



**BAULKHAM HILLS HIGH SCHOOL**

**2013  
YEAR 11  
YEARLY EXAMINATIONS**

# Mathematics

## General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen  
Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 – 14
- Marks may be deducted for careless or badly arranged work

## Total marks – 70

**Section I** Pages 2 – 4

### 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

**Section II** Pages 5 – 9

### 60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

## Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 If  $2x^2 - 12x + 11$  is expressed in the form  $2(x - b)^2 + c$ , what is the value of  $c$ ?

(A)  $-25$

(B)  $-7$

(C)  $2$

(D)  $29$

2 For what values of  $k$  does the equation  $x^2 - 3x + k = 0$  have equal roots?

(A)  $-\frac{9}{4}$

(B)  $-\frac{1}{12}$

(C)  $0$

(D)  $\frac{9}{4}$

3 What is the derivative of  $(x^3 + 4)^3$ ?

(A)  $3(x^3 + 4)^2$

(B)  $3(3x^2 + 4)^2$

(C)  $9x^2(x^3 + 4)^2$

(D)  $9x^2(3x^2 + 4)^2$

- 4 A function is given by  $f(x) = \sqrt{9 - x^2}$ . What is a suitable domain of  $f(x)$  ?
- (A)  $x \geq 3$
- (B)  $x \leq 3$
- (C)  $-3 \leq x \leq 3$
- (D)  $-9 \leq x \leq 9$
- 5 The line through  $P(7, p)$  and  $Q(4, -5)$  has a gradient of 3. What is the value of  $p$ ?
- (A)  $-14$
- (B)  $4$
- (C)  $6$
- (D)  $8$
- 6 What is the gradient of the perpendicular to the line with equation  $3y = -2x + 1$  ?
- (A)  $-2$
- (B)  $-\frac{2}{3}$
- (C)  $\frac{1}{2}$
- (D)  $\frac{3}{2}$
- 7 Find all the values of  $x$  in the interval  $0^\circ \leq x \leq 360^\circ$  for which  $\tan x = -\sqrt{3}$ .
- (A)  $30^\circ, 330^\circ$
- (B)  $120^\circ, 240^\circ$
- (C)  $120^\circ, 300^\circ$
- (D)  $150^\circ, 330^\circ$

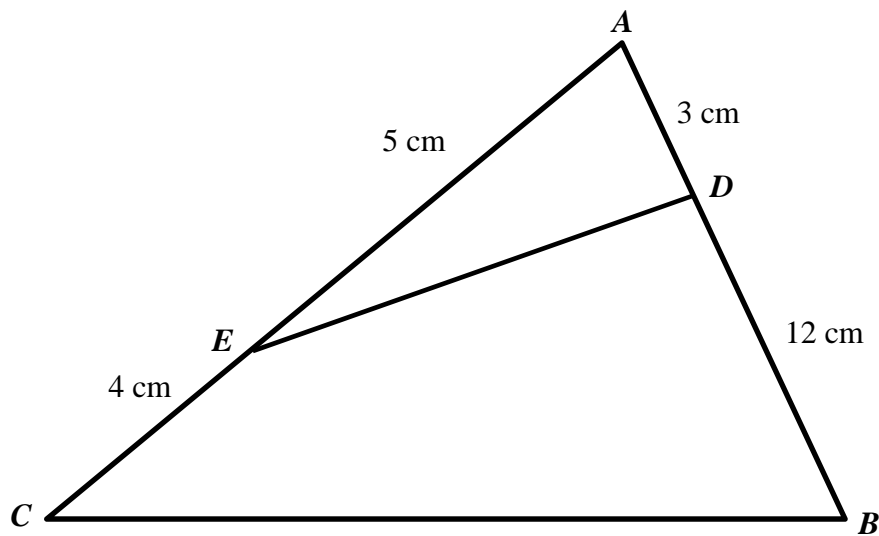
8 Which of the following is true for the function  $g(x)$  where  $g'(x) = x^2 + 2x + 1$  ?

- (A)  $g(x)$  is never decreasing.
- (B)  $g(x)$  is increasing then decreasing.
- (C)  $g(x)$  is decreasing then increasing.
- (D)  $g(x)$  is never increasing.

9 If  $a > b$  , which of the following is always true?

- (A)  $a^2 > b^2$
- (B)  $\frac{1}{a} > \frac{1}{b}$
- (C)  $-a > -b$
- (D)  $2^a > 2^b$

10 If  $\triangle ABC$  has area  $36 \text{ cm}^2$  , then the area of  $\triangle ADE$  is



- (A)  $4 \text{ cm}^2$
- (B)  $8 \text{ cm}^2$
- (C)  $12 \text{ cm}^2$
- (D)  $16 \text{ cm}^2$

**END OF SECTION I**

## Section II

90 marks

Attempt Questions 11 – 14

Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.

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*Marks*

**Question 11** (15 marks) Use a *separate* answer sheet

(a) Express  $0.\dot{6}0\dot{4}$  as a fraction in its simplest form. 2

(b) Factorise  $x^3 - 2x^2 - 15x$  2

(c) Solve  $|5x - 9| > 21$  2

(d) Write  $\frac{\sqrt{2}}{\sqrt{3} + 2}$  with a rational denominator 2

(e) Find the coordinates of the focus of the parabola  $x^2 = 12(y - 4)$  2

(f) Simplify  $\frac{1}{x^2 + x} - \frac{1}{x^2 - x}$  2

(g) Solve the following pair of simultaneous equations 3

$$\begin{aligned}xy &= 2 \\x - 2y &= 3\end{aligned}$$

**Question 12** (15 marks) Use a *separate* answer sheet

(a) Find the derivative of

(i)  $5x^3 + 4x - 7 + \frac{3}{x^2}$  2

(ii)  $\frac{2x-5}{3x+7}$  2

(iii)  $\sqrt{4-x^2}$  2

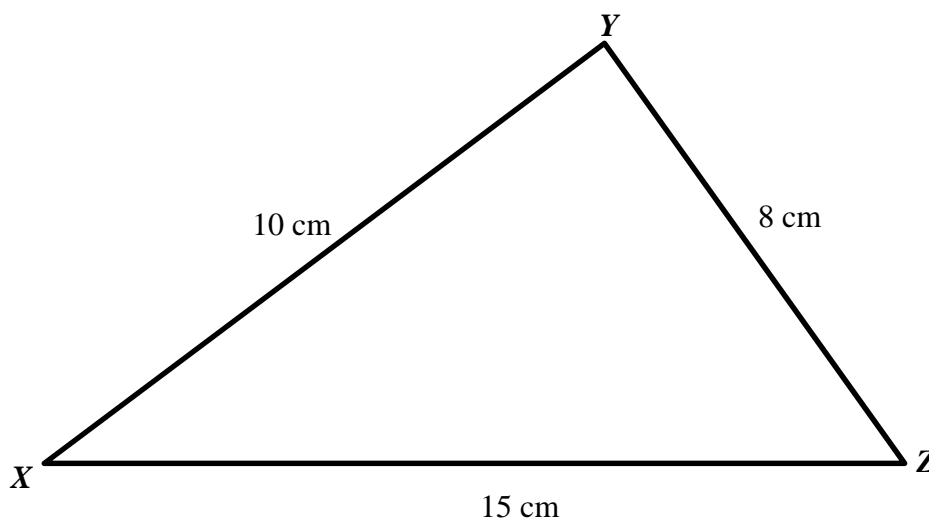
(b) If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 2x + 5 = 0$ , find

(i)  $\frac{1}{\alpha} + \frac{1}{\beta}$  2

(ii)  $\alpha^2 + \beta^2$  2

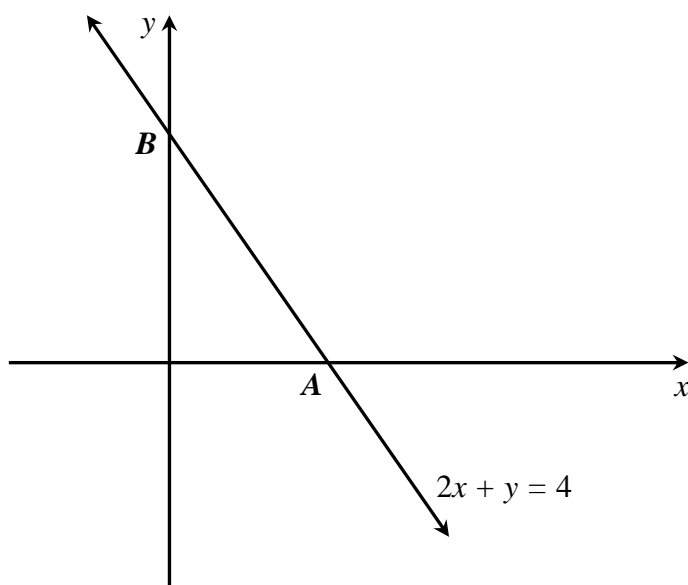
(c) If  $\theta$  is a reflex angle such that  $\tan\theta = \frac{2}{3}$ , find the value of  $\cos\theta$ . 2

(d) Use the cosine rule to find the largest angle in this triangle, correct to the nearest minute. 3



**Question 13** (15 marks) Use a *separate* answer sheet

(a) In the diagram, the line  $2x + y = 4$  cuts the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$

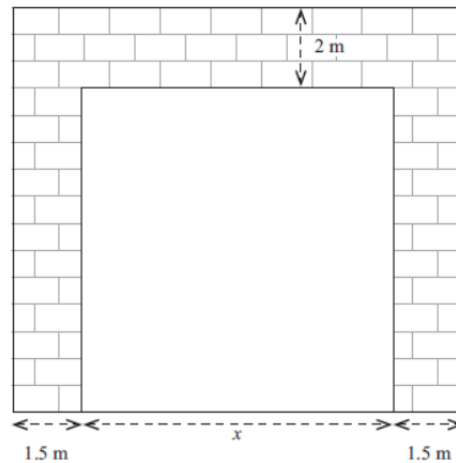


Copy the diagram onto your answer sheet.

- (i) Find the coordinates of  $A$  and  $B$ . 1
- (ii) Find the perpendicular distance of the point  $C(5, 2)$  from the line  $2x + y = 4$ . 2
- (iii) Show that the gradient of the line  $AC$  is  $\frac{2}{3}$ . 1
- (iv) Hence, or otherwise, show that the equation of the line  $AC$  is  $2x - 3y = 4$ . 1
- (v) Find the distance  $AB$ . 2
- (vi) Find the exact area of  $\triangle ABC$ . 1
- (vii) On your diagram, sketch the region described by the inequalities  $2x + y \geq 4$  and  $2x - 3y \leq 4$ . 1
- (b) The function  $y = x^3 - 3x^2 - 9x + 1$  is defined in the domain  $-2 \leq x \leq 5$ .
- (i) Find the coordinates of any turning points and determine their nature. 3
- (ii) Find the coordinates of any points of inflection. 1
- (iii) Draw a neat sketch of the curve, clearly labelling the important features. 2

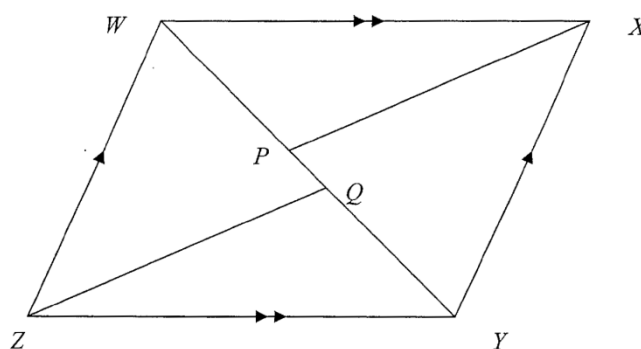
**Question 14** (15 marks) Use a *separate* answer sheet

- (a) A square garage door of length  $x$  metres is surrounded on three sides, by brickwork. The brickwork is 1.5 metres wide on each side of the door and of height 2 metres above the door, as shown below.



In order to meet building regulations, the area of brickwork must be more than twice the area of the garage door.

- (i) Show that  $2x^2 - 5x - 6 < 0$  2
- (ii) Hence find the range of values of  $x$ , in order to satisfy building regulations. 2
- (b)  $WXYZ$  is a parallelogram.  $XP$  bisects  $\angle WXY$  and  $ZQ$  bisects  $\angle WZY$ .



Copy or trace the diagram onto your answer sheet.

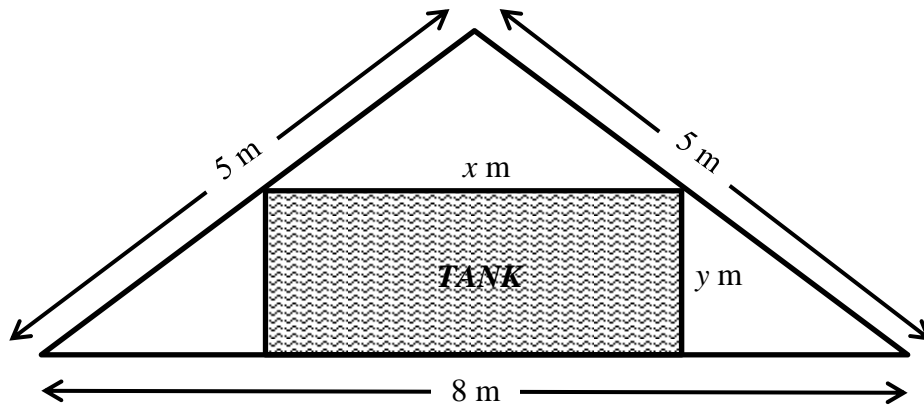
- (i) Explain why  $\angle WXY = \angle WZY$  1
- (ii) Prove  $\triangle WXP \equiv \triangle YZQ$  2
- (iii) Hence find the length of  $PQ$ , given  $WY = 20$  cm and  $QY = 8$  cm 1

**Question 14 continues on page 9**



**Question 14 (continued)**

- (c) A rectangular hot water tank  $x$  metres wide,  $y$  metres high and 1.5 metres long, which fits exactly into the roof of the house.  
The cross-section of the roof is an isosceles triangle with base 8 metres and equal sides 5 metres in length, as shown below.



- (i) Explain why the roof of the house is 3 metres high. 1
- (ii) Show that  $y = \frac{3}{8}(8 - x)$ . 2
- (iii) Show that the volume of the tank is given by  $V = \frac{9}{16}x(8 - x)$ . 1
- (iv) Calculate the maximum volume of the tank. 3

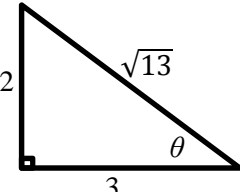
**End of paper**

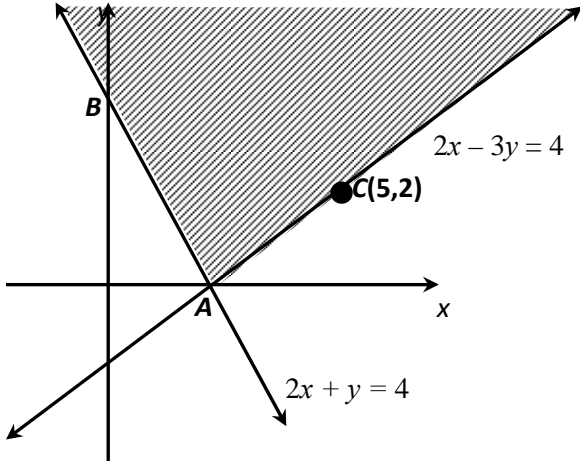
**BAULKHAM HILLS HIGH SCHOOL**  
**YEAR 11 MATHEMATICS YEARLY EXAMINATION 2013 SOLUTIONS**










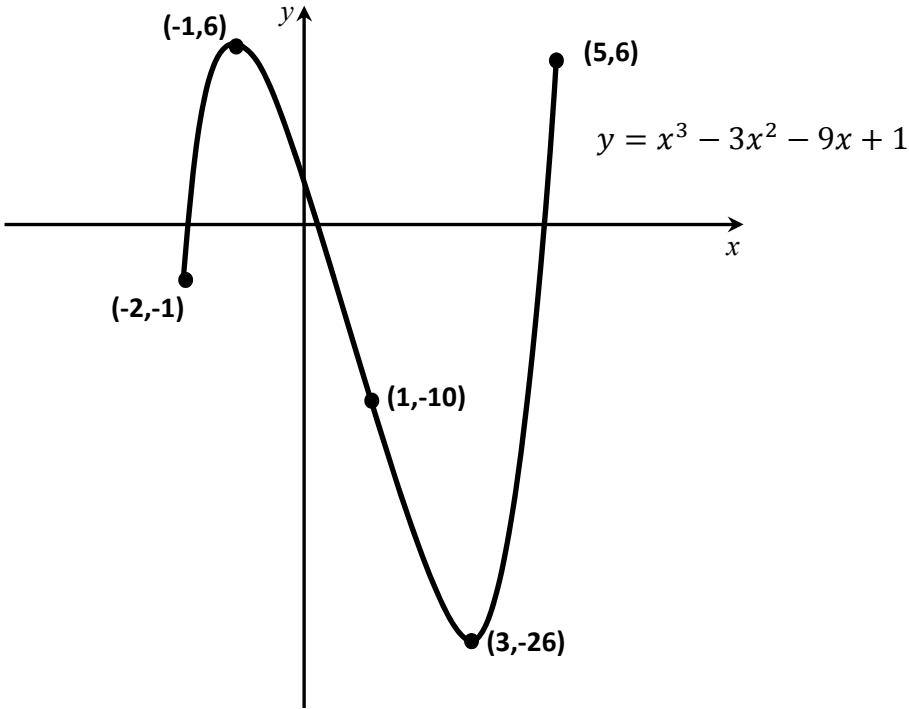
Solution	Marks	Comments
<b>SECTION I</b>		
<p><b>1. B</b> - <math>2x^2 - 12x + 11 = 2(x^2 - 6x) + 11</math>  <math>= 2(x^2 - 6x + 9) - 18 + 11</math>  <math>= 2(x - 3)^2 - 7</math>  <math>\therefore c = -7</math></p>	<b>1</b>	
<p><b>2. D</b> - equal roots occur when <math>\Delta = 0</math>  i.e. <math>b^2 - 4ac = 0</math>  <math>(-3)^2 - 4(1)(k) = 0</math>  <math>9 - 4k = 0</math>  <math>4k = 9</math>  <math>k = \frac{9}{4}</math></p>	<b>1</b>	
<p><b>3. C</b> - <math>f(x) = (x^3 + 4)^3</math>  <math>f'(x) = 3(x^3 + 4)^2(3x^2)</math>  <math>= 9x^2(x^3 + 4)^2</math></p>	<b>1</b>	
<p><b>4. C</b> - domain: <math>9 - x^2 \geq 0</math>  <math>x^2 \leq 9</math>  <math>-3 \leq x \leq 3</math></p>	<b>1</b>	
<p><b>5. B</b> - <math>m_{PQ} = 3</math>  <math>\frac{p+5}{7-4} = 3</math>  <math>\frac{p+5}{3} = 3</math>  <math>p+5 = 9</math>  <math>p = 4</math></p>	<b>1</b>	
<p><b>6. D</b> - <math>3y = -2x + 1</math>  <math>y = -\frac{2}{3}x + \frac{1}{3}</math>  gradient = <math>-\frac{2}{3}</math>  <math>\therefore</math> gradient of perpendicular = <math>\frac{3}{2}</math></p>	<b>1</b>	
<p><b>7. C</b> - <math>\tan x = -\sqrt{3}</math> <math>0^\circ \leq x \leq 360^\circ</math>  Q2 &amp; Q4  <math>\tan \alpha = \sqrt{3}</math>  <math>\alpha = 60^\circ</math>  <math>x = 180 - \alpha, 360 - \alpha</math>  <math>x = 120^\circ, 300^\circ</math></p>	<b>1</b>	
<p><b>8. A</b> - <math>g'(x) = x^2 + 2x + 1</math>  <math>= (x+1)^2 \geq 0</math>  <math>\therefore g'(x)</math> is never negative  Thus <math>g(x)</math> is never decreasing</p>	<b>1</b>	
<p><b>9. D</b> - (A) <math>-1 &gt; -2</math> however <math>(-1)^2 &gt; (-2)^2</math> is FALSE  (B) <math>3 &gt; 2</math> however <math>\frac{1}{3} &gt; \frac{1}{2}</math> is FALSE  (C) <math>3 &gt; 2</math> however <math>-3 &gt; -2</math> is FALSE  (D) as <math>2^x</math> increases as <math>x</math> increases then if <math>a &gt; b</math> then <math>2^a &gt; 2^b</math></p>	<b>1</b>	

Solution	Marks	Comments
<p><b>10. A -</b> <math>\angle DAE = \angle CAB</math> (common <math>\angle</math>)</p> $\frac{AD}{AC} = \frac{3}{9} = \frac{1}{3}$ $\frac{AE}{AB} = \frac{5}{15} = \frac{1}{3}$ <p><math>\therefore \frac{AD}{AC} = \frac{AE}{AB}</math></p> <p>thus <math>\triangle DAE \parallel \triangle CAB</math> (SAS - in ratio 1:3)</p> <p>If sides are in the ratio 1:3 then the areas are in the ratio <math>1:3^2 = 1:9</math></p> <p><math>\therefore</math> area <math>\triangle DAE = 4 \text{ cm}^2</math></p>	<b>1</b>	



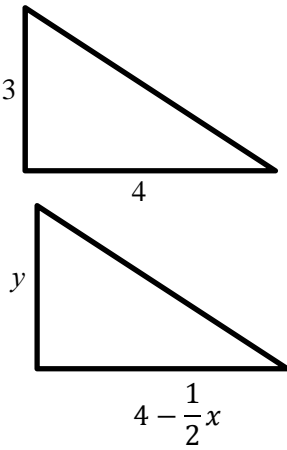
Solution		Marks	Comments	
<b>QUESTION 12</b>				
<b>12(a) (i)</b>	$f(x) = 5x^3 + 4x - 7 + \frac{3}{x^2}$ $= 5x^3 + 4x - 7 + 3x^{-2}$ $f'(x) = 15x^2 + 4 - 6x^{-3}$ $= 15x^2 + 4 - \frac{6}{x^3}$	<b>2</b>	<b>2 marks</b> • Correct answer <b>1 mark</b> • Correctly differentiates at least two terms <i>Note: may be left in index form</i>	
<b>12(a) (ii)</b>	$f(x) = \frac{2x-5}{3x+7}$ $f'(x) = \frac{(3x+7)(2) - (2x-5)(3)}{(3x+7)^2}$ $= \frac{6x+14-6x+15}{(3x+7)^2}$ $= \frac{29}{(3x+7)^2}$	<b>2</b>	<b>2 marks</b> • Correct answer <b>1 mark</b> • Correct use of the quotient rule or equivalent merit	
<b>12(a) (iii)</b>	$f(x) = \sqrt{4-x^2}$ $= (4-x^2)^{\frac{1}{2}}$	$f'(x) = \frac{1}{2}(4-x^2)^{-\frac{1}{2}}(-2x)$ $= -x(4-x^2)^{-\frac{1}{2}}$ $= -\frac{x}{\sqrt{4-x^2}}$	<b>2</b>	<b>2 marks</b> • Correct answer <b>1 mark</b> • Correct use of the chain rule or equivalent merit <i>Note: may be left in index form</i>
<b>12(b) (i)</b>	$\alpha + \beta = 2 \quad \alpha\beta = 5$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$ $= \frac{2}{5}$	<b>2</b>	<b>2 marks</b> • Correct answer. <b>1 mark</b> • Correctly finds $\alpha + \beta$ and $\alpha\beta$ • Rewrites $\frac{1}{\alpha} + \frac{1}{\beta}$ in terms of sum and product of roots	
<b>12(b) (ii)</b>	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= (2)^2 - 2(5)$ $= 4 - 10$ $= -6$	<b>2</b>	<b>2 marks</b> • Correct answer. <b>1 mark</b> • Rewrites $\alpha^2 + \beta^2$ in terms of sum and product of roots	
<b>12(c)</b>	$\theta$ is reflex and $\tan\theta > 0$ , $\therefore$ 3rd quadrant and thus $\cos\theta < 0$		<b>2</b>	$\therefore \cos\theta = -\frac{3}{\sqrt{13}}$ <b>2 marks</b> • Correct answer <b>1 mark</b> • Establishes the sign of the answer • Finds the correct ratio, disregarding sign
<b>12(d)</b>	$y^2 = x^2 + z^2 - 2xz\cos Y$ $\cos Y = \frac{x^2 + z^2 - y^2}{2xz}$ $= \frac{8^2 + 10^2 - 15^2}{2 \times 8 \times 10}$ $Y = \cos^{-1}\left(-\frac{61}{160}\right)$ $= 112^\circ 25'$	<b>3</b>	<b>3 marks</b> • Correct solution <b>2 marks</b> • Correctly finds one of the other angles • Uses the cosine rule in an attempt to find the largest angle <b>1 mark</b> • Correctly identifies $Y$ as the largest angle • Uses the cosine rule in an attempt to find one of the other angles <i>Note: do not penalise for the incorrect rounding of minutes</i>	

Solution	Marks	Comments
<b>QUESTION 13</b>		
<b>13(a) (i)</b> $x$ -intercept occurs when $y = 0$ $2x = 4$ $x = 2$ $A$ is $(2, 0)$	<b>1</b>	<b>1 mark</b> <ul style="list-style-type: none"> <li>• Correct answer for both <math>A</math> and <math>B</math></li> </ul>
<b>13(a) (ii)</b> $d = \frac{ 2(5) + (2) - 4 }{\sqrt{2^2 + 1^2}}$ $= \frac{8}{\sqrt{5}}$ units	<b>2</b>	<b>2 marks</b> <ul style="list-style-type: none"> <li>• Correct answer</li> </ul> <b>1 mark</b> <ul style="list-style-type: none"> <li>• Correctly substitutes into the perpendicular distance formula</li> </ul>
<b>13(a) (iii)</b> $m_{AB} = \frac{2-0}{5-2}$ $= \frac{2}{3}$	<b>1</b>	<b>1 mark</b> <ul style="list-style-type: none"> <li>• Correct solution</li> </ul>
<b>13(a) (iv)</b> $y - 0 = \frac{2}{3}(x - 2)$ $3y = 2x - 4$ $2x - 3y = 4$	<b>1</b>	<b>1 mark</b> <ul style="list-style-type: none"> <li>• Correct solution</li> </ul>
<b>13(a) (v)</b> $d_{AB} = \sqrt{(2-0)^2 + (0-4)^2}$ $= \sqrt{4 + 16}$ $= \sqrt{20}$ $= 2\sqrt{5}$ units	<b>2</b>	<b>2 marks</b> <ul style="list-style-type: none"> <li>• Correct answer</li> </ul> <b>1 mark</b> <ul style="list-style-type: none"> <li>• Correctly substitutes into the distance formula or equivalent merit</li> </ul> <i>Note: Need to use answers found in part (i)</i>
<b>13(a) (vi)</b> Area $\Delta ABC = \frac{1}{2} \times 2\sqrt{5} \times \frac{8}{\sqrt{5}}$ $= 8 \text{ units}^2$	<b>1</b>	<b>1 mark</b> <ul style="list-style-type: none"> <li>• Correct answer</li> </ul> <i>Note: May use answers found in (ii) and (v)</i>
<b>13(a) (vii)</b> 	<b>1</b>	<b>1 mark</b> <ul style="list-style-type: none"> <li>• Correct answer</li> </ul>

Solution	Marks	Comments												
<p><b>13(b) (i)</b> <math>y = x^3 - 3x^2 - 9x + 1</math>  <math>\frac{dy}{dx} = 3x^2 - 6x - 9</math>  <math>\frac{d^2y}{dx^2} = 6x - 6</math>  <math>\frac{d^3y}{dx^3} = 6</math></p> <p>Stationary points occur when <math>\frac{dy}{dx} = 0</math>  i.e. <math>3x^2 - 6x + 9 = 0</math>  <math>x^2 - 2x + 3 = 0</math>  <math>(x - 3)(x + 1) = 0</math>  <math>x = 3</math> or <math>x = -1</math></p> <p>when <math>x = -1</math>, <math>\frac{d^2y}{dx^2} = 6(-1) - 6 = -12 &lt; 0</math>  when <math>x = 3</math>, <math>\frac{d^2y}{dx^2} = 6(3) - 6 = 12 &gt; 0</math></p> <p><math>\therefore (-1, 6)</math> is a maximum turning point <math>\therefore (3, -26)</math> is a minimum turning point</p>	3	<p><b>3 marks</b></p> <ul style="list-style-type: none"> <li>• Correct solution</li> </ul> <p><b>2 marks</b></p> <ul style="list-style-type: none"> <li>• Finds both stationary points</li> <li>• Finds one stationary point and determines its nature</li> </ul> <p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• Acknowledges the condition for stationary points</li> </ul>												
<p><b>13(b) (ii)</b> possible inflection points occur when <math>\frac{d^2y}{dx^2} = 0</math>  i.e. <math>6x - 6 = 0</math>  <math>x = 1</math></p> <p style="text-align: center;"><b>OR</b></p> <table border="1" data-bbox="108 936 563 1070"> <tr> <td style="text-align: center;"><math>x</math></td> <td style="text-align: center;"><math>1^- (0)</math></td> <td style="text-align: center;"><math>1</math></td> <td style="text-align: center;"><math>1^+ (2)</math></td> </tr> <tr> <td style="text-align: center;"><math>\frac{d^2y}{dx^2}</math></td> <td style="text-align: center;"><math>-6</math></td> <td style="text-align: center;"><math>0</math></td> <td style="text-align: center;"><math>6</math></td> </tr> <tr> <td></td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> </table> <p style="text-align: center;">when <math>x = 1</math>, <math>\frac{d^3y}{dx^3} = 6 \neq 0</math></p> <p><math>\therefore</math> there is a change in concavity  thus <math>(1, -10)</math> is a point of inflection</p>	$x$	$1^- (0)$	$1$	$1^+ (2)$	$\frac{d^2y}{dx^2}$	$-6$	$0$	$6$					1	<p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• Finds the possible inflection point and explains the change in concavity</li> </ul>
$x$	$1^- (0)$	$1$	$1^+ (2)$											
$\frac{d^2y}{dx^2}$	$-6$	$0$	$6$											
														
<p><b>13(b) (iii)</b></p> 	2	<p><b>2 marks</b></p> <ul style="list-style-type: none"> <li>• Neat sketch with key points labelled</li> </ul> <p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• A complete sketch with a poor scale</li> <li>• Neat sketch with correct basic shape, with only some key features labelled</li> <li>• A correct sketch that extends outside the given domain</li> </ul>												

Solution		Marks	Comments
<b>QUESTION 14</b>			
<b>14(a) (i)</b>	<p>The total face of the garage must be <math>&gt; 3x^2</math></p> $\therefore (x+3)(x+2) > 3x^2$ $x^2 + 5x + 6 > 3x^2$ $0 > 2x^2 - 5x - 6$ $2x^2 - 5x - 6 < 0$	<b>2</b>	<p><b>2 marks</b></p> <ul style="list-style-type: none"> <li>Obtains desired inequality using a valid argument</li> </ul> <p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>Makes some logical statement that would lead to a correct solution</li> </ul>
<b>14(a) (ii)</b>	$2x^2 - 5x - 6 < 0$ $\frac{5 - \sqrt{5^2 - 4 \times 2 \times (-6)}}{2 \times 2} < x < \frac{5 + \sqrt{5^2 + 4 \times 2 \times (-6)}}{2 \times 2}$ $\frac{5 - \sqrt{73}}{4} < x < \frac{5 + \sqrt{73}}{4}$ <p>However <math>x &gt; 0</math>, <math>\therefore 0 &lt; x &lt; \frac{5 + \sqrt{73}}{4}</math></p>	<b>2</b>	<p><b>2 marks</b></p> <ul style="list-style-type: none"> <li>Correct solution</li> </ul> <p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>Finds the two roots of <math>2x^2 - 5x - 6</math></li> <li>Discounts the negative solutions of their inequality</li> </ul>
<b>14(b) (i)</b>	Opposite $\angle$ 's in a parallelogram are =	<b>1</b>	<p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>Correct explanation</li> </ul>
<b>14(b) (ii)</b>	$\angle WXP = \frac{1}{2}\angle WXY \quad (\text{XP bisects } \angle WXY)$ $\angle YZQ = \frac{1}{2}\angle WZY \quad (\text{ZQ bisects } \angle WZY)$ $\angle WXY = \angle WZY \quad (\text{proven in (i)})$ $\therefore \angle WXP = \angle YZQ \quad (A)$ $\angle PWX = \angle QYZ \quad (\text{alternate } \angle\text{'s are } =, \text{WX} \parallel \text{ZY}) (A)$ $WX = YZ \quad (\text{opposite sides in } \parallel\text{gram are } =) (S)$ $\therefore \triangle WXP \cong \triangle YZQ \quad (AAS)$	<b>2</b>	<p><b>2 marks</b></p> <ul style="list-style-type: none"> <li>Correct solution</li> </ul> <p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>Significant progress towards a correct solution</li> </ul>
<b>14(b) (iii)</b>	$WP = YQ \quad (\text{matching sides in } \cong \Delta\text{'s})$ $WY = WP + PQ + QY \quad (\text{common side})$ $WY = 2QY + PQ$ $PQ = WY - 2QY$ $= 20 - 2 \times 8$ $= 4 \text{ cm}$	<b>1</b>	<p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>Correct answer</li> </ul>



Solution	Marks	Comments
<p><b>14(c) (i)</b> Using Pythagoras</p> $h^2 = 5^2 - 4^2$ $= 9$ $h = 3 \text{ m}$	1	<p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• Correct explanation</li> </ul>
<p><b>14(c) (ii)</b></p> $\frac{y}{3} = \frac{4 - \frac{1}{2}x}{4} \quad (\text{ratio of sides in } \triangle\text{'s})$ $4y = 12 - \frac{3}{2}x$ $y = 3 - \frac{3}{8}x$ $y = \frac{3}{8}(8 - x)$ <div style="text-align: center;">  </div>	2	<p><b>2 marks</b></p> <ul style="list-style-type: none"> <li>• Correct solution</li> </ul> <p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• Shows without using similar figures</li> <li>• Uses similar triangles to establish a relationship between <math>x</math> and <math>y</math></li> </ul>
<p><b>14(c) (iii)</b> <math>V = 1.5 \times xy</math></p> $= \frac{3}{2} \times x \times \frac{3}{8}(8 - x)$ $= \frac{9}{16}x(8 - x)$	1	<p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• Correct solution</li> </ul>
<p><b>14(c) (iv)</b> <math>V = \frac{9}{2}x - \frac{9}{16}x^2</math>      stationary points occur when <math>\frac{dV}{dx} = 0</math></p> $\frac{dV}{dx} = \frac{9}{2} - \frac{9}{8}x \quad \text{i.e. } \frac{9}{2} - \frac{9}{8}x = 0$ $\frac{d^2V}{dx^2} = -\frac{9}{8} \quad \frac{1}{8}x = \frac{1}{2}$ $x = 4$ <p>when <math>x = 4</math>, <math>\frac{d^2V}{dx^2} = -\frac{9}{8} &lt; 0</math></p> <p><math>\therefore</math> Volume is a maximum when <math>x = 4</math></p> $\text{maximum } V = \frac{9}{2}(4) - \frac{9}{16}(4)^2$ $= 9 \text{ m}^3$ <p style="text-align: center;"><b>OR</b></p> <p>Function is a concave down parabola, thus the maximum is the <math>y</math>-value of the vertex</p> $AOS = -\frac{\frac{9}{2}}{2 \times -\frac{9}{16}}$ $= \frac{9}{2} \times \frac{8}{9}$ $= 4$ $\text{maximum } V = \frac{9}{2}(4) - \frac{9}{16}(4)^2$ $= 9 \text{ m}^3$ <p style="text-align: center;"><b>OR</b></p> $\text{maximum } V = -\frac{A}{4a}$ $= -\frac{\left(\frac{9}{2}\right)^2 - 0}{4 \times -\frac{9}{16}}$ $= \frac{81}{4} \times \frac{4}{9}$ $= 9 \text{ m}^3$	3	<p><b>3 marks</b></p> <ul style="list-style-type: none"> <li>• Correct solution</li> </ul> <p><b>2 marks</b></p> <ul style="list-style-type: none"> <li>• Proves that <math>x = 4</math> gives the maximum volume</li> <li>• Finds the volume without proving it is a maximum</li> <li>• Finds the axis of symmetry</li> <li>• Finds the discriminant of the function</li> </ul> <p><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• Obtains the value of <math>x</math> that will produce a maximum volume</li> <li>• Attempts to show that a maximum is obtained</li> <li>• Recognises the significance of the concavity of the function.</li> </ul>