BAULKHAM HILLS HIGH SCHOOL

2016

YEAR 11 YEARLY

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions

Total marks – 70

This paper consists of TWO sections.

Section 1 – Multiple Choice
10 marks

Section 2 – Extended Response
60 marks
Attempt all questions
Start a new page for each question

Section 1 –Multiple Choice (10 marks)
Attempt all questions.

Answer the following on the booklet provided.

1 A yacht sailed from A to B on a bearing of 196°T. To sail from B directly back to A, the bearing it should travel is:

(A) 016°T (B) 074°T (C) 164°T (D) 196°T

2 Simplify $\frac{1}{5-\sqrt{3}}$

(A) $\frac{5+\sqrt{3}}{2}$ (B) $\frac{5+\sqrt{3}}{22}$ (C) $\frac{5-\sqrt{3}}{2}$ (D) $\frac{5-\sqrt{3}}{22}$

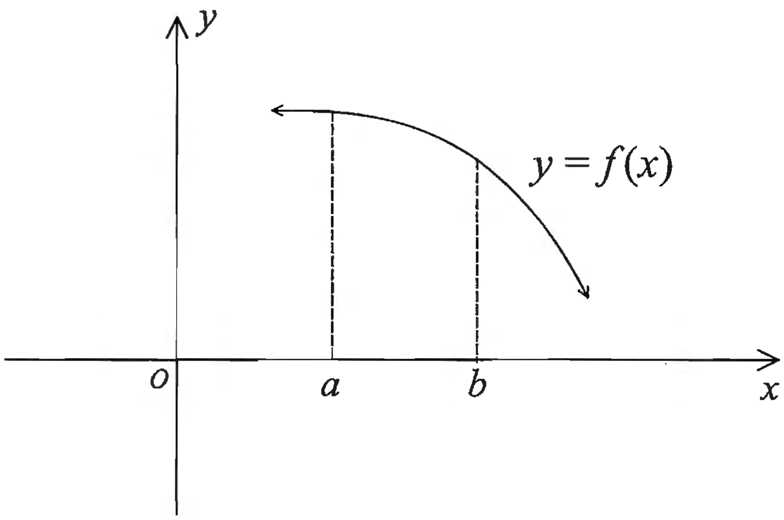
3 If α and β are the roots of $5x^2 + 15x - 3 = 0$, then $\alpha^2 + 3\alpha\beta + \beta^2$ is :

(A) $\frac{42}{5}$ (B) $-\frac{42}{5}$ (C) $\frac{48}{5}$ (D) $-\frac{48}{5}$

4 Correct to the nearest degree, the angle of inclination from the positive x-axis of a line with x-intercept 2 and y-intercept $2\sqrt{2}$ is:

(A) 35° (B) 55° (C) 125° (D) 155°

5 For the function $y = f(x)$ and the domain $a \le x \le b$, which of the following is true?



(A) $f'(x) > 0$ and $f''(x) < 0$ (B) $f'(x) < 0$ and $f''(x) < 0$
(C) $f'(x) > 0$ and $f''(x) > 0$ (D) $f'(x) < 0$ and $f''(x) > 0$

6 Which of the following expressions is the correct simplification of $\frac{\operatorname{cosec}\theta \sec\theta}{\tan\theta}$?

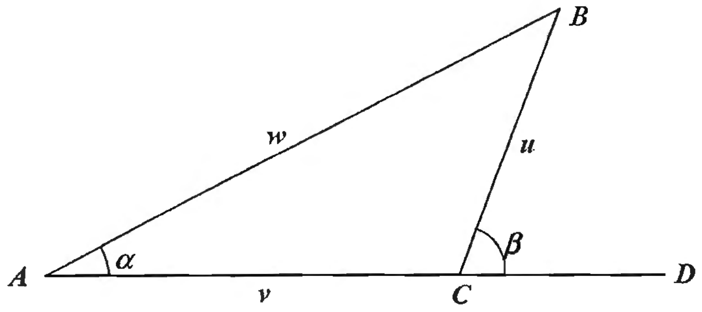
(A) $\sin^2\theta$ (B) $\cos^2\theta$ (C) $\sec^2\theta$ (D) $\operatorname{cosec}^2\theta$

7 The solutions of $\sqrt{3}\tan x + 1 = 0$ for the domain $0^\circ \le x \le 360^\circ$ are:

(A) $x = 30^\circ, 210^\circ$ (B) $x = 150^\circ, 210^\circ$

(C) $x = 150^\circ, 330^\circ$ (D) $x = 210^\circ, 330^\circ$

8



In the diagram above, ACD is a straight angle. Which of the following statements is true?

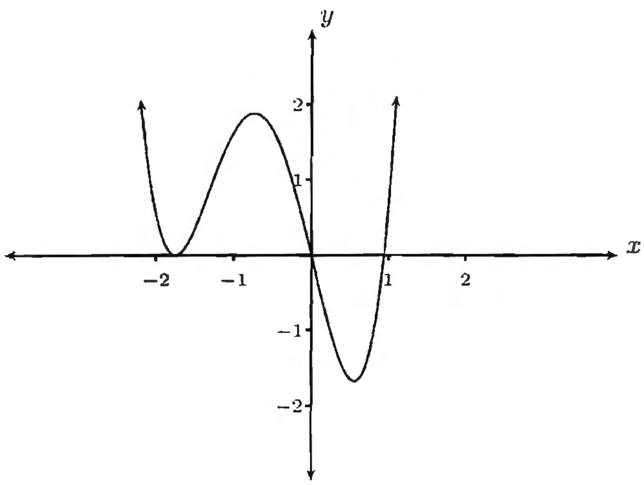
(A) $w^2 = u^2 + v^2 - 2uv\cos\beta$

(B) $w^2 = u^2 + v^2 + 2uv\cos\beta$

(C) $u^2 = v^2 + w^2 - 2vw\cos\beta$

(D) $\frac{u}{\sin\alpha} = \frac{w}{\cos\beta}$

9



The diagram above shows the function $y = f(x)$.

A student wishes to solve the equation $f(x) = 2 - |x|$ graphically. The correct number of solutions is:

(A) 1 (B) 2 (C) 3 (D) 4

10 Given $f(x+1) = x^2 - x + 1$, then $f(x)$ in its simplest form is:

(A) $x^2 + x + 1$ (B) $x^2 + x + 3$

(C) $x^2 - 3x + 3$ (D) $x^2 - 3x + 1$

End of Section I

Section II – Extended Response
All necessary working should be shown in every question.

Marks

Question 11 (15 marks) - Start on the appropriate page in your answer booklet

a) Simplify **2**

$$\frac{x+3}{5} - \frac{x-2}{4}$$

b) Solve **2**

$$|3x - 2| = 7$$

c) Factorise **2**

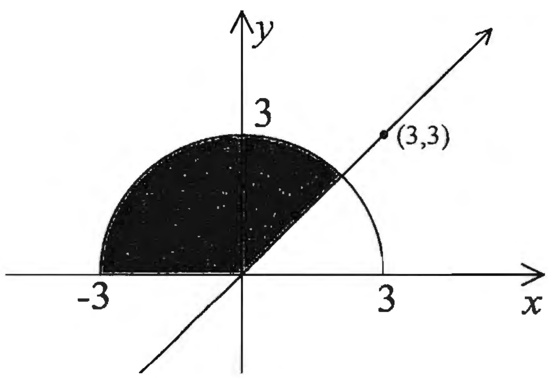
$$6x^2 + 11x - 10$$

d) For the function $f(x) = \sqrt{x^2 - 16}$:

(i) Find the domain and range **2**

(ii) Is the function odd, even or neither? **2**
(Justify your answer by showing working)

e) Write a set of inequalities which define the shaded region. **2**



f) Find the solutions of the equation $2\cos^2\theta = 3\sin\theta + 3$, for $0° \leq \theta \leq 360°$. **3**

End of Question 11

Question 12 (15 marks) - Start on the appropriate page in your answer booklet **Marks**

a) Simplify $\frac{3^{2n}-1}{6^n-2^n}$ **3**

b) Differentiate with respect to x

(i) $\frac{4}{1-x}$ **2**

(ii) $2x\sqrt{3x+1}$ **2**

(iii) $\frac{4x-1}{2x+1}$ **2**

c) Evaluate, showing all working:

$$\lim_{x\to -2} \frac{x^3+8}{2x+4}$$ **2**

d) Find the equation of the tangent to $y = x^2 - 4x$ at the point $(1, -3)$ on it. **2**

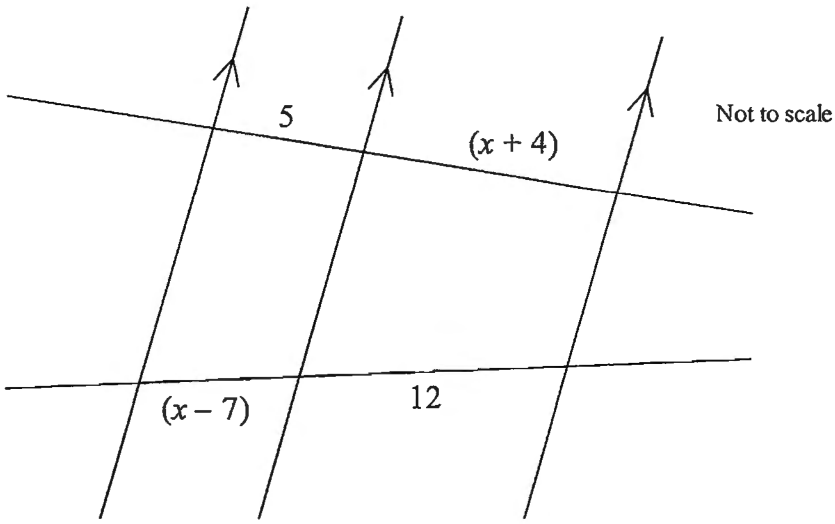
e) Solve $|2x-3| = 4x+1$ **2**

End of Question 12

Question 13 (15 marks) - Start on the appropriate page in your answer booklet **Marks**

a) Find the exact value of $\cos\theta$, given that $\tan\theta = 7$ and $\sin\theta < 0$. **2**

b) Find the value of x. **2**



c) If $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$, find the values of A, B and C. **3**

d) For which values of k does the equation $x^2 - 2x + 3 - k = 0$ have real roots? **2**

e) Consider the function $f(x) = \frac{1}{2}x^4 - 2x^3 + 2$.

(i) Find the coordinates of the stationary points of $y = f(x)$. **2**

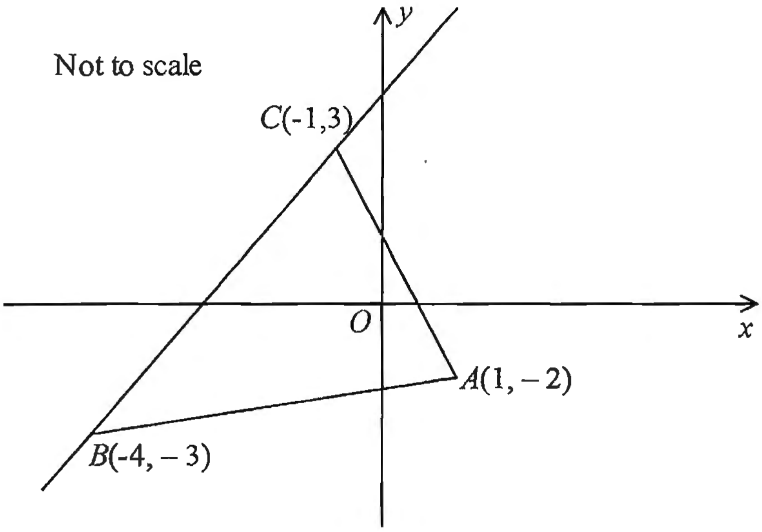
(ii) Determine the nature of these stationary points. **2**

(iii) Sketch the graph of $y = f(x)$, clearly showing the stationary points and y-intercept. (You need not find the x-intercepts). **2**

End of Question 13

Question 14 (15 marks) - Start on the appropriate page in your answer booklet

a) The diagram shows the points $A(1,-2)$, $B(-4,-3)$ and $C(-1,3)$, and the line BC.

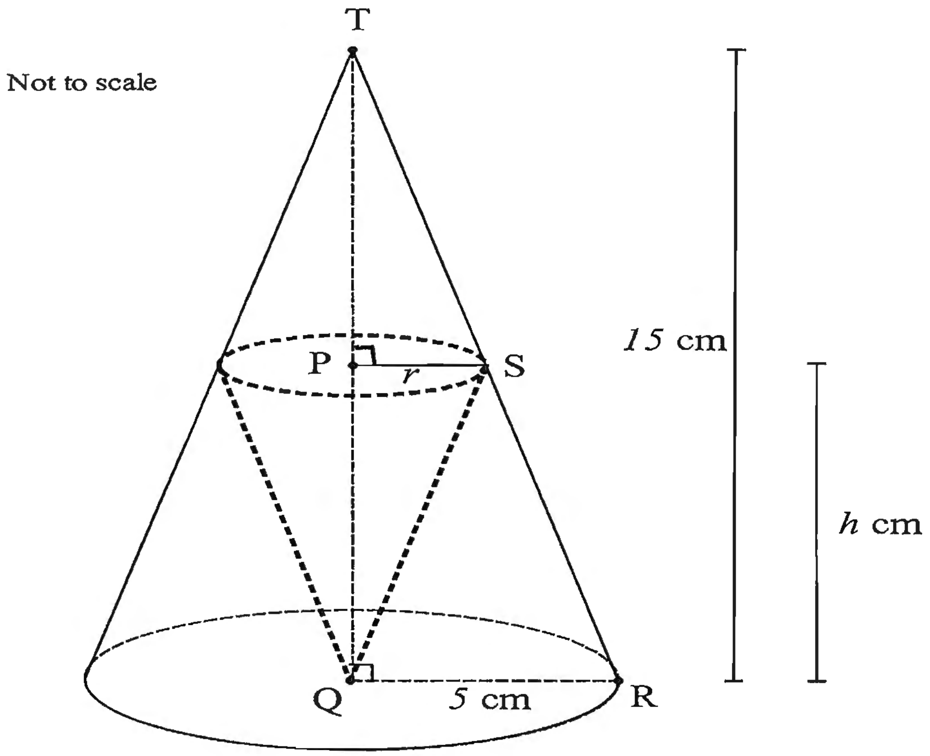


(i)	Calculate the exact length of BC.	**1**
(ii)	Show that the equation of BC is $2x - y + 5 = 0$.	**2**
(iii)	Find the exact area of triangle ABC.	**3**

b) Solve $x^2 - 2x - 1 < 0$. **3**

Question 14 continues on the next page

c) The diagram shows two right cones. The small cone of radius *PS=r cm* is enclosed within a larger cone of radius *QR=5cm*, as shown. The larger cone has height 15cm and the small cone has height *h cm*.



(i) Prove that $\Delta TPS|||\Delta TQR$ **2**

(ii) Show that $h = 15 - 3r$ **1**

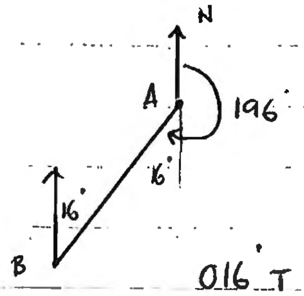
(iii) Find the radius of the small cone such that its volume is a maximum. **3**

End of Examination

P3 2U 2016 Soln

Mult. Choice

1.



016°T (A)

2. $\frac{1}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{5+\sqrt{3}}{22}$ (B)

3. $\alpha+\beta = \frac{-15}{5} = -3$

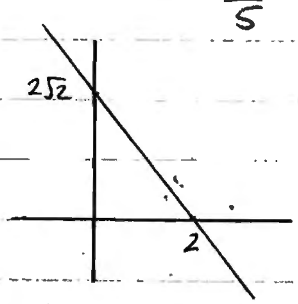
$\alpha\beta = \frac{-3}{5}$

$\alpha^2 + 3\alpha\beta + \beta^2 = (\alpha+\beta)^2 + \alpha\beta$

$= (-3)^2 + \left(\frac{-3}{5}\right)$

$= \frac{42}{5}$ (A)

4.



$m = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

$\tan\theta = -\sqrt{2}$

$\therefore \angle$ of inclination $= 125°$ (C)

5. Decreasing : $f'(x) < 0$

Conc. down : $f''(x) < 0$ (B)

6. $\dfrac{\frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$= \frac{1}{\sin\theta} \cdot \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta}$

$= \frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta$ (D)

7. $\tan x = \frac{-1}{\sqrt{3}}$

(Related $\angle = 30°$)

$x = 150°, 330°$ (C)

8. $\angle ACB = 180° - \beta$

$w^2 = u^2 + v^2 - 2uv\cos(180° - \beta)$

but $\cos(180° - \beta) = -\cos\beta$

$\therefore w^2 = u^2 + v^2 + 2uv\cos\beta$ (B)

9. Draw $y = 2 - |x|$

4 points of intersection

$\therefore$ 4 solutions. (D)

10. $f(x+1) = x^2 - x + 1$

$f(x) = (x-1)^2 - (x-1) + 1$

$= x^2 - 2x + 1 - x + 1 + 1$

$= x^2 - 3x + 3$ (C)

cover	1
Inst time	1
Q11	3
12	3
13	4
14	4

Q11

a) $\frac{x+3}{5} - \frac{x-2}{4} = \frac{4(x+3)-5(x-2)}{20}$ ← (1)

$= \frac{4x+12-5x+10}{20}$

$= \frac{-x+22}{20}$ or $\frac{22-x}{20}$ ← (1)

b) $|3x-2| = 7$

$3x-2=7$ or $-3x+2=7$

$3x = 9$ $\quad$ $-3x = 5$

$x = 3$ ← (1) $\quad$ $x = \frac{-5}{3}$ ← (1)

c) $6x^2+11x-10 = (3x-2)(2x+5)$ ← (2)

[Correct terms, Signs wrong ← 1]

d) $y = \sqrt{x^2-16}$

(i) D: $x^2-16 \geq 0$

$\therefore x \leq -4$, $x \geq 4$ ← (1)

R: $y \geq 0$ ← (1)

(ii) $f(x) = \sqrt{x^2-16}$

$f(-x) = \sqrt{(-x)^2-16}$

$= \sqrt{x^2-16}$ ← (1)

$= f(x)$ $\quad$ $\therefore$ Even ← (1)

e) $x^2+y^2 \leq 9$ or $y \leq \sqrt{9-x^2}$

$y \geq x$

$y \geq 0$

← all three (2)

[Any two ← 1]

f) $4(1-\sin^2\theta) - 6\sin\theta - 6 = 0$ ← (1)

$4 - 4\sin^2\theta - 6\sin\theta - 6 = 0$

$$-4\sin^2\theta - 6\sin\theta - 2 = 0$$

$(\div -2)$ $\quad 2\sin^2\theta + 3\sin\theta + 1 = 0$

$$(2\sin\theta + 1)(\sin\theta + 1) = 0 \longleftarrow (1)$$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = -1$

$$\underline{\theta = 210°, 330°, 270°} \longleftarrow (1)$$

Q12

a) $\dfrac{3^{2n}-1}{6^n - 2^n} = \dfrac{(3^n-1)(3^n+1)}{2^n(3^n-1)}$ $\longleftarrow$ (1) factorise numerator and/or denominator

$= \dfrac{3^n+1}{2^n}$ $\longleftarrow$ (1) correct answer

b) i) $y = 4(1-x)^{-1}$

$\cancel{y' = 4 \cdot -(1-x)^{-2} \cdot -1}$

$= 4(1-x)^{-2}$ or $\dfrac{4}{(1-x)^2}$

ii) $y = 2x.\sqrt{3x+1}$

$u = 2x \qquad u' = 2$

$v = (3x+1)^{1/2} \qquad v' = \frac{1}{2}(3x+1)^{-1/2}.3 = \dfrac{3}{2\sqrt{3x+1}}$

$\dfrac{dy}{dx} = \cancel{2}x \times \dfrac{3}{\cancel{2}\sqrt{3x+1}} + \sqrt{3x+1} \times 2$

$= \dfrac{3x}{\sqrt{3x+1}} + 2\sqrt{3x+1}$

(1) — uses product rule

(1) — correct ans.

$\left(\text{or } \quad 3x(3x+1)^{-1/2} + 2(3x+1)^{1/2}\right)$

(iii) $y = \dfrac{4x-1}{2x+1}$ $\qquad u = 4x-1 \quad u' = 4$

$\qquad v = 2x+1 \quad v' = 2$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(2x+1).4 - (4x-1).2}{(2x+1)^2}$ ← (1) uses quotient rule

$= \dfrac{8x+4-8x+2}{(2x+1)^2}$

$= \dfrac{6}{(2x+1)^2}$ ← (1) correct answer

c) $\lim\limits_{x \to -2} \dfrac{x^3+8}{2x+4} = \lim\limits_{x \to -2} \dfrac{\cancel{(x+2)}(x^2-2x+4)}{2\cancel{(x+2)}}$ ← (1) factorise numerator

$= \dfrac{(-2)^2 - 2(-2) + 4}{2}$

$= \dfrac{4+4+4}{2}$

$= 6$ ← (1) correct answer

d) $m = \dfrac{dy}{dx} = 2x - 4$

$= 2(1) - 4$ at $(1, -3)$

$= -2$ ← (1) correct gradient

$y - y_1 = m(x - x_1)$

$y + 3 = -2(x-1)$

$y + 3 = -2x + 2$

$y = -2x - 1$ or $2x + y + 1 = 0$ ← (1) correct eqn.

e) $|2x-3| = 4x+1$

$2x - 3 = 4x + 1$

$-4 = 2x$

$\cancel{x = -2}$

but LHS $= |-7| = 7$ } not equal

and RHS $= -7$

∴ not a solution

$-2x + 3 = 4x + 1$

$2 = 6x$

$x = 1/3$

LHS $= |-2\tfrac{1}{3}| = 2\tfrac{1}{3}$ } equal

RHS $= 2\tfrac{1}{3}$

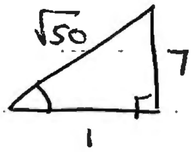
∴ Soln.

Solution : $x = \frac{1}{3}$ only

(1) for $x = 1/3$

(1) for show and reject $x = -2$

Q13.

a) 

$\cos\theta = -\frac{1}{\sqrt{50}}$ (1) for neg.

(1) for $\frac{1}{\sqrt{50}}$.

b) $\frac{x+4}{5} = \frac{12}{x-7}$

$(x+4)(x-7) = 5\times 12$

$\left.\begin{aligned} x^2-3x-28 &= 60 \\ x^2-3x-88 &= 0 \end{aligned}\right\} \longleftarrow$ (1)

$(x+8)(x-11)=0$

$x = -8, 11$ but $x>0$

$\therefore \underline{x=11} \longleftarrow$ (1)

c) $RHS = A(x^2+4x+4) + Bx + C$

$= Ax^2 + 4Ax + 4A + Bx + C$

$= Ax^2 + (4A+B)x + (4A+C)$

$LHS = 6x^2 + 0x - 11$

$\underline{A=6}$ (1) $\quad 4(6)+B=0$, $\underline{B=-24}$ (1) $\quad 4(6)+C=-11$, $\underline{C=-35}$ (1)

d) For real roots, $\Delta \geq 0$

$\Delta = b^2-4ac$

$=(-2)^2 - 4.1.(3-k) \longleftarrow$ (1)

$= 4-12+4k$

$= 4k-8 \geq 0$ when $\underline{k\geq 2} \longleftarrow$ (1)

e) $y = \frac{1}{2}x^4 - 2x^3 + 2$

$y' = 2x^3 - 6x^2$
$y'' = 6x^2 - 12x$

(i) For stationary points, $y' = 2x^3 - 6x^2 = 0$

$2x^2(x-3) = 0$

$x = 0$ or $x = 3$

$y = 2$ $\quad$ $y =$

$(0, 2)$ (1) $\quad$ $(3, -11.5)$ (1)

(ii) At $(0,2)$:

$y'' = 6(0)^2 - 12(0) = 0$ ∴ Possible pt. of inflexion

x	-1	0	1
y''	$+18$	0	-6

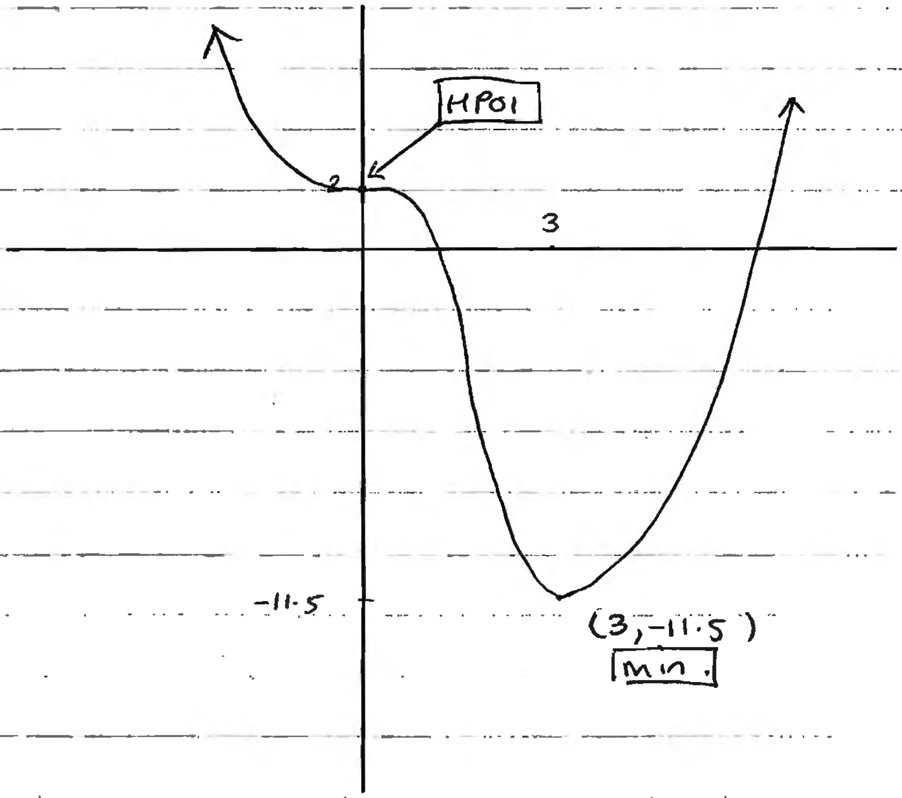
$6(-1)^2 - 12(-1)$ $\quad$ $6(1)^2 - 12(1)$

∴ Since concavity change $(0, 2)$ is a horizontal pt. of inflexion (1)

At $(3, -11.5)$

$y'' = 6(3)^2 - 12(3)$
$= +18$
> 0 ∴ $(3, -11.5)$ is a minimum turning point (1)

(iii)



(2)

b) $x^2 - 2x - 1 < 0$

Roots: $x^2 - 2x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

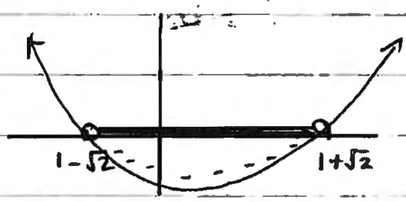
$$= \frac{2 \pm \sqrt{(-2)^2 - 4.1.-1}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{8}}{2} \quad 2\sqrt{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 \pm \sqrt{2}$$ ← (1)

Sketch



← (1)

Soln: $\underline{1 - \sqrt{2} < x < 1 + \sqrt{2}}$ ← (1)

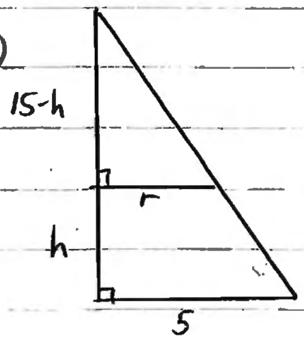
c) i) In ΔTPR, ΔTQS

$\angle TPR = \angle TQS$ (given, both 90°)

$\angle T$ is common

$\therefore \Delta TPR \parallel\!\mid \Delta TQS$ (matching $\angle$s equal) or (AA)

(ii)



$$\frac{15-h}{15} = \frac{r}{5}$$

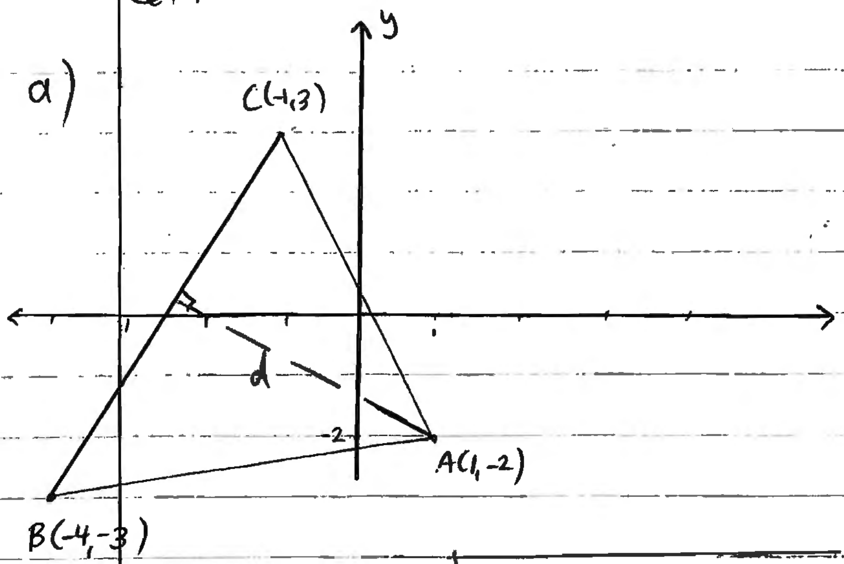
$$15r = 75 - 5h$$ — (1)

(÷5) $3r = 15 - h$

$\underline{h = 15 - 3r}$

Q14.

a)



(i) $BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(\cancel{3+3}\ -1+4)^2 + (3+3)^2}$

$= \sqrt{9 + 36}$

$= \underline{\sqrt{45} \text{ or } 3\sqrt{5} \text{ units}}$

(ii) BC has $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3+3}{-1+4}$

$= \dfrac{6}{3}$ ⟵ (1)

$= 2$ and passes through $(-1,3)$

$\therefore y - y_1 = m(x - x_1)$

$y - 3 = 2(x+1)$ ⟵ (1)

$y - 3 = 2x + 2$

$\underline{2x - y + \cancel{4}5 = 0}$

(iii) $d = \left| \dfrac{2(1) - (-2) + 5}{\sqrt{2^2 + (-1)^2}} \right|$ ⟵ (1) $= \dfrac{9}{\sqrt{5}}$ ⟵ (1)

Area of $\triangle ABC = \frac{1}{2}bh = \frac{1}{2} \times 3\cancel{\sqrt{5}} \times \dfrac{9}{\cancel{\sqrt{5}}}$

$= \dfrac{27}{2}$ units^2 ⟵ (1)

(iii) $V = \pi r^2 h$ but $h = 15 - 3r$

$V = \pi r^2(15 - 3r)$

$= 15\pi r^2 - 3\pi r^3 \longleftarrow (1)$

For maximum volume, $V' = 0$ and $V'' < 0$

$V' = 30\pi r - 9\pi r^2$

$= 3\pi r(10 - 3r)$

$= 0$ when $\cancel{r = 0}$ or $\underline{r = \frac{10}{3}} \longleftarrow (1)$

$V'' = 30\pi - 9\pi(2r)$

$= 30\pi - 18\pi r$

$= 30\pi - 18\pi\left(\frac{10}{3}\right)$

$= 30\pi - 60\pi$

< 0, as required $\longleftarrow (1)$

$\therefore$ $\underline{r = \frac{10}{3} \text{ cm gives max. volume}}$