y' = 2x(2x+1) + x^2 - 5)(2)$ $= 6x^2 + 2x - 10$	2
	ii) $y' = 4(3x^3-7)^3 \times 9x^2$ $= 36x^2(3x^3-7)^3$	2
	iii) $y' = \frac{2x(2x+3)-(x^2-7)\times 2}{(2x+3)^2}$ $= \frac{4x^2+6x-2x^2+14}{(2x+3)^2}$ $= \frac{2x^2+6x+14}{(2x+3)^2}$	2
b)	At $x = 2,\ y = 8 - 12 + 6 - 1$ $= 1$ $y' = 3x^2 - 6x + 3$ at $x = 2,\ y' = 12 - 12 + 3 = 3$ $\therefore$ tangent is $y - 1 = 3(x-2)$ $y = 3x - 5$	3
c)	$PH^2 = PT^2 + HT^2 - 2.PT.HT.\cos\angle PTH$ OR $(q^2 = p^2 + h^2 - 2ph\cos T)$ $PH^2 = 211^2 + 387^2 - 2 \times 211 \times 387 \cos 12^\circ$ $PH^2 = 34544.80273$ $PH = 185.862322\ldots$	3

Question 4 (12 marks)

a)	 $\angle PQB + \angle BQR = 180^\circ$ ($\angle$sum of a line) $\therefore \angle PQB = 180 - 126 = 54^\circ$ $\angle BPQ = \angle PQB = 54^\circ$(base $\angle$ of isos Δ) $\angle DPQ = \angle BQR = 126^\circ$ (corresp $\angle$ on // lines) $\therefore \angle DPB = \angle DPQ - \angle BPQ$ $= 126 - 54 = 72^\circ$	3
b)	$x^2 = 4 \times 3(y-2)$ i) $V\ (0,2)$ ii) equation of directrix is $y = 2 - 3 = -1$ $y = -1$	3

c)	$\Delta = 4k^2 - 36 = 0$ $k = \pm 3$	**3**
d)	$x = 7,\ y = 3$ $m_T = \frac{1}{2}(x+2)^{-\frac{1}{2}}$ $x = 7\ \ m_T = \frac{1}{2\sqrt{9}} = \frac{1}{6}$ $\therefore m_n = -6$ $y - 3 = -6(x-7),\quad 6x + y - 45 = 0$	**3**

Question 5 (12 marks)

a)	$\sin\theta > 0\ \ \tan\theta < 0$ $\therefore$ *2nd quad* $\cos\theta = -\frac{\sqrt{40}}{7}$ $\therefore \sin\theta = -\frac{7}{\sqrt{40}}$	**3**
b)	$x^2 - \frac{1}{x} = (\sqrt{a}-1)^2 - \frac{1}{\sqrt{a}-1} \times \frac{\sqrt{a}+1}{\sqrt{a}+1}$ $= a - 2\sqrt{a} + 1 - \left(\frac{\sqrt{a}+1}{a-1}\right)$	**2**
c)	i) $D: x \le 2\ \ or -2 \le x \le 2$ $R: y \ge 0$ ii) D: all x except $x = \mp 1$ R: $y > 0, y < -1$	**2** **2**
d)	$x(x-1) = x + 4$ $x^2 - 2x - 4 = 0$ $x = \frac{x^2 \pm \sqrt{4+16}}{2}$ $x = 1 \pm \sqrt{5}$	**3**

Question 6 (12 marks)

a)	$(1) \times 5 \to 15x - 10y = 30 \quad \to (3)$ $(2) \times 2 \to 8x + 10y = 12 \quad \to (4)$ $(3) + (4) \to 23x = 92$ $x = 4,\ \ y = 3$	**3**
b)	$d = \left\lvert\frac{7 \times 3 + -1 \times 4 - 7}{\sqrt{3^2 + 4^2}}\right\rvert$ $d = 2$	**3**
c)	$\frac{3^n.2^n + 3^n}{2(2^n+1)} = \frac{3^n(2^n+1)}{2(2^n+1)}$ $= \frac{3^n}{2}$	**2**
d)	i) $\frac{AE}{AC} = \frac{8}{16} = \frac{1}{2}$ $\frac{AB}{AD} = \frac{6}{12} = \frac{1}{2}$ $\therefore AE{:}AC = AB{:}AD$ $\angle DAC$ is common $\therefore \Delta ABE /// \Delta ADC$ *(2 matching sides in the same ratio and included ∠ equal)* ii) $\frac{EB}{DC} = \frac{1}{2}$ *(matching sides in similar Δ's are =)* $\frac{4}{DC} = \frac{1}{2}$ $\therefore DC = 8$	**2** **2**

Question 7 (12 marks)

a)	i) $\alpha + \beta = -\frac{3}{5}$ ii) $\alpha\beta = \frac{6}{5}$ iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -\frac{3}{6} = -\frac{1}{2}$ iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= \left(-\frac{3}{5}\right)^2 - 2 \times \frac{6}{5}$ $= -\frac{51}{25}$	**1** **1** **2** **2**
b)	i) $\angle CDB = 10°$ *(ext ∠= sum interior opp ∠'s)* $\frac{DB}{\sin 30°} = \frac{CB}{\sin 10°}$ $DB = 158.366188\ldots$ 158 OK ii) $\sin 40° = \frac{x}{DB}$ $x = 101.795823\ldots$	**2** **2**
c)	$-6 \le (3x-2) \le 6$ $-4 \le 3x \le 8$ $-\frac{4}{3} \le x \le \frac{8}{3}$	**2**

Question 8 (12 marks)

a)	$x^2 + x - \frac{3}{4} = 0$ $\left(x^2 + x - \frac{1}{4}\right) - \frac{3}{4} = \frac{1}{4}$ $\left(x + \frac{1}{2}\right)^2 = 1$ $x = -\frac{1}{2} \pm 1$ $x = -\frac{3}{2}\ or\ \frac{1}{2}$	**3**
b)	$LHS = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ $= \frac{1}{\cos\theta\sin\theta}$ $= \sec\theta\,\mathrm{cosec}\,\theta$ $= RHS$	**3**
c)	$y = 3^x$ $\therefore y^2 - 12y + 27 = 0$ $y = 9\ \ or\ y = 3\ x = 1\ or\ 2$ $x = 1\ or\ 2$	**3**

~~$\therefore 3^x = 9 \ or \ 3^x = 1$~~
~~$x = 1 \ or \ 2$~~

d) (3 marks)

$$AP = 2PB$$
$$AP^2 = 4PB^2 \quad 1$$
$$(x-5)^2 + (y-1)^2 = 4(x+1)^2 + 4(y-4)^2$$
$$x^2 - 10x + 25 + y^2 - 2y + 1 = 4x^2 + 8x + 4 + 4y^2 - 32y + 64 \quad 1$$
$$\therefore 3x^2 + 3y^2 - 18x - 30y + 42 = 0$$
$$\therefore x^2 + y^2 - 6x - 10y + 14 = 0 \quad 1$$

Question 9 (12 marks)

a) (4 marks)

i) $f(x) = 3x^4 - 4x^3$
$f'(x) = 12x^3 - 12x^2$ 1
$f''(x) = 36x^2 - 24x$
$f'(x) = 0 \quad \therefore 12x^3 - 12x^2 = 0$
$12x^2(x-1) = 0$
$x = 0 \ or \ 1$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
$f'(x)$			

must show values

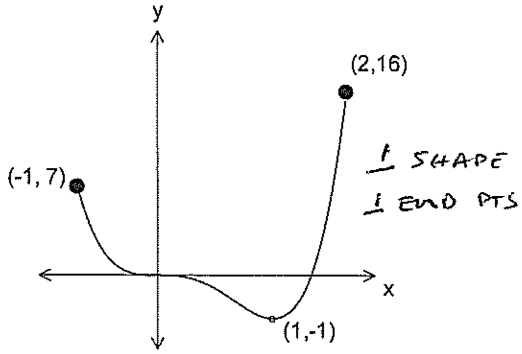
x	$\frac{1}{2}$	1	$\frac{3}{2}$
$f'(x)$			

$\therefore$ (0,0) is a horizontal inflexion 1
$(1,-1)$ is a min 1

ii) $f''(x) = 0$ (2 marks)
$36x^2 - 24x = 0$
$12x(3x-2) = 0$ 1
$x = 0, \quad y = 0$ *horzontal inflexion*
$x = \frac{2}{3} \quad f''\left(\frac{2}{3}\right) = 0$
$f''(1) > 0$
$f''\left(\frac{1}{3}\right) < 0$
$\therefore x = \frac{2}{3} \quad y = 3\left(\frac{2}{3}\right)^4 - 4\left(\frac{2}{3}\right)^3$
$= -\frac{16}{27}$
$\left(\frac{2}{3}, -\frac{16}{27}\right)$ inflexion 1

iii) $f(-1) = 7 \quad f(2) = 16$ (2 marks)
greatest is 16 1
least is -1 1

iv) (2 marks)



b) (2 marks)

$$\frac{\sin^3\theta}{\cos\theta} + \frac{\sin\theta\cos^2\theta}{\cos\theta}$$
$$= \frac{\sin^2\theta\,(\sin^2\theta + \cos^2\theta)}{\cos\theta} \quad 1$$
$$= \tan\theta$$
$$\tan\theta = 1$$
$$\theta = 45° \ or \ 225° \quad 1$$

Question 10 (12 marks)

a) (2, 2, 3 marks)

area of rectangle $= xy$
But $x^2 + y^2 = 24^2$
$$\therefore y = \sqrt{576 - x^2} \quad 1$$
$$\therefore A = x\sqrt{576 - x^2}$$
$$= x(576 - x^2)^{\frac{1}{2}} \quad 1$$
$$\frac{dA}{dx} = (576 - x^2)^{\frac{1}{2}} + x.\frac{1}{2}.-\frac{2x}{(576 - x^2)^{\frac{1}{2}}} \quad 1$$
$$= \frac{(576 - x^2) - x^2}{\sqrt{576 - x^2}}$$
$$= \frac{576 - 2x^2}{\sqrt{576 - x^2}} \quad 1$$
$= 0$ ~~min~~ MAX 1
$\therefore x = 124\sqrt{2} \quad y = 124\sqrt{2}$ 1
$\therefore A = 288 \ sq \ metres$ 1

b) (3 marks)

$$LHS = \frac{1}{\sec x \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} \quad 1$$
$$= \frac{\sec x - \tan x}{\sec^2 x - \tan^2 x}$$
$$= \frac{\sec x - \tan x}{1} \quad 1$$
$$= \sec x - \tan x = RHS \quad 1$$

OTHER METHODS

c) (2 marks)

$$2\sqrt{x}\sqrt{y} - y + x^2 = x$$
$$y - 2\sqrt{x}\sqrt{y} + x = x^2$$
$$\left(\sqrt{y} - \sqrt{x}\right)^2 = x^2$$
$$\sqrt{y} - \sqrt{x} = \pm x$$
$$\sqrt{y} = \sqrt{x} \pm x$$
$$y = \left(\sqrt{x} \pm x\right)^2 \quad 2$$

OTHER METHODS

End of Exam