BAULKHAM HILLS HIGH SCHOOL

2012

YEAR 11 YEARLY

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions

Total marks – 70

This paper consists of TWO sections.

Section 1 – Multiple Choice
10 marks

Section 2 – Extended Response
60 marks
Attempt all questions
Start a new page for each question

Section 1 –Multiple Choice (10 marks)
Attempt all questions.

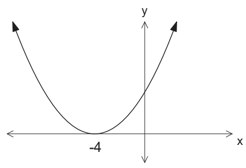
Answer the following on the answer sheet provided. **Marks**

1 What is the solution of $|1-2x|<7$? **1**

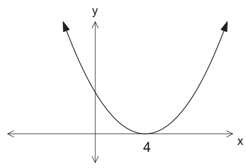
(A) $x<-3,\ x>4$ (B) $x<-4,\ x>3$ (C) $-3<x<4$ (D) $-4<x<3$

2 Which of the following could be the graph of $=(4-x)^2$? **1**

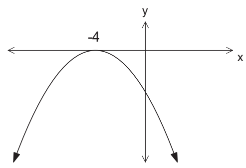
(A)



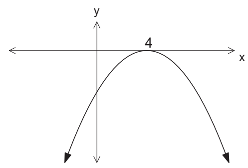
(B)



(C)



(D)



3 Factorise fully: $x^2-4y^2+16y-16$ **1**

(A) $(x+2y-4)(x-2y-4)$
(B) $(x-2y-4)(x+2y+4)$
(C) $(x-2y+4)^2$
(D) $(x-2y+4)(x+2y-4)$

4 If $a^b=3$, then $a^{4b}-5$ equals **1**

(A) 76 (B) 7 (C) 22 (D) 86

5 $(2,5)$ is the midpoint of $(5,y)$ and $(x,7)$. **1**

$x+y=$

(A) 2 (B) 3 (C) 4 (D) 18

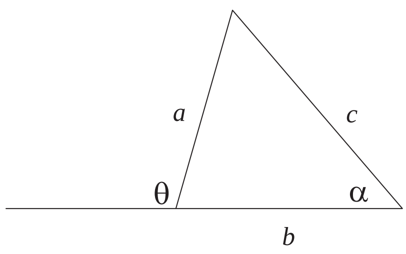
6 If the equation $(1-3k)x^2+3x-4=0$ has real roots, and k is an integer, what is the largest possible value for k? **1**

(A) -2 (B) -1 (C) 0 (D) 1

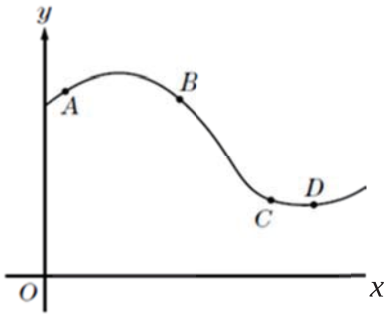
7 If a>b, which statement is always true? **1**

(A) $a^2>b^2$ (B) $\frac{1}{a}>\frac{1}{b}$ (C) $-a>-b$ (D) $2^a>2^b$

8 Which of the following statements is true for this diagram? **1**



(A) $c^2=a^2+b^2+2ab\cos\theta$
(B) $c^2=a^2+b^2-2ab\cos\theta$
(C) $a^2=b^2+c^2-2bc\cos\theta$
(D) $\frac{a}{\sin\alpha}=\frac{c}{\cos\theta}$

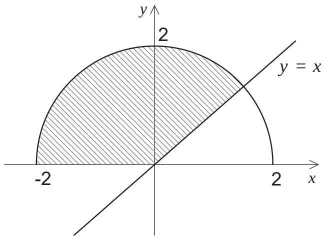
9	 At which of the points shown are y' and y'' both negative? (A) A (B) B (C) C (D) D	1
10	The area of a rectangle is 80 m^2 and its length is x metres. Its perimeter, in metres, is: (A) $2x + 160$ (B) $2x + \frac{80}{x}$ (C) $2x + \frac{40}{x}$ (D) $2x + \frac{160}{x}$	1
	End of Section I	

Section II – Extended Response
Attempt all questions. Show all necessary working.
Start each question on a new page. Clearly indicate question number.
Write your name and teacher's name at the top of each new page.

Question 11 (15 marks) - Start a new page		**Marks**
a)	If $\left(2+\sqrt{3}\right)^2 = a + \sqrt{b}$. where and b are rational, find the values of a and b	2
b)	Solve $8^{2-x} = 4^{2x}$	2
c)	Solve $\frac{3x-1}{4} - \frac{2x-5}{8} = x$	2
d)	Differentiate (i) $y = \frac{2x-1}{x+1}$ (ii) $f(x) = (2x-1)(3x+2)^5$	2 3
e)	State the domain and range of the function $f(x) = 2\sqrt{x+2} - 3$	2
f)	Evaluate $\lim\limits_{x \to 2} \dfrac{4 - x^2}{x^3 - 8}$	2

Question 12 (15 marks) - Start a new page **Marks**

a) Write the set of inequalities whose intersection describes the shaded region **3**



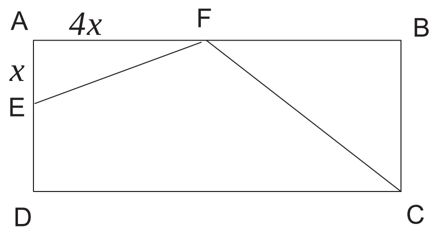
b) Find the exact value of $\cos 240°$ **1**

c) Solve for $0° \leq x \leq 360°$:

(i) $\cos 2x = \frac{1}{\sqrt{2}}$ **2**

(ii) $3sin^2x + 2sinx - 1 = 0$ **3**

d) ABCD is a rectangle with AB=12 cm and AD=6 cm. F is a point on AB such that AE=x and AF=$4x$.



(i) Show that the area of quadrilateral EFCD is $(36 + 12x - 2x^2)\ cm^2$ **2**

(ii) Without using calculus, find the value of x for which the quadrilateral EFCD has maximum area. (Clearly show your working) **1**

e) If the function $f(x) = (k-1)x^2 + (k+2)x + 4$ is positive definite, find all possible values of k **3**

Question 13 (15 marks) - Start a new page **Marks**

a) Find the equation of the tangent to the curve $y = x^3 - 3x$ at the point on the curve where $x = 2$ **3**

b) Let α and β be the roots of the equation $2x^2 - 5x - 2 = 0$.

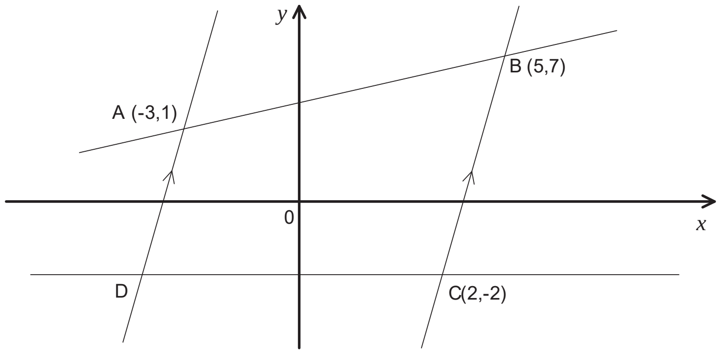
Evaluate:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha - 2)(\beta - 2)$ **1**

c)	A, B and C are the points (-3,1), (5,7) and (2,-2) respectively. The equation of the line AB is $3x - 4y + 7 = 0$ and the equation of DC is $y = -2$. AD is parallel to BC.	



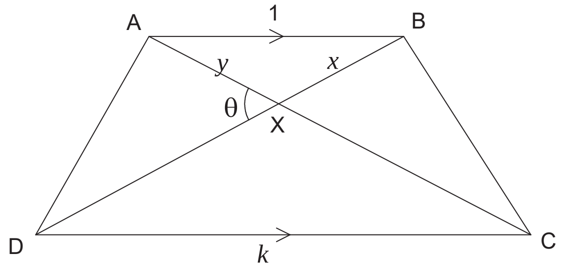
(i) Find the length of AB — 1
(ii) Calculate the perpendicular distance from C to AB — 2
(iii) Find the gradient of BC — 1
(iv) Show that the equation of AD is $y = 3x + 10$ — 2
(v) State the coordinates of D — 1
(vi) Find the area of quadrilateral ABCD — 2

Question 14 (15 marks) - Start a new page

a) For the curve $y = x^3 - 2x^2 - 4x$:

(i) Find the stationary points and determine their nature — 3
(ii) Show that there is a point of inflexion and determine its coordinates — 2
(iii) Neatly sketch the curve clearly showing its important features (x-intercepts are not required) — 2

b) ABCD is a trapezium with AB//DC, AB=1, DC=k and $k > 1$. Its diagonals intersect at X, with BX=x and AX=y.
Let $\angle AXD = \theta$.



(i) Prove that ΔABX is similar to ΔDCX — 2
(ii) Explain why $DX = kx$ and $CX = ky$. — 1
(iii) Show that $AD^2 - BC^2 = (k^2 - 1)(x^2 - y^2)$ — 3
(iv) Find the ratio $\frac{AD^2 - BC^2}{DB^2 - AC^2}$ in its simplest form — 2

End of Examination

SOLUTIONS : Year 11 Yearly Exam 2012 (2-unit)

SECTION I

1. $-7 < 1-2x < 7$

$-8 < -2x < 6$

$4 > x > -3$

$-3 < x < 4$ (C)

2. (B)

3. $x^2 - (4y^2 - 16y + 16)$

$= x^2 - (2y-4)^2$

$= (x - (2y-4))(x + (2y-4))$

$= (x - 2y + 4)(x + 2y - 4)$ (D)

4.

$(a^b)^4 - 5 = 3^4 - 5$

$= 76.$ $\therefore$ (A)

5. $\frac{5+x}{2} = 2$ $x = -1$

$\frac{y+7}{2} = 5$ $y = 3$

$\therefore x + y = -1 + 3 = 2$ (A)

6. $\Delta = 3^2 - 4(1-3k).-4$

$= 9 + 16 - 48k$

$= 25 - 48k \geq 0$

when $-48k \geq -25$

$k \leq \frac{25}{48}$

Largest integer $k = 0$ (C)

7. (D)

8. $c^2 = a^2 + b^2 - 2ab\cos(180° - \theta)$

$= a^2 + b^2 - 2ab.-\cos\theta$

$= a^2 + b^2 + 2ab\cos\theta$ (A)

9. Decreasing + conc. down (B)

10. 

$xy = 80 \therefore y = \frac{80}{x}$

$P = 2x + 2y = 2x + \frac{160}{x}$ (D)

SECTION II

Q11.

a) $(2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3$

$= 7 + 4\sqrt{3}$ — 1

$= 7 + \sqrt{48}$ — 1

$\therefore a = 7, \quad b = 48$

b) $(2^3)^{2-x} = (2^2)^{2x}$

$2^{6-3x} = 2^{4x}$ — 1

$6 - 3x = 4x$

$7x = 6$

$x = 6/7.$ — 1

c) $\frac{2(3x-1) - 1(2x-5)}{8} = x$

$6x - 2 - 2x + 5 = 8x$ — 1

$4x + 3 = 8x$

$3 = 4x$

$x = \frac{3}{4}$ — 1

d) (i) $u = 2x - 1$ $u' = 2$

$v = x + 1$ $v' = 1$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{(x+1).2 - (2x-1).1}{(x+1)^2}$ — 1

$= \frac{2x + 2 - 2x + 1}{(x+1)^2}$

$= \frac{3}{(x+1)^2}$ — 1

(ii) $u = 2x - 1$ $u' = 2$

$v = (3x+2)^5$ $v' = 5(3x+2)^4.3$

$= 15(3x+2)^4$

$f'(x) = uv' + vu'$

$= \underbrace{15(2x-1)(3x+2)^4}_{1} + \underbrace{2(3x+2)^5}_{1}$

e) Domain : $x \geq -2$ $\leftarrow 1$

Range : $f(x) \geq -3$ (or $y \geq -3$) $\leftarrow 1$

f) $\lim\limits_{x \to 2} \dfrac{\cancel{(2-x)}^{-1}(2+x)}{\cancel{(x-2)}(x^2+2x+4)}$ $\leftarrow 1$

$= \lim\limits_{x \to 2} \dfrac{-(2+x)}{x^2+2x+4}$ — 1

$= \dfrac{-4}{4+4+4}$

$= -\dfrac{1}{3}$ $\leftarrow 1$

Q12.

a) $y \leq \sqrt{4-x^2}$ or $x^2+y^2 \leq 4$ $\leftarrow 1$

$y \geq x$ $\leftarrow 1$

$y \geq 0$ (or $y > 0$) $\leftarrow 1$

b) $\cos 240^\circ = -\cos 60^\circ = -\dfrac{1}{2}$ 1

c) i) $2x = 45^\circ, 315^\circ, 405^\circ, 675^\circ$

$x = \underbrace{22.5^\circ, 157.5^\circ}_{1}, \underbrace{202.5^\circ, 337.5^\circ}_{1}$

ii) $(3\sin x - 1)(\sin x + 1) = 0$ 1

$\sin x = \frac{1}{3}$ or $\sin x = -1$

$x = \underbrace{19^\circ 28', 160^\circ 32'}_{1}, \underbrace{270^\circ}_{1}$

d) i) Area shaded

$= (12 \times 6) - \frac{1}{2}.4x.x - \frac{1}{2}.(12-4x).6$ 1

$= 72 - 2x^2 - 3(12-4x)$

$= 72 - 2x^2 - 36 + 12x$ 1

$= \underline{36 + 12x - 2x^2}$

ii) Max. value of a quadratic fn occurs when $x = -\frac{b}{2a}$

ie. $x = \dfrac{-12}{2 \times -2} = 3$ 1

[must have clear working]

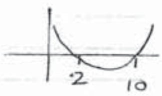
e) For pos. definite $a > 0$, $\Delta < 0$

$\Delta = (k+2)^2 - 4(k-1).4$

$= k^2+4k+4-16k+16$

$= k^2 - 12k + 20$ — (1)

$= (k-2)(k-10)$



< 0 when

$\therefore 2 < k < 10$

but also $a = k-1 > 0$

$k > 1$ (1)

$\therefore$ $\underline{\text{Solution } \; 2 < k < 10}$ (1)

Q13.

a) $y = x^3 - 3x$

$\dfrac{dy}{dx} = 3x^2 - 3$

$= 3(2)^2 - 3$ when $x = 2$

$= 9$ — (1)

Point of contact is $(2, 2)$ — (1)

$y - 2 = 9(x-2)$

$y - 2 = 9x - 18$

$y = 9x - 16$ — (1)

Q13 (cont)

b) i) $\alpha+\beta = \frac{-b}{a} = \frac{5}{2}$ (1)

ii) $\alpha\beta = \frac{c}{a} = \frac{-2}{2} = -1$ (1)

iii) $(\alpha-2)(\beta-2)$

$= \alpha\beta - 2\alpha - 2\beta + 4$

$= \alpha\beta - 2(\alpha+\beta) + 4$

$= -1 - 2\left(\frac{5}{2}\right) + 4$

$= -2$ (1)

c) i) $AB = \sqrt{(5+3)^2 + (7-1)^2}$

$= \sqrt{64+36}$

$= \sqrt{100}$

$= 10$ units (1)

ii) $d = \left|\frac{3(2)-4(-2)+7}{\sqrt{3^2+(-4)^2}}\right|$ ← (1)

$= \left|\frac{21}{5}\right|$

$= \frac{21}{5}$ units. ← (1)

iii) $m_{BC} = \frac{7+2}{5-2} = \frac{9}{3} = 3$ (1)

iv) $\therefore$ AD has $m = 3$, and passes through $(-3, 1)$ (1)

AD // BC

$y - 1 = 3(x+3)$ (1)

$y = 3x + 10$

v) At D: $y = -2$

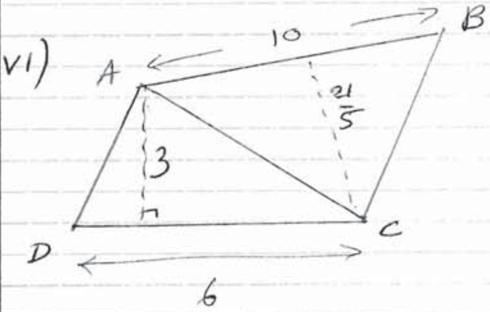
$-2 = 3x + 10$

$3x = -12$

$x = -4$

$\therefore D(-4, -2)$ (1)

vi)



$A_1 = \frac{1}{2} \times 6 \times 3 = 9$ (1)

$A_2 = \frac{1}{2} \times 10 \times \frac{21}{5} = 21$ (1)

Total area $= 30$ units^2

[Also accept 36 units^2, because given equation for AB is incorrect]

Q14.

a) $y = x^3 - 2x^2 - 4x$

i) $y' = 3x^2 - 4x - 4 = 0$ for stationary pts.

$(3x+2)(x-2) = 0$

$\left.\begin{array}{l} x = -\frac{2}{3} \\ y = \frac{40}{27} \end{array}\right\}$ $\qquad$ $\left.\begin{array}{l} x = 2 \\ y = -8 \end{array}\right\}$

$\left(-\frac{2}{3}, \frac{40}{27}\right)$ and $(2,-8)$ are stationary pts (1)

$y'' = 6x - 4$

At $\left(-\frac{2}{3}, \frac{40}{27}\right)$ $\quad y'' = 6\left(-\frac{2}{3}\right) - 4 = -4-4 < 0$

$\therefore$ Maximum turning pt (1)

At $(2,-8)$ $\quad y'' = 6(2) - 4 = 12 - 4 > 0$

$\therefore$ Minimum turning pt. (1)

ii) For a POI, $y'' = 0$ and changes sign

$y'' = 6x - 4 = 0$ when $x = \frac{2}{3}$, $y = -\frac{88}{27}$

$\therefore \left(\frac{2}{3}, -\frac{88}{27}\right)$ is a possible POI $\leftarrow$ (1)

Now check

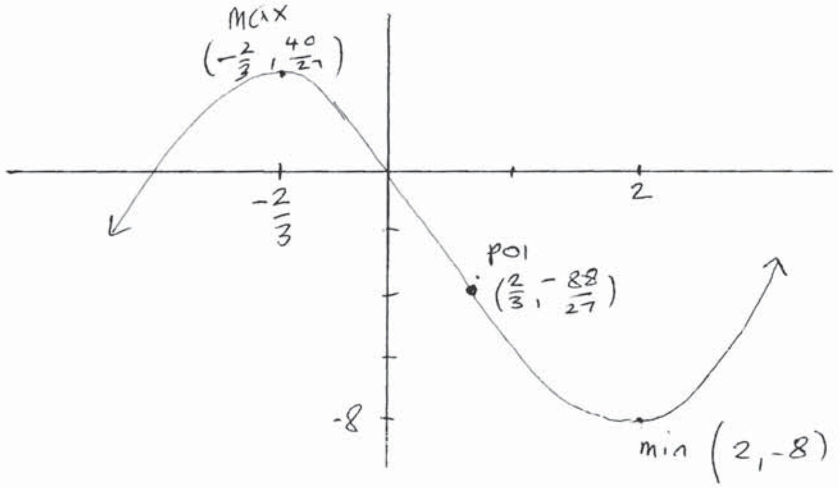
x	0	$\frac{2}{3}$	1
y''	-4	0	$+2$
	$\uparrow$ $6(0)-4$		$\uparrow$ $6(1)-4$

Sign / concavity changes

$\therefore \left(\frac{2}{3}, -\frac{88}{27}\right)$ is a POI

$\leftarrow$ (1)

iii)



(2)

~~Question 15.~~

b) i) In ΔABX, ΔDCX

$\angle AXB = \angle DXC$ (vertically opposite $\angle$s equal)

$\angle BAX = \angle XCD$ (alternate $\angle$s, $AB \parallel DC$) ← (1)

$\Delta ABX \,|||\, \Delta DCX$ (matching $\angle$s equal) ← (1)

ii) $\frac{DX}{x} = \frac{CX}{y} = \frac{k}{1}$ (matching sides of similar Δs proportional) ← (1)

$\therefore DX = kx$ and $CX = ky$

iii) $AD^2 = (kx)^2 + y^2 - 2.kx.y.\cos\theta$

$= k^2x^2 + y^2 - 2kxy\cos\theta$ — (1)

Use Cosine Rule (1)

$\angle BXC = \theta$ (vertically opposite $\angle$s equal)

$BC^2 = (ky)^2 + x^2 - 2.ky.x.\cos\theta$

$= k^2y^2 + x^2 - 2kxy\cos\theta$ — (2)

Subtracting, (1) − (2)

$AD^2 - BC^2 = k^2x^2 + y^2 - k^2y^2 - x^2$

Subtracting (1)

$$= (k^2-1)x^2 + (1-k^2)y^2$$
$$= (k^2-1)x^2 - (k^2-1)y^2 \quad (1)$$
$$= (k^2-1)(x^2-y^2)$$

iv) $DB^2 = (kx+x)^2$
$$= (x(k+1))^2$$
$$= x^2(k+1)^2$$

$AC^2 = (y+ky)^2$
$$= (y(1+k))^2$$
$$= y^2(k+1)^2$$

Find DB^2 or AC^2 (1)

$\therefore DB^2 - AC^2 = (k+1)^2x^2 - (k+1)^2y^2$
$$= (k+1)^2(x^2-y^2)$$

Ratio required $= \dfrac{(k^2-1)\cancel{(x^2-y^2)}}{(k+1)^2\cancel{(x^2-y^2)}}$

$$= \frac{(k-1)(k+1)}{(k+1)^2}$$

$$= \frac{k-1}{k+1}$$

Answer - fully simplified (1)