BAULKHAM HILLS HIGH SCHOOL

2011

YEAR 11 YEARLY

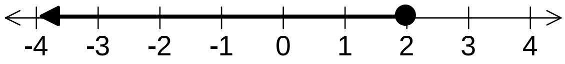
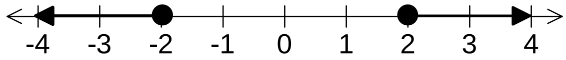
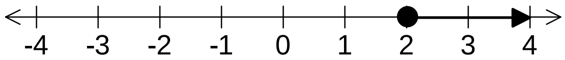
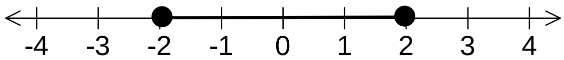
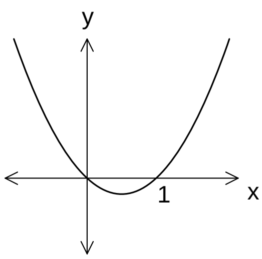
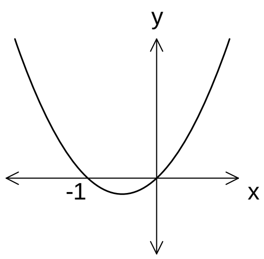
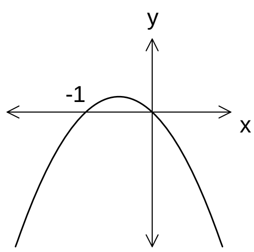
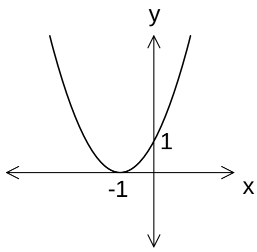
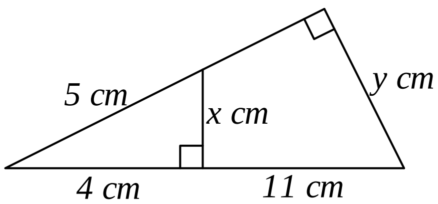
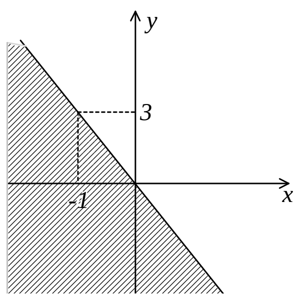
Mathematics Advanced

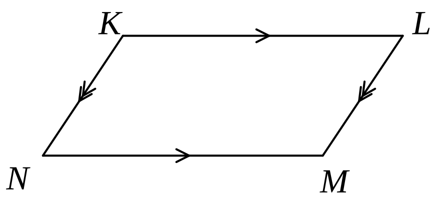
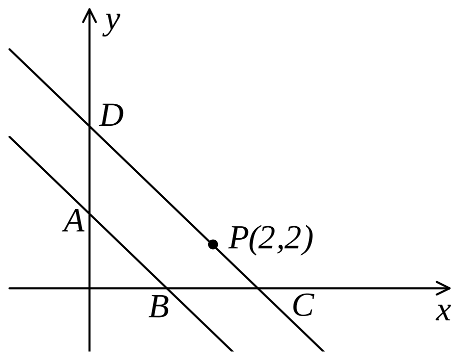
General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work

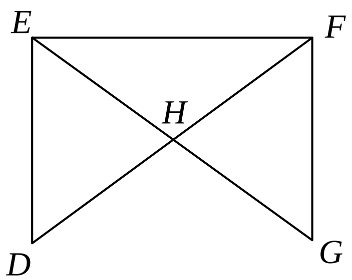
Total marks – 100

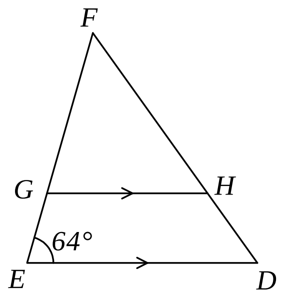
- Attempt Questions 1 – 9
- All questions are of equal value
- Start each question on a new sheet of paper.
- Write your name and the question number at the top of each sheet.

Question 1 (10 marks) - Answer the following on the answer sheet provided.		**Marks**
a)	Evaluate $\sqrt{\dfrac{19.28}{4.25 \times 3.5}}$ correct to 3 significant figures	**2**
b)	Which of the following represents the solution of $x^2 \le 4$? (A)  (B)  (C)  (D) 	**1**
c)	Which of the following is the graph of $y = x^2 + x$ (A)  (B)  (C)  (D) 	**1**
d)	For which value of x is $\|2x + 5\| \le 7$ <u>false?</u> (A) -1 (B) 0 (C) 1 (D) 4	**1**
e)	Find the distance AB for $A(-3,-4)$, $B(5,7)$ to one decimal place	**1**
Questions (f) and (g) refer to the information below. 	f) The value of x is: (A) 1 (B) 2 (C) 3 (D) 4	**1**
	g) The value of y is: (A) 4 (B) 6 (C) 8 (D) 9	**1**
h)	The shaded region is best described by which inequality: (A) $y < -\frac{1}{3}x$ (B) $y \ge -3x$ (C) $y \le -3x$ (D) $y \le -\frac{1}{3}x$ 	**1**
i)	If a curve is always increasing which <u>must</u> occur for all values of x: (A) $\frac{dy}{dx} > 0$ (B) $\frac{dy}{dx} < 0$ (C) $\frac{d^2y}{dx^2} > 0$ (D) $\frac{d^2y}{dx^2} < 0$	**1**

Question 2	**(11 marks) - Start on a new page**	**Marks**
a)	Factorise fully $x^4 - 27xy^3$	**2**
b)	Simplify and express your answer as a rational number $$\frac{1}{3\sqrt{2}+1}+\frac{1}{1-3\sqrt{2}}$$	**2**
c)	Solve $$(y-5)^2 - 1 = (y-4)(y+3)$$	**2**
d)	Find the exact value of $\tan 120°$	**2**
e)	M is the midpoint of $A(5, k) and\ B(11,8)$. Find k, if M lies on the line $x - 2y = 4$	**3**
Question 3	**(13 marks) - Start a new page**	**Marks**
a)	Solve $3^{6x+3} = 27^{x+2}$	**2**
b)	 In a parallelogram $KLMN$ $\angle NKL = 3x - 30°$ and $\angle KLM = x + 10°$ Find the size of $\angle KNM$ (with working and no reasons required)	**3**
c)	 The diagram shows the straight line AB with equation $x + 2y = 2$	
	i) Show that the equation of DC parallel to AB passing through (2,2) is $x + 2y = 6$	**2**
	ii) Calculate the perpendicular distance from P to AB	**2**
	iii) Calculate the area of trapezium $ABCD$	**2**
d)	Derive the equation of the tangent to the curve $y = 2x^2 + 5$ at the point (2,13)	**2**

Question 4 (11 marks) - **Start a new page**		
a)	Solve the inequality $$3x^2 - 5x - 12 \geq 0$$	2
b)	Differentiate i) $y = \dfrac{3x+2}{4x-3}$ ii) $f(x) = x\sqrt{x}$ iii) $y = (2x+1)(5x-3)^4$	 2 2 2
c)	Show that i) $\cos\theta \tan\theta = \sin\theta$ ii) Hence solve $$\cos\theta \tan\theta = -\frac{1}{2} \quad \text{for} \quad 0° \leq \theta \leq 360°$$	 1 2

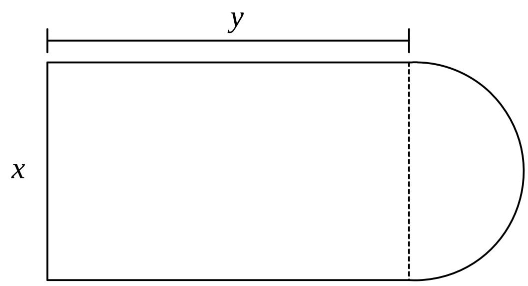
Question 5 (11 marks) - **Start a new page**		**Marks**
a)	If $y = ax^2 + bx$ has a maximum at $(2,3)$, find the values for a and b	2
b)	 H is the point of intersection of DF and EG. If $DH = GH$ and $\angle HEF = \angle HFE$ Show that $\Delta EHD \equiv \Delta FHG$ (Draw your own diagram and mark all given information)	4
c)	Find the domain and range for $y = \dfrac{1}{2x-3}$	2
d)	Sketch $y = 9x - 11 - 2x^2$ by first finding the vertex and any intercepts	3

Question 6 (11 marks) - Start a new page		
a)	The roots of the equation $2x^2 - 4x + 1 = 0$ are α and β. Without solving the equation find: i) $\alpha + \beta$ ii) $\alpha\beta$ iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ iv) $\alpha^3 + \beta^3$	1 1 1 2
b)	 Given that $\Delta EDF \mid\mid\mid \Delta GHF$ i) With reasons, find HF when $ED = 6m$, $GH = 4m$ and $DH = 4m$ ii) Find $\angle HFG$ to the nearest min iii) Find area of ΔEDF to 1 decimal place	2 2 2

Question 7 (11 marks) - Start a new page		Marks
a)	Find A, B and C such that $$3x^2 - x + 3 \equiv A(x-1)(x+2) + B(x-2) + C$$	3
b)	The function $f(x)$ is defined by $f(x) = x^3 - 3x^2 - 9x$ i) Find the turning points for the curve $y = f(x)$ and determine their nature ii) Show that there is a point of inflexion and find its coordinates iii) Sketch the graph of $y = f(x)$ showing relevant information for the domain $-3 \le x \le 4$ iv) What is the maximum value of this curve in this domain	3 1 3 1

Question 8 (11 marks) - Start a new page		
a)	Find the values of x for which $2x - 5 \leq x + 7 \leq 4x + 1$	2
b)	Solve $4\sin^2 x = 3$ for $0° \leq x \leq 360°$	2

c)



A garden bed is to be constructed in the shape of a semi-circle added to a rectangle.
The width of the rectangle is x and the length y metres as shown in the diagram.

Forty metres of edging material is available to use around the edge of the garden bed

i) Show that $y = 20 - \frac{1}{2}x - \frac{1}{4}\pi x$ **2**

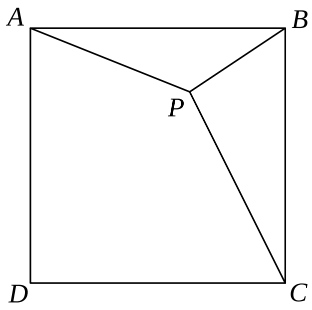
ii) Show that the area of the garden bed can be expressed as **1**

$$A = 20x - \frac{1}{2}x^2 - \frac{1}{8}\pi x^2$$

iii) Find the dimensions of the rectangle such that the area contained in the garden bed is a maximum. **4**

Question 9 (11 marks) - Start a new page		Marks
a)	Show that the constants k and l have different signs, if the quadratic equation: $(k^2 + l^2)x^2 - 2m(k + l)x + m^2 = 0$ has no real roots Where $m \neq 0$	3

b)



$ABCD$ is a square of side x cm, with P a point within the square such that $PC = 6cm, PB = 2cm$ and $AP = 2\sqrt{5}$cm.
Let $\angle PBC = \alpha$

i) By using triangle PBC show that $\cos\alpha = \frac{x^2-32}{4x}$ **1**

ii) By considering triangle PBA, show that $\sin\alpha = \frac{x^2-16}{4x}$ **2**

iii) Hence or otherwise, show that the value of x is a solution of **2**

$$x^4 - 56x^2 + 640 = 0$$

iv) Find x. Give reasons for your answer. **3**

End of Exam

[redacted]

Year 11 2 unit Yearly 2011 SOLNS

Question one. (10 marks)

a) 1.14 — 2 for correct answer; 1 for 3 sig fig, or 1.138...

b) D c) B d) D e) 13.6 f) C

g) D h) C i) A.

Question two (11 marks)

a) $x^4 - 27xy^3 = x(x^3 - 27y^3)$ — 1

$= x(x-3y)(x^2+3y+y^2)$ — 1

b) $\frac{1}{3\sqrt{2}+1} + \frac{1}{1-3\sqrt{2}} = \frac{1}{3\sqrt{2}+1} - \frac{1}{3\sqrt{2}-1}$

$= \frac{3\sqrt{2}-1-3\sqrt{2}-1}{9\times 2 - 1}$ 1

$= \frac{-2}{17}$ 1

c) $(y-5)^2 - 1 = (y-4)(y+3)$ 1

$\cancel{y^2} - 10y + 24 = \cancel{y^2} - y - 12$

$-9y = -36$

$y = 4$ 1

d) $\tan 120^\circ = -\tan 60^\circ$

1 $= \frac{-\sqrt{3}}{2}$ — 1

e) $M: \left(\frac{16}{2}, \frac{k+8}{2}\right) = \left(8, \frac{k+8}{2}\right)$ 1

sub in

$8 - 2\left(\frac{k+8}{2}\right) = 4$ 1

$8 - k - 8 = 4$

$k = -4$ 1

Question 3 (13 marks)

a) $3^{6x+3} = 3^{3x+6}$ ①

$\therefore \quad 6x+3 = 3x+6$

$3x = 3$

$x = 1$ ①

b) $3x - 30 + x + 10 = 180$ ✓ ①

$4x = 200$

$x = 50$ ✓ ①

$\angle KLM = 50+10$

$= 60$

$\angle KNM = \angle KLM$

$= 60°$ ✓ ①

c) i) AB: $2y = 2 - x$

$y = 1 - \frac{1}{2}x \qquad m_1 = -\frac{1}{2}$ ①

$P(2,2)$

DC: $y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x-2)$ ①

$2y - 4 = -x + 2$

$x + 2y = 6$ or $x + 2y - 6 = 0$

ii) $pd = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}} \qquad x + 2y - 2 = 0$: AB

$P(2,2)$

$= \frac{|1\times 2 + 2\times 2 - 2|}{\sqrt{1+4}}$ ①

$= \frac{4}{\sqrt{5}}$ or $\frac{4\sqrt{5}}{5}$ ① $= 1.79$ (2d.p)

iii) Area ABCD

D: $(0,3)$ C: $(6,0)$

$A_{\triangle_1} = \frac{1}{2} \times 1 \times 2$ ①

$= 1$

$A_{\triangle_2} = \frac{1}{2} \times 3 \times 6$

$= 9$

$A_{ABCD} = 9 - 1$

$= 8u^2$ ①

or

A $(0,1)$ B $(0,2)$

$\overline{AB} = \sqrt{5}$

$\overline{CD} = \sqrt{9+36}$

$= \sqrt{45}$ ①

$A_{trap} = \frac{1}{2} \times \frac{4}{\sqrt{5}}\left(\sqrt{5} + 3\sqrt{5}\right)$

$= 8u^2$ ①

d) $y = 2x^2 + 5$ at $(2,7)$

mistake should be 13

no worry

$\frac{dy}{dx} = 4x$ ①

at $x = 2 \qquad m = 8$

$\therefore y - y_1 = m(x - x_1)$ ①

$y - 7 = 8(x-2)$

$y = 8x - 9$

OR if at $x = 2$

$\therefore y = 13$

$\therefore y - 13 = 8(x-2)$

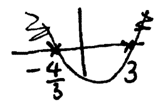
$y = 8x - 3$

Nigel, Pam. Jason, Liz, Gemi, Bob, sam,

Question 4 (11 marks)

a) $(3x+4)(x-3) \geq 0$

$x = -\frac{4}{3}, 3$



(2) both answers

(1) working towards soln.

$\therefore \quad x \leq -\frac{4}{3}, \quad x \geq 3$

b) i) (1)

$$y' = \frac{(4x-3)\times 3 - 4(3x+2)}{(4x-3)^2}$$

$$= \frac{12x - 9 - 12x - 8}{(4x-3)^2}$$

(1) $$= \frac{-17}{(4x-3)^2}$$

ii) $f(x) = x\sqrt{x} = x^{\frac{3}{2}}$ (2)

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$ or $\frac{3\sqrt{x}}{2}$

or $f'(x) = \frac{x}{2\sqrt{x}} + \sqrt{x}$

iii) (1) (1)

$$\frac{dy}{dx} = 2(5x-3)^4 + 20(5x-3)^3(2x+1)$$

$$= 2(5x-3)^3(5x-3+20x+10)$$

$$= 2(5x-3)^3(25x+7)$$

c) i) $\cos\theta \tan\theta = \sin\theta$

$L.H.S. = \cancel{\cos}\theta \times \frac{\sin\theta}{\cancel{\cos\theta}}$ (1)

$= \sin\theta = R.H.S.$

ii) $\cos\theta \tan\theta = -\frac{1}{2}$

$\therefore \quad \sin\theta = -\frac{1}{2}$

$\theta = 210, 330$

(1) (1)

Question 5 (11 marks)

a) $y = ax^2 + bx$

$\frac{dy}{dx} = 2ax + b = 0$ at stat pt

$(2, 3)$: $4a + b = 0$ --- (1)

(1)

$\rightarrow \quad 3 = 4a + 2b$ --- (2)

$4a + 2b = 3$ sub (1) from (2)

$4a + b = 0$

$b = 3$

sub into (1)

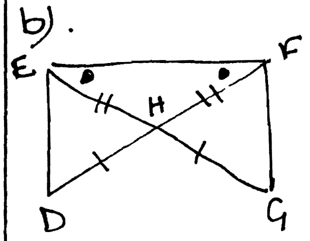
$4a = -3$

$a = -\frac{3}{4}$

$\therefore \quad a = -\frac{3}{4} \quad b = 3$ (1)

quest 5 cont.

b).



In $\triangle HEF$

$H\hat{E}F = H\hat{F}E$ (given)

$\therefore \triangle$ is isosceles (base $\angle$s equal) ①

then $EH = HF$ (sides opposite equal $\angle$s)

In $\triangle EHD$ and $\triangle FHG$

$EH = FH$ (above)

$E\hat{H}D = F\hat{H}G$ (vertically opposite $\angle$s)

$DH = HG$ (given)

$\therefore \triangle EHD \equiv \triangle FHG$ (S.A.S) ③ marks

c) $\mathbb{D}$: all real x, $x \neq \frac{3}{2}$ ①

$\mathbb{R}$: all real y, $y \neq 0$ ①

d) $y = 9x - 11 - 2x^2$

axis of symmetry : $x = \frac{-b}{2a}$

$= \frac{-9}{-4}$

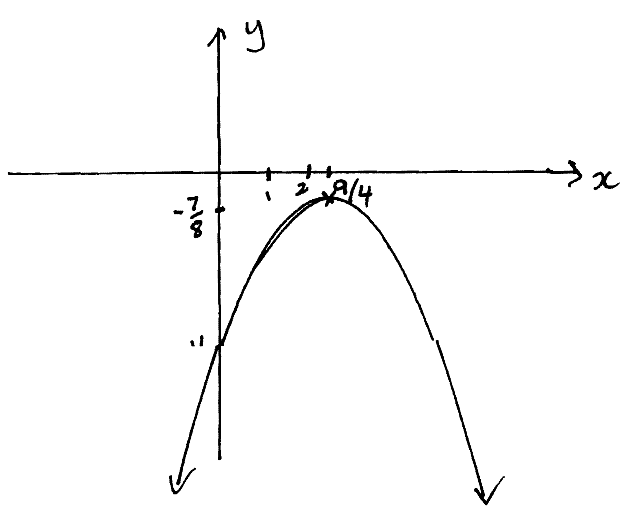
$= \frac{9}{4}$.

$y = (9 \times \frac{9}{4}) - 11 - 2 \times (\frac{9}{4})^2$

$= -0.875$

Vertex : $(\frac{9}{4}, -0.875)$

$(\frac{9}{4}, -\frac{7}{8})$



Question 6 (11 marks)

a) $2x^2 - 4x + 1 = 0$

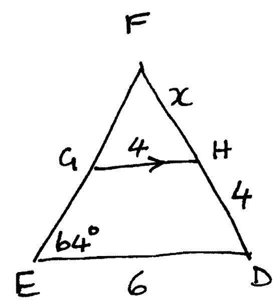
i) $\alpha + \beta = \frac{-b}{a}$
$= \frac{4}{2} = 2$ 1

ii) $\alpha\beta = \frac{c}{a}$
$= \frac{1}{2}$ 1

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$
$= \frac{2}{\frac{1}{2}} = 4$ 1

iv) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$
$= (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)$ 1
$= 2\left(2^2 - \frac{3}{2}\right)$
$= 5$ 1

b)



i) $\frac{FH}{FD} = \frac{GH}{ED}$ (sides in ratio in similar Δs) 1

$\frac{x}{x+4} = \frac{4}{6}$
$6x = 4x + 16$
$2x = 16$
$FH = x = 8$ m 1

ii) $\frac{\sin\theta}{4} = \frac{\sin 64}{8}$ 1
$\sin\theta = \frac{4\sin 64}{8}$
$= 0.44939\ldots$
$\theta = 26°42'$ 1

iii) Area $= \frac{1}{2}ab\sin C$
$= \frac{1}{2} \times 12 \times 6 \times \sin 89°18'$
$= 35.997\ldots$
$= 36.0$ m^2 1

$F\hat{D}E = 180 - 26°42' - 64$
$= 89°18'$ 1

Question 7 (11 marks)

a) $3x^2 - x + 3 \equiv A(x^2+x-2) + B(x-2) + C$ ①

RHS $= Ax^2 + x(A+B) - 2A - 2B + C$

equate: $3 = A$ $\quad -1 = A+B$ ① $\quad 3 = -2A - 2B + C$

$-1 = 3 + B \quad 3 = -6 + 8 + C$

$B = -4 \quad 1 = C$

$\therefore A = 3,\ B = -4,\ C = 1$ ①

b) $f(x) = x^3 - 3x^2 - 9x$

i) $f'(x) = 3x^2 - 6x - 9$

$3(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$ ①

$x = -1, 3$

$x = -1$	$x = 3$
$y = (-1)^3 - 3(-1)^2 - 9\times -1$	$y = 3^3 - 3\times 3^2 - 9\times 3$
$= 5$	$= -27$
$(-1, 5)$ ①	$(3, -27)$
$\boxed{f''(x) = 6x - 6}$	
test	
$f''(-1) = -6 - 6$	$f''(3) = 18 - 6$
$= -12 < 0$	$= 12 > 0$
max ①	min

ii) $f''(x) = 6x - 6 = 0$ for P.O.I

$x = 1$ $\quad y = 1^3 - 3(1) - 9\times 1$

$= -11$

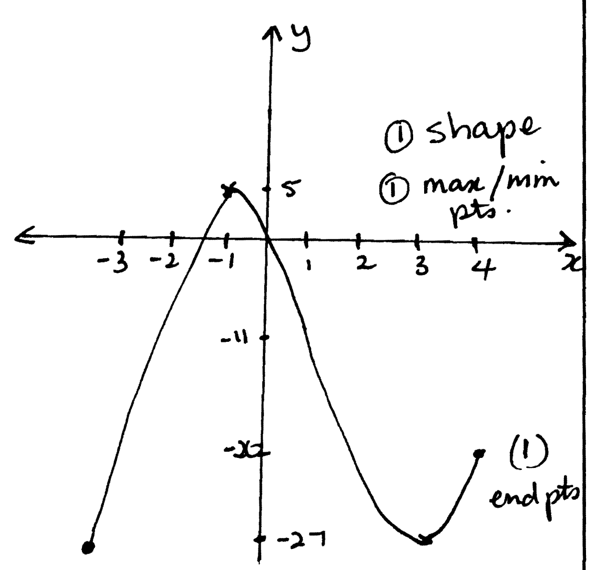
$(1, -11)$ ①

Test

$f''(0) = -6 \quad f''(2) = 12 - 6$

$= 6$

$\therefore$ change in concavity.



① shape
① max/min pts.
(1) end pts

iii) End pts

$-3 = x \quad (-3, -27) \quad x = 4$

$f(-3) = -27 \quad f(4) = -20$

$(4, -20)$

iv) max value is 5 ①

Question 8 11 marks

a) $2x-5 \le x+7 \le 4x+1$

$2x-5 \le x+7$ $\quad$ $x+7 \le 4x+1$

$x \le 12$ (1) $\quad$ $6 \le 3x$

$2 \le x$

$\therefore 2 \le x \le 12$ (1)

b) $4\sin^2 x = 3$

$\sin x = \pm\frac{\sqrt{3}}{2}$ (1)

$x = 60, 120, 240, 300$ (1)

c)

i) $P = \frac{1}{2}\pi x + 2y + x$ (1)

$40 = 2y + \frac{1}{2}\pi x + x$

$2y = 40 - \frac{1}{2}\pi x - x$ (1)

$y = 20 - \frac{1}{2}x - \frac{1}{4}\pi x$

ii) $A = xy + \frac{1}{2}\pi \times \left(\frac{x}{2}\right)^2$

$= x\left(20 - \frac{1}{2}x - \frac{1}{4}\pi x\right) + \frac{\pi x^2}{8}$ (1)

$= 20x - \frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{\pi x^2}{8}$

$= 20x - \frac{1}{2}x^2 - \frac{\pi x^2}{8}$

iii) $\frac{dA}{dx} = 20 - x - \frac{\pi x}{4}$ (1)

let $\frac{dA}{dx} = 0$

$20 - x - \frac{\pi x}{4} = 0$

$20 = x\left(1 + \frac{\pi}{4}\right)$

$x = \frac{80}{4+\pi}$ (1)

$\frac{d^2A}{dx^2} = -1 - \frac{1}{4}\pi < 0$ (1)

Max for all values.

dimensions

$x = \frac{80}{4+\pi} = 11.2$ (1dp)

$y = \frac{40}{4+\pi} = 5.6$ (1dp)

(1)

Question 9 (11 marks)

a) $\Delta < 0$ no real roots 1

$(k^2+l^2)x^2 - 2m(k+l)x + m^2 = 0$

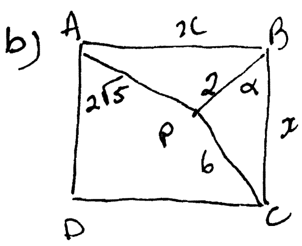
$\Delta = 4m^2(k+l)^2 - 4m^2(k^2+l^2) < 0$ 1

$(k+l)^2 - (k^2+l^2) < 0$

$\cancel{k^2} + 2kl + \cancel{l^2} - \cancel{k^2} - \cancel{l^2} < 0$

$2kl < 0$ 1

$kl < 0$ $\therefore$ either k or l is negative and $\therefore$ opposite signs

b) 

In $\triangle PBC$

i) $\cos\alpha = \dfrac{2^2 + x^2 - 6^2}{2 \times 2 \times x}$ 1

$= \dfrac{x^2 - 32}{4x}$

ii) In $\triangle PBA$ $\quad A\hat{B}P = 90 - \alpha$

$\cos(90-\alpha) = \sin\alpha$ 1

$\therefore \quad \sin\alpha = \dfrac{x^2 + 2^2 - (2\sqrt{5})^2}{2 \times 2 \times x}$ 1

using cos rule

$= \dfrac{x^2 + 4 - 20}{4x} = \dfrac{x^2 - 16}{4x}$

iii) now $\sin^2\alpha + \cos^2\alpha = 1$ 1

$\left(\dfrac{x^2-16}{4x}\right)^2 + \left(\dfrac{x^2-32}{4x}\right)^2 = 1$

$(x^2-16)^2 + (x^2-32)^2 = 16x^2$

$x^4 - 32x^2 + 256 + x^4 - 64x^2 + 1024 = 16x^2$

$2x^4 - 112x^2 + 1280 = 0$ 1

$\therefore x^4 - 56x^2 + 640 = 0$

iv) $x^2 = \dfrac{56 \pm \sqrt{56^2 - 4 \times 640}}{2}$

$= \dfrac{56 \pm 24}{2}$ 1

$= 40, 16$

$\therefore x = \pm\sqrt{40}, \pm 4$

but $x > 0$ — for length

$x = \sqrt{40}, 4$ 1

For α to be acute $x^2 > 32$ — or $\cos\alpha = \dfrac{x^2-32}{4x}$ would be obtuse 1

$\therefore x = \sqrt{40} = 2\sqrt{10}$

alternate

(If $x = 4$ $\quad \sin\alpha = 0, \ \alpha = 0$; $\cos\alpha = -1, \ \alpha = 180$ $\Big\}$ no solution here.

if $x = \sqrt{40}$ $\quad \sin\alpha = \frac{3}{\sqrt{10}} \quad \alpha = 71°34'$; $\cos\alpha = 1/\sqrt{10} \quad \alpha = 71°34'$ ok.)