BAULKHAM HILLS HIGH SCHOOL

2015 YEAR 11 YEARLY EXAMINATION

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 70
Exam consists of 8 pages.

This paper consists of TWO sections.

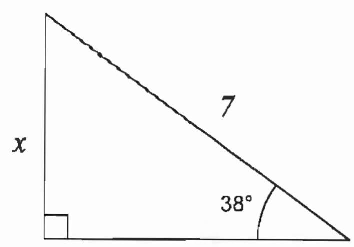
Section 1 – Page 2-5 (10 marks)
- Attempt Question 1-10
- Allow about **15** minutes for this section

Section II – Pages 5-8 (60 marks)
- Attempt questions 11-14
- Allow about **1 hours and 45** minutes for this section

Section 1 –Multiple Choice (10 marks)
Attempt all questions.

Answer the following on the booklet provided.

1 The value of x to 3 significant figures is:



(A) 4.31

(B) 4.310

(C) 11.4

(D) 11.36

2 Which of the following is equal to $\frac{1}{2\sqrt{5}-\sqrt{3}}$?

(A) $\frac{2\sqrt{5}-\sqrt{3}}{7}$

(B) $\frac{2\sqrt{5}+\sqrt{3}}{7}$

(C) $\frac{2\sqrt{5}-\sqrt{3}}{17}$

(D) $\frac{2\sqrt{5}+\sqrt{3}}{17}$

3 The quadratic equation $3x^2 - x - 4 = 0$ has roots α and β :

What is the value of $12\alpha + 12\beta$?

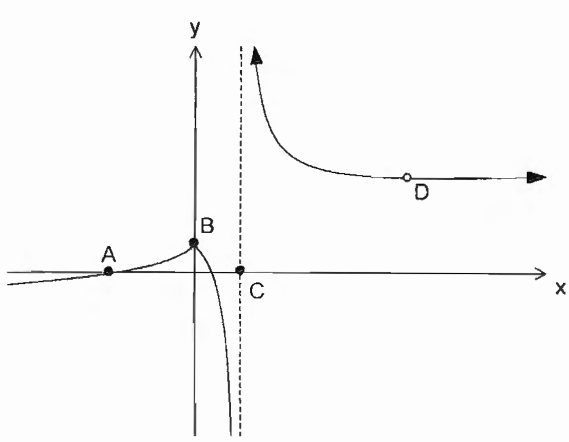
(A) -16

(B) -4

(C) 4

(D) 16

4 At which of the following points is the function continuous but not differentiable?



(A) A

(B) B

(C) C

(D) D

5 The function $f(x) = \dfrac{x^2 - 1}{x}$ is :

(A) an even function

(B) an odd function

(C) neither odd nor even function

(D) a zero function

6 Which inequality defines the domain of the function $f(x) = \dfrac{1}{\sqrt{x+3}}$?

(A) $x > -3$

(B) $x \geq -3$

(C) $x < -3$

(D) $x \leq -3$

7 What is the solution for $x^2 > 4$?

(A) $x > \pm 2$

(B) $x < -2$ or $x > 2$

(C) $x < 2$ or $x > -2$

(D) $-2 < x < 2$

8 Which expression is the correct simplification of $\dfrac{9^{2x}}{3^x}$?

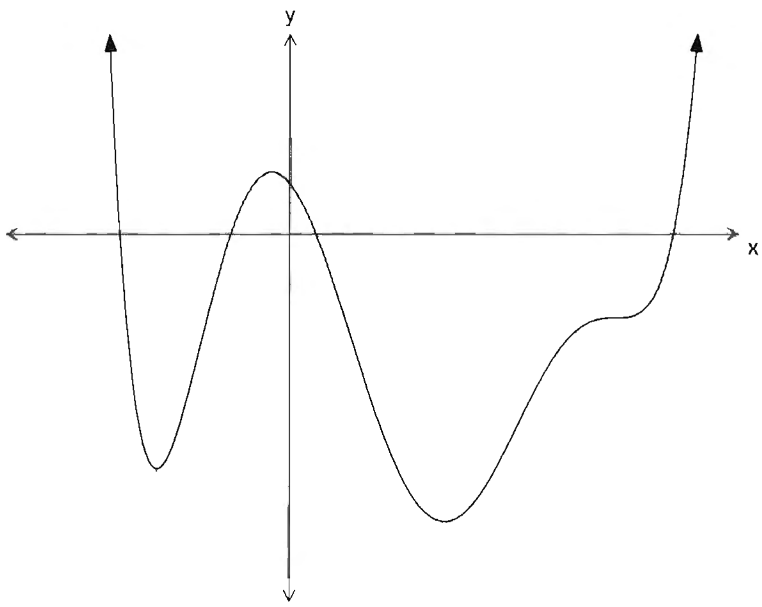
(A) 6^2

(B) 3^4

(C) 3^x

(D) 3^{3x}

9 The diagram shows the graph of $y = f(x)$



How many points of inflection are on the graph?

(A) 1

(B) 2

(C) 3

(D) 4

10 If $f(x) = a^x + a^{-x}$, then $[f(x)]^2 =$:

(A) $1 - f(2x)$

(B) $1 + f(2x)$

(C) $2 - f(2x)$

(D) $2 + f(2x)$

End of Section 1

Section II – Extended Response
All necessary working should be shown in every question.

Marks

Question 11 (15 marks) - Start on the appropriate page in your answer booklet

a) Simplify $3\sqrt{2} + 3\sqrt{18} - \sqrt{8}$ **2**

b) Factorise $2x^3 - 16y^3$ **2**

c) Expand and simplify $5m(m-7) - (m-1)^2$ **2**

d) Draw a neat sketch of $y = 3^{-x} + 1$ showing all important features. **2**

e) Find the perpendicular bisector of the points $(1, -2)$ and $(3, -8)$ leaving your answer in general form. **3**

f) If $f(x) = (3x-1)^4$, find $f'(2)$. **2**

g) Find the equation of the tangent to the curve $y = x^2 + 3x$ at the point $(1,4)$ **2**

End of Question 11

Question 12 (15 marks) - Start on the appropriate page in your answer booklet **Marks**

a) Find the values of P, Q and R if $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$ **2**

b) Find the value(s) of k in the equation $(k-1)x^2 + (2k+1)x - 2 = 0$ so that it has equal roots **2**

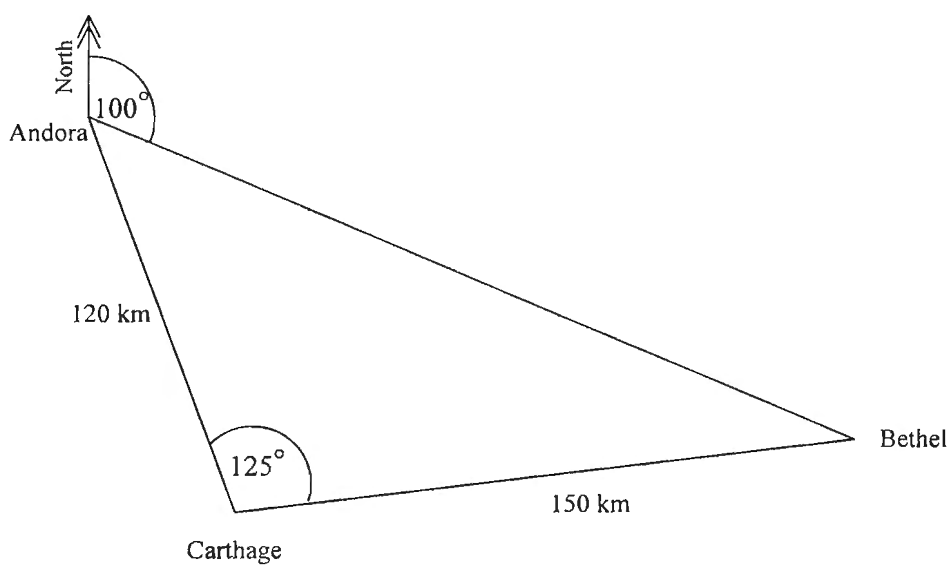
c) Differentiate

(i) $7x^4 - 3x + 1$ **1**

(ii) $\frac{2x+3}{(x-1)^2}$ **2**

(iii) $\frac{3x+5}{1-x^2}$ **2**

d) The diagram below shows the relative positions of three towns called Andora, Bethel and Carthage.



(i) Copy the diagram into your booklet.
Calculate the distance from Andora to Bethel to the nearest kilometre. **2**

(ii) Given the bearing of Bethel from Andora is 100°, find the true bearing of Andora from Carthage to the nearest degree **2**

e) Solve for x if $0^\circ \leq x \leq 360^\circ$ **2**

$$2\sin x = \tan x$$

End of Question 12

Question 13 (15 marks) - Start on the appropriate page in your answer booklet **Marks**

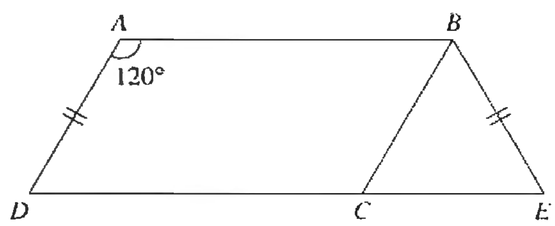
a) If α and β are roots of the equation $2x^2 - 4x + 1 = 0$
Find the value of

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 1

b) 3

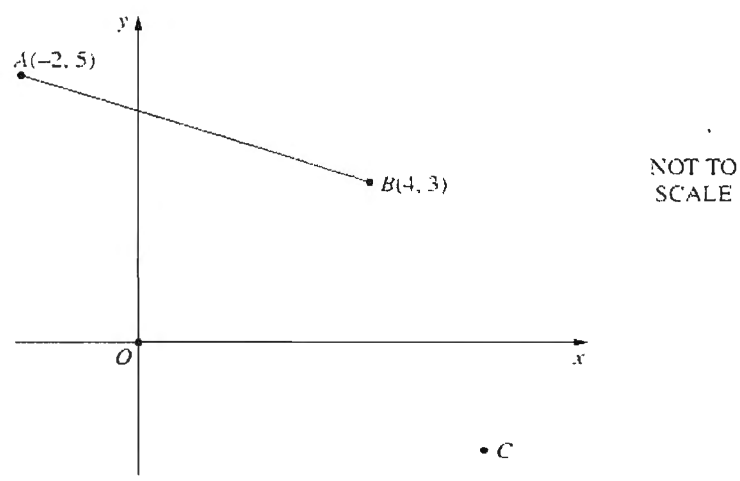


The diagram shows a parallelogram $ABCD$ with $\angle DAB = 120°$. The side DC is produced to E so that $AD = BE$.

Copy or trace the diagram into your writing booklet.

Prove that ΔBCE is equilateral.

c)



The diagram shows the points $A(-2, 5)$, $B(4, 3)$ and $O(0, 0)$.
The point C is the fourth vertex of the parallelogram $OABC$.

(i) Show that the equation of AB is $x + 3y - 13 = 0$. 2

(ii) Show that the length of AB is $2\sqrt{10}$. 1

(iii) Calculate the perpendicular distance from O to the line AB. 2

(iv) Find the coordinates of C 1

(v) Calculate the area of parallelogram $OABC$. 1

(vi) Hence or otherwise, find the perpendicular distance from O to the line BC 2

End of Question 13

Question 14 (15 marks) - Start on the appropriate page in your answer booklet

a) Prove that **2**

$$\frac{\cos x\,(\sin x + \cos x)}{(1 + \sin x)(1 - \sin x)} = 1 + \tan x$$

b) Consider the function $f(x) = x^4 - 4x^3$.

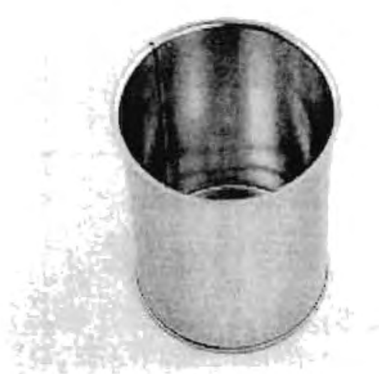
(i) Show that $f'(x) = 4x^2(x - 3)$ **1**

(ii) Find the coordinates of the stationary points of the curve $y = f(x)$, and determine their nature. **3**

(iii) Sketch the graph of the curve $y = f(x)$, showing the stationary points and intercepts. **2**

(iv) Find the values of x for which the graph $y = f(x)$ is concave down. **2**

c) An open cylindrical can with radius r and height h is made so that its surface area is $300\pi\text{cm}^2$.



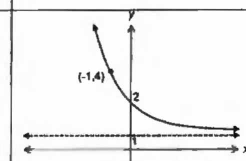
(i) Show that the volume of the can is given by $V = 150\pi r - \frac{\pi r^3}{2}$. **2**

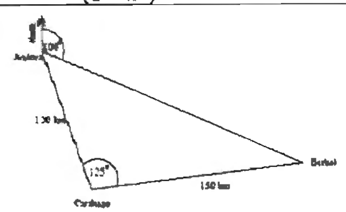
(ii) Find the radius of the cylinder that gives the maximum volume. **3**

End of Examination

Qn	Solutions	Mks	Comments
Section I			
1	A $\sin 38° = \frac{x}{7}$ $x = 4.30963$ $x = 4.31$	1	
2.	D $\frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{4\times 5 - 3}$ $= \frac{2\sqrt{5}+\sqrt{3}}{17}$	1	
3.	C $12(\alpha+\beta) = 12 \times \left(\frac{1}{3}\right) = 4$	1	
4	B The curve is not continuous at points C and D. Point A is both differentiable and continuous.	1	
5.	B $f(-x) = \frac{(-x)^2 - 1}{(-x)}$ $= \frac{x^2-1}{-x}$ $= -f(x)$	1	
6	A Denominator≠0 and inside square root must be positive. $x + 3 > 0$ $x > -3$	1	
7.	B $x^2 - 4 > 0$ $(x-2)(x+2) > 0$ $x < 0 \; or \; x > 0$	1	
8.	D $\frac{(3^2)^{2x}}{3^x} = \frac{3^{4x}}{3^x}$ $= 3^{3x}$	1	
9.	D There are 4 points of inflections.	1	
10.	D $f(2x) = a^{2x} + a^{-2x}$ $[f(x)]^2 = (a^x + a^{-x})^2$ $= (a^x)^2 + 2(a^x)(a^{-x}) + (a^{-x})^2$ $= a^{2x} + a^{-2x} + 2$ $= f(2x) + 2$	1	

Adv Yr11 Yearly 2015 Solutions

Qn	Solutions	Mks	Comments
Section II			
11a)	$3\sqrt{2} + 3\sqrt{18} - \sqrt{8} = 3\sqrt{2} + 3\times 3\sqrt{2} - 2\sqrt{2}$ $= 10\sqrt{2}$	2	**2 marks** •Correct answer **1 mark** • Makes progress towards finding values using correct methods
11b)	$2(x^3 - 8y^3) = 2(x-2y)(x^2+2xy+4y^2)$	2	**2 marks** •Correct answer **1 mark** • Factorises HCF or cubic
11c)	$5m^2 - 35m - (m^2 - 2m + 1) = 5m^2 - 35m - m^2 + 2m - 1$ $= 4m^2 - 33m - 1$	2	**2 marks** •Correct answer **1 mark** • Makes progress towards finding the correct answer
11d)		2	**2 marks** •Shows the correct curve and labels the y-int and the asymptote. **1 mark** • Draws an exponential curve.
11e)	$Midpoint = \left(\frac{1+3}{2}, \frac{-2+-8}{2}\right)$ $Midpoint = (2, -5)$ $m_1 = \frac{-8 - -2}{3-1}$ $m_1 = -3$ $m_2 = \frac{1}{3}$ Equation of the line $y - -5 = \frac{1}{3}(x-2)$ $3y + 15 = x - 2$ $x - 3y - 17 = 0$	3	**3 marks** •Correct solution **2 marks** • Makes progress towards finding equation of the line using correct methods **1 mark** • Finds midpoint • Finds m_1
11f)	$f'(x) = 4(3x-1)^3 \times 3$ $f'(2) = 12(3(2)-1)^3 = 1500$	2	**2 marks** •Correct solution **1 mark** • Correct derivative • Substitutes $x = 2$ into an incorrect derivative
11g)	$\frac{dy}{dx} = 2x + 3$ $m_T = 2(1) + 3$ $m_T = 5$ Equation of the tangent $y - 4 = 5(x-1)$ $y = 5x - 1$	2	**2 marks** •Correct solution **1 mark** • Finds the gradient of the tangent •Find the equation of the tangent with an incorrect gradient
12a)	$3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$ Let $x = -1$ $3(-1)^2 + 5(-1) - 1 \equiv P(-1+1)^2 + Q(-1+1) + R$ $-3 = R$ Equating x^2: $3 = P$ Let $x = 0$ $3(0)^2 + 5(0) - 1 \equiv 3(0+1)^2 + Q(0+1) - 3$ $-1 = Q$	2	**2 marks** •Correct answer **1 mark** • Find one correct value • Finds two correct values if their first value is incorrect.

Qn	Solutions	Mks	Comments
	$\therefore P = 3, Q = -1, R = -3$		
12b)	Equal roots when $\Delta = 0$ $(2k+1)^2 - 4(k-1)(-2) = 0$ $4k^2 + 4k + 1 + 8k - 8 = 0$ $4k^2 + 12k - 7 = 0$ $(2k+7)(2k-1) = 0$ $k = -\frac{7}{2}$ or $k = \frac{1}{2}$	2	**2 marks** •Correct solution **1 mark** • Correct expression for the discriminant.
12c)(i)	$28x^3 - 3$	1	**1 mark** •Correct answer
12c)(ii)	$u = 2x + 3$ $v = (x-1)^2$ $u' = 2$ $v' = 2(x-1)$ $\frac{d}{dx} = \frac{2 \times (x-1)^2 - 2(x-1) \times (2x+3)}{(x-1)^4}$ $= \frac{2x - 2 - 4x - 6}{(x-1)^3}$ $= \frac{-2x - 8}{(x-1)^3}$	2	**2 marks** • Correct solution **1 mark** • Makes progress towards finding the correct answer
12c)(iii)	$u = 3x + 5$ $v = 1 - x^2$ $u' = 3$ $v' = -2x$ $\frac{d}{dx} = \frac{3 \times (1-x^2)^2 - -2x \times (3x+5)}{(1-x^2)^2}$ $= \frac{3x^2 + 10x + 3}{(1-x^2)^2}$	2	**2 marks** • Correct solution **1 mark** • Makes progress towards finding the correct answer
12d)(i)	 $AB^2 = 120^2 + 150^2 - 2 \times 120 \times 150 \times \cos(125°)$ $AB = 239.89 \ldots$ $AB = 240km$	2	**2 marks** • Correct solution **1 mark** • Shows correct substitution to the cosine formula
12d)(ii)	$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{150} = \frac{\sin 125°}{239.89}$ $\theta = 30°81'$ $\theta = 31°$ (*nearest degree*) Bearing $= 360° - (180° - 100° - 31°)$ $= 360° - 49°$ $= 311°T$	2	**2 marks** • Correct solution **1 mark** • Finds θ correctly • Finds the correct bearing with an incorrect θ

Qn	Solutions	Mks	Comments
12e)	$2\sin x = \tan x$ for $0° \le x \le 360°$ $2\sin x = \frac{\sin x}{\cos x}$ $2\sin x \cos x - \sin x = 0$ $\sin x(2\cos x - 1) = 0$ $\sin x = 0$ $\cos x = \frac{1}{2}$ $x = 0°, 180°, 360°$ Acute angle $= 60°$ In the 1st and 4th quadrant $x = 60°, 300°$ $\therefore x = 0°, 180°, 360°, 60°, 300°$	2	**2 marks** • Correct solution **1 mark** • Finds $x = 0°, 180°, 360°$ • Finds $x = 60°, 300°$
13a)(i)	$\alpha + \beta = \frac{-b}{a}$ $= \frac{4}{2} = 2$	1	**1 mark** • Correct answer
13a)(ii)	$\alpha\beta = \frac{c}{a}$ $= \frac{1}{2}$	1	**1 mark** • Correct answer
13a)(iii)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$ $= \frac{2}{\left(\frac{1}{2}\right)}$ $= 4$	1	**1 mark** • Correct answer
13b)	$\angle BCD = \angle DAB = 120°$ (opposite angles of a parallelogram) $BC = AD$ (opposite sides of a parallelogram) $\angle BCE = 180° - \angle BCD$ (angle sum of a straight angle) $= 180° - 120°$ $= 60°$ $\angle BCE = \angle BEC$ (angles opposite equal sides of Δ) $\therefore \angle BCE = \angle BEC = 60°$ (angles opposite equal sides; $BC = BE$) $\angle BCE + \angle BEC + \angle CBE = 180°$ (angle sum of a ΔBCE) $60° + 60° + \angle CBE = 180°$ $\angle CBE = 60°$ $\therefore \Delta BCE$ is equilateral (all angles 60°)	3	**3 marks** • Correct solution **2 marks** • Significant progress towards a correct solution **1 mark** • Shows $\angle BCE = 60°$ using logical steps and reasoning.
13c)(i)	$m_{AB} = \frac{3-5}{4--2} = -\frac{1}{3}$ Equation of the line AB through $(-2,3)$ *and* $(4,3)$ is $y - 5 = -\frac{1}{3}(x+2)$ $3y - 15 = -x - 2$ $x + 3y - 13 = 0$	2	**2 marks** • Correct solution **1 mark** • Finds the correct gradient • Finds the correct equation using a wrong gradient

Qn	Solutions	Mks	Comments
13c)(ii)	$AB=\sqrt{(4--2)^2+(3-5)^2}$ $=\sqrt{40}$ $=2\sqrt{10}$	1	**1 mark** • Correct answer
13c)(iii)	$d=\left\|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right\|$ $d=\left\|\frac{1(0)+3(0)-13}{\sqrt{1^2+3^2}}\right\|$ $=\frac{13}{\sqrt{10}}$	2	**2 marks** • Correct solution **1 mark** • Shows correct substitution to the perp. distance formula
13c)(iv)	Since $AO//BC$, Shifting A to O by moving 2 right and 5 down Shifting $B(4,3)$ the same to $C=(4+2,3-5)$ $\therefore C=(6,-2)$	1	**1 mark** • Correct answer
13c)(v)	$A=$ base $\times$ perpendicular height $A=2\sqrt{10}\times\frac{13}{\sqrt{10}}$ $A=26$ unit^2	1	**1 mark** • Correct answer
13c)(vi)	$BC=AO$ (opposite sides of a parallelogram equal) $AO=\sqrt{(-2-0)^2+(5-0)^2}$ $AO=\sqrt{29}$ $\therefore BC=\sqrt{29}$ Area of $OABC=26$ from (iv) $\therefore BC\times$ perp. height $=26$ perp. height $=\frac{26}{BC}$ $=\frac{26}{\sqrt{29}}$	2	**2 marks** • Correct solution **1 mark** • Significant progress towards a correct solution
14a)	RTP: $\frac{\cos x(\sin x+\cos x)}{(1+\sin x)(1-\sin x)}=1+\tan x$ $LHS=\frac{\cos x\sin x+\cos^2 x}{1-\sin^2 x}$ $=\frac{\cos x\sin x+\cos^2 x}{\cos^2 x}\ \frac{\div\cos x}{\div\cos x}$ $=\frac{\sin x}{\cos x}+\frac{\cos x}{\cos x}$ $=\tan x+1$ $=RHS$	2	**2 marks** • Correct solution **1 mark** • Significant progress towards a correct solution

Qn	Solutions	Mks	Comments
14b)(i)	$f'(x)=4x^3-12x^2$ $=4x^2(x-3)$	1	**1 mark** • Correct answer
14b)(ii)	see below	3	**3 marks** • Correct solution **2 marks** • Finds one stationary point and determines its nature **1 mark** • Finds $x=0,3$

14b)(ii) Stat points when $f'(x)=0$

$$4x^2(x-3)=0$$
$$4x^2=0 \quad or \quad x-3=0$$
$$x=0 \quad or \quad x=3$$

When $x=0$
$f(0)=(0)^4-4(0)^3$
$f(0)=0$
$\therefore (0,0)$

When $x=3$
$f(3)=(3)^4-4(3)^3$
$f(3)=-27$
$\therefore (3,-27)$

x	-1	0	1	3	4
$f'(x)$	-16	0	-8	0	64
Gradient	\	---	\	---	/

$\therefore (0,0)$ is a horizontal point of inflection and $(3,-27)$ is a min. turning point.

OR

$$f''(x)=12x^2-24x$$

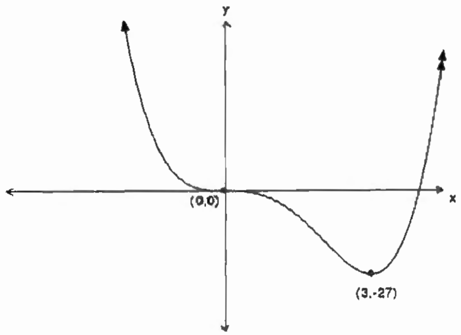
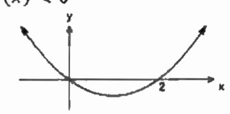
When $x=0$
$f''(0)=12(0)^2-24(0)$
$f''(0)=0$

Test concavity:
$f''(-0.1)=12(-0.1)^2-24(-0.1)$
$=$
>0

$f''(0.1)=12(0.1)^2-24(0.1)$
$=$
<0

$\therefore$ Concavity changes
$\therefore$ A horizontal point of inflection at $(0,0)$

When $x=3$
$f''(3)=12(3)^2-24(3)$
$f''(3)=36>0$
$\therefore$ A min. turning point at $(3,-27)$

Qn	Solutions	Mks	Comments
14b)(iii)		2	**2 marks** • Correct solution **1 mark** • Correct graph without the correct information labelled.
14b)(iv)	The curve is concave down when $f''(x) < 0$ $12x^2 - 24x < 0$ $12x(x-2) < 0$ $\therefore 0 < x < 2\ (from\ graph)$ 	2	**2 marks** • Correct solution **1 mark** • Finds $x = 0,2$ • Shows understanding The curve is concave down when $f''(x) < 0$
14c)(i)	$SA = \pi r^2 + 2\pi rh$ $300\pi = \pi r^2 + 2\pi rh$ $300 - r^2 = 2rh$ $h = \frac{300 - r^2}{2r}$ Volume of cylinder $= \pi r^2 h$ $V = \pi r^2 \times \frac{300 - r^2}{2r}$ $V = 150\pi r - \frac{\pi r^3}{2}$	2	**2 marks** • Correct solution **1 mark** • Finds the correct expression for h.
14c(ii)	Max Volume when $\frac{dV}{dr} = 0$ and $\frac{d^2V}{dr^2} < 0$ $\frac{dV}{dr} = 150\pi - \frac{3\pi r^2}{2}$ $0 = 150\pi - \frac{3\pi r^2}{2}$ $\frac{3\pi r^2}{2} = 150\pi$ $r^2 = 100$ $r = 10\ (r > 0\ as\ it\ is\ a\ length)$ $\frac{d^2V}{dr^2} = -3\pi r$ $< 0\ (as\ r > 0\ since\ r\ is\ a\ length)$ $\therefore r = 10$ give a minimum value	3	**2 marks** • Correct solution **1 mark** • Finds the value of r.