BAULKHAM HILLS HIGH SCHOOL

2017 YEAR 11
YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 70
Exam consists of 8 pages.

This paper consists of TWO sections.

Section I – Page 2-4 (10 marks)
- Attempt Question 1-10
- Allow about **15** minutes for this section

Section II – Pages 5-8 (60 marks)
- Attempt questions 11-14
- Allow about **1 hours and 45** minutes for this section

Section 1 –Multiple Choice (10 marks)
Answer the following on the booklet provided.

1 What are the solutions of $2x^2 - 5x - 1 = 0$?

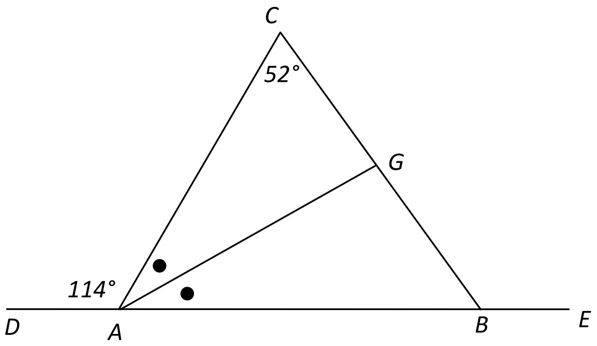
(A) $x = \dfrac{-5 \pm \sqrt{17}}{4}$

(B) $x = \dfrac{5 \pm \sqrt{17}}{4}$

(C) $x = \dfrac{-5 \pm \sqrt{33}}{4}$

(D) $x = \dfrac{5 \pm \sqrt{33}}{4}$

2 In the diagram, $\angle CAD = 114°$ and $\angle ACB = 52°$. DE is a straight line. AG bisects $\angle CAB$.



What is the value of $\angle AGB$?

(A) 33°

(B) 52°

(C) 62°

(D) 85°

3 The quadratic equation $3x^2 - 5x - 4 = 0$ has roots α and β.

What is the value of $\alpha\beta$?

(A) $-\frac{5}{3}$

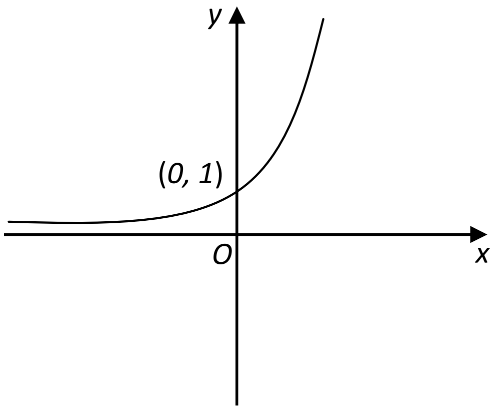
(B) $\frac{5}{3}$

(C) $-\frac{4}{3}$

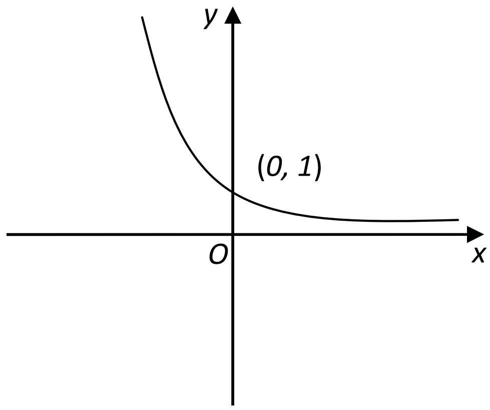
(D) $\frac{4}{3}$

4 Which graph best represents the function $\quad = 3^{-x}$?

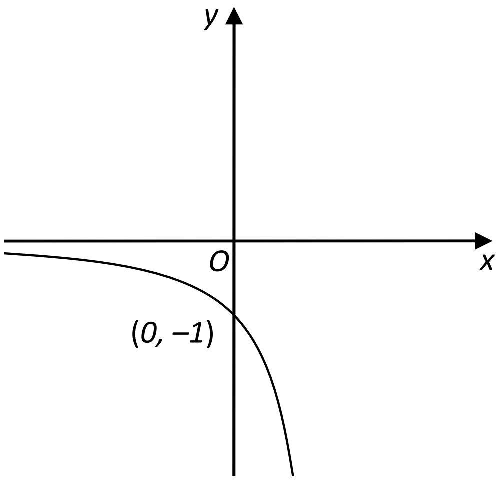
(A)



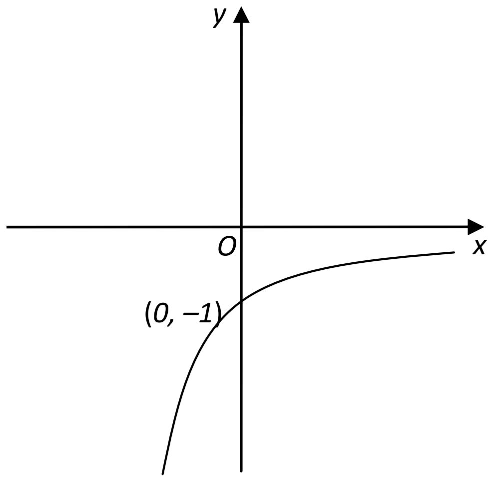
(B)



(C)



(D)



5 What are the solutions of $\sqrt{3}\tan x = -1$ for $0° \le x \le 360°$?

(A) $120°$ and $240°$

(B) $120°$ and $300°$

(C) $150°$ and $210°$

(D) $150°$ and $330°$

6 A parabola has focus $(5,0)$ and directrix $x = 1$. What is the equation of the parabola?

(A) $y^2 = 16(x-5)$

(B) $y^2 = 8(x-3)$

(C) $y^2 = -16(x-5)$

(D) $y^2 = -8(x-3)$

7 What is the gradient of the tangent to the curve $y = (x+1)\sqrt{x+1}$ at $x = 0$?

(A) $-\frac{1}{2}$

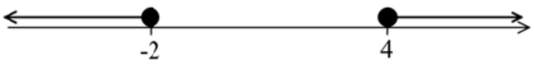
(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

8 Which of the following represents the solution to $|x-1| \geq 3$?

(A)

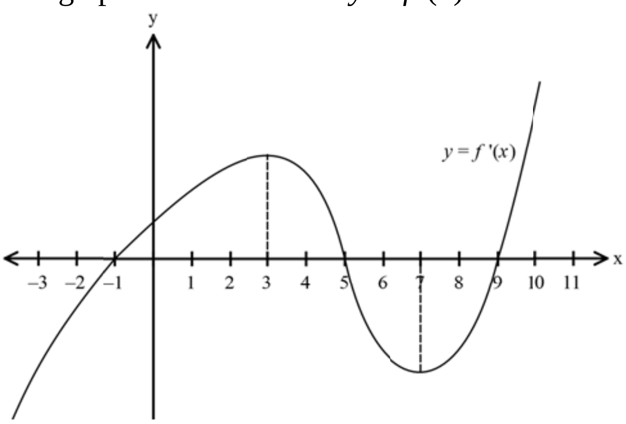


(B)

(C)

(D)

9 The graph of the derivative $y = f'(x)$ is drawn below.



A maximum turning point on $y = f(x)$ occurs at:

(A) $x = -1$

(B) $x = 3$

(C) $x = 5$

(D) $x = 7$

10 Line l has an x-intercept of -4 and y-intercept of -2.
Line k is perpendicular to line l and passes through the point (4,5).
Line m passes through the point (1,3).
Which of the following does not represent any of the lines k, l and m?

(A) $2x + y - 3 = 0$

(B) $x + 2y + 4 = 0$

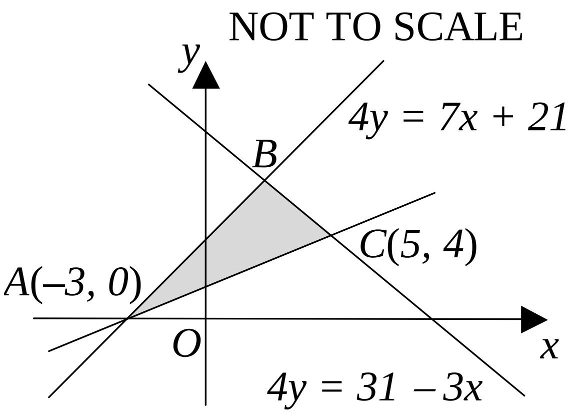
(C) $2x + y - 5 = 0$

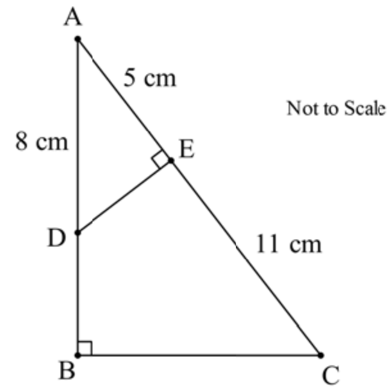
(D) $x + y - 2 = 0$

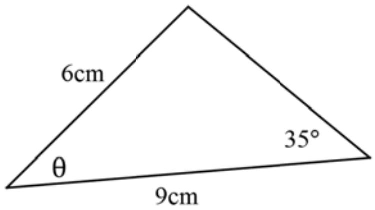
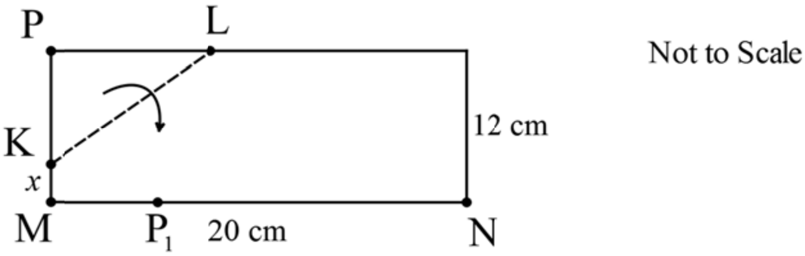
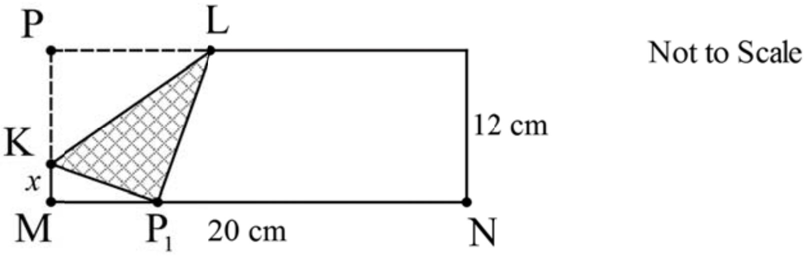
End of Section I

Section II – Extended Response
All necessary working should be shown in every question.

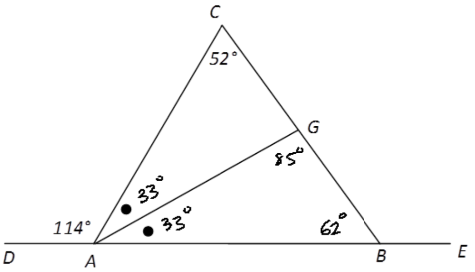
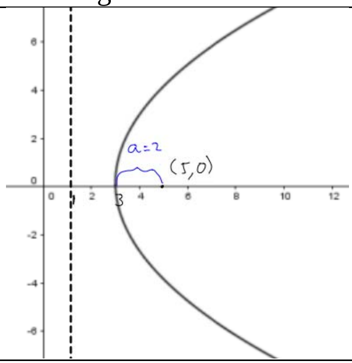
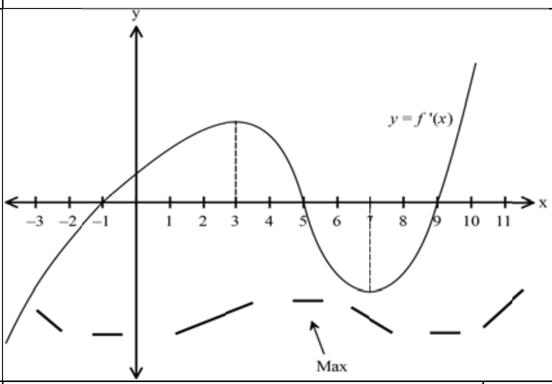
Question 11 (15 marks) - Start on the appropriate page in your answer booklet		**Marks**
a)	Rationalise the denominator of $$\frac{3}{\sqrt{5}-2}$$	**2**
b)	Differentiate the following with respect to x	
	(i) $x^2+\frac{2}{x}-7$	**2**
	(ii) $\sqrt{3x-1}$	**2**
	(iii) $\frac{x^2}{x+1}$	**2**
c)	Given $5x^2+7x+6\equiv A(x+1)^2+B(x+1)+C$, find the value of the constants A, B and C.	**2**
d)	Find the equation of the normal to the curve $y=2x^7-11x+1$ at $x=1$.	**3**
e)	Let $g(x)=x^2+1$	
	(i) Evaluate $g(-3)$.	**1**
	(ii) For what value(s) of x is $g(x)=2$?	**1**
	End of Question 11	

Question 12 (15 marks) - Start on the appropriate page in your answer booklet		**Marks**
a)	Given that the point $(-3, k)$ lies on the line $x + 5y - 7 = 0$, evaluate k.	1
b)	Evaluate $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$.	2
c)	Given $g(x) = \sqrt{36 - x^2}$	
	(i) Show that $g(x)$ is an even function.	1
	(ii) Find the range of $g(x)$.	1
d)	In the diagram below, the lines $4y = 7x + 21$ and $4y = 31 - 3x$ intersect at the point B. Point A has co-ordinates $(-3, 0)$ and point C has co-ordinates $(5, 4)$.	
		
	(i) Show that the line AC has equation $2y = x + 3$.	2
	(ii) Show that B has co-ordinates $(1, 7)$.	2
	(iii) Show that the perpendicular distance from B to the line AC is $2\sqrt{5}$ units.	2
	(iv) Find the exact length of the interval AC. Express your answer as a simplified surd.	1
	(v) Hence or otherwise, find the area of ΔABC.	1
e)	For what values of k does $kx^2 - 4x + k + 1 = 0$ have equal roots?	2
	End of Question 12	

Question 13 (15 marks) - Start on the appropriate page in your answer booklet		**Marks**
a)	Given $0° \le \theta \le 180°$ and $\tan\theta = -\frac{5}{3}$, find the exact value of $\cos\theta$.	2
b)	ABC is a right-angled triangle in which $\angle ABC = 90°$. Points D and E lie on AB and AC respectively such that DE is perpendicular to AC. $AD = 8$ cm, $EC = 11$ cm, and $AE = 5$ cm. 	
	(i) Prove that ΔABC is similar to ΔAED.	2
	(ii) Hence, or otherwise find the length of DB.	3
c)	Consider the function $f(x) = 1 - 3x + x^3$, in the domain $-2 \le x \le 3$.	
	(i) Find the co-ordinates of the stationary points and determine their nature.	3
	(ii) Find the point of inflection.	2
	(iii) Draw a sketch of the curve $y = f(x)$ clearly showing all turning points and the point of inflection.	2
	(iv) What is the maximum value of the function $f(x)$ in the given domain?	1
	End of Question 13	

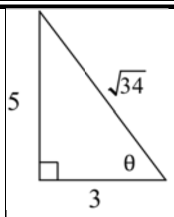
Question 14 (15 marks) - Start on the appropriate page in your answer booklet		Marks
a)	Find all the possible values of θ for the triangle shown below. 	2
b)	Prove that $\dfrac{\sin\theta}{1-\cos\theta}+\dfrac{\sin\theta}{1+\cos\theta}=2\operatorname{cosec}\theta$.	2
c)	Factorise and simplify $$\frac{4^x-2^x}{6^x-3^x}$$	2
d)	The point $P(x, y)$ moves so that its distance from point $A(1, 5)$ is always twice its distance from point $B(4, -1)$. Show that the locus of P is a circle stating its centre.	3
e)	 Tegan has rectangle sheet of paper 12cm wide by 20 cm long as shown above. Tegan folds the corner of the sheet along the dotted line KL so that the corner at P lies on MN. Let the new position of P be P_1 as shown:  At the bottom of the left rectangle there is a small triangle ΔMKP_1 Let the length of KM be x cm. (i) Explain why KP_1 is $(12-x)$ cm long. (ii) Show that the area of ΔMKP_1 is given by $A = x\sqrt{36-6x}$ (iii) Find the maximum area of ΔMKP_1.	 1 2 3
	End of Examination	

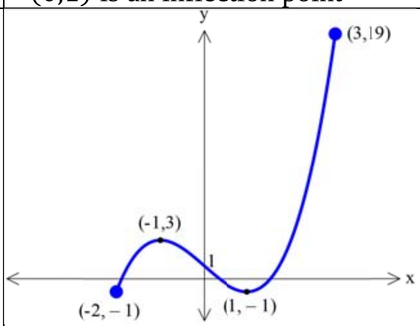
Year 11 Advanced Yearly 2017

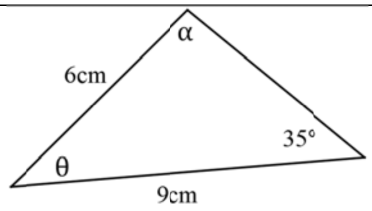
	Solutions	Mks	Comments
1	$x = \frac{-(-5) \pm \sqrt{5^2 - 4 \times 2 \times -1}}{2 \times 2}$ $x = \frac{5 \pm \sqrt{33}}{4}$	**1**	D
2		**1**	D
3	Product of roots = $\frac{c}{a} = -\frac{4}{3}$	**1**	C
4		**1**	B
5	$\tan x = -\frac{1}{\sqrt{3}}$ Acute angle = 150° and in the 2nd and 4th quadrant	**1**	D
6	 Vertex = (3,0)	**1**	B
7	$y = (x+1)^{\frac{3}{2}}$ $\frac{dy}{dx} = \frac{3}{2}(x+1)^{\frac{1}{2}}$ $at\ x = 0$ $m_T = \frac{3}{2}$	**1**	D
8	$x = 0$ is not a solution	**1**	A
9		**1**	C
10	Line l: $y = mx - 2$ since it passes $(-4,0)$ $0 = -4m - 2$ $m = -\frac{1}{2}$ $y = -\frac{1}{2}x - 2$ $x + 2y + 4 = 0$ Line k: $m = 2$ $y - 5 = 2(x-4)$ $y - 5 = 2x - 8$ $2x - y - 3 = 0$ Line m: using equation C: $2(1) + 3 - 5 = 0$ $\therefore 2x + y - 5 = 0$	**1**	D

	Solutions	Mks	Comments
11a	$\frac{3}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{3\sqrt{5}+6}{1}$	**2**	**2 marks** • correct solution. **1 mark** • correct conjugate
11b(i)	$y=x^2+2x^{-1}-7$ $\frac{dy}{dx}=2x-\frac{2}{x^2}$	**2**	**2 marks** • correct solution. **1 mark** • derives one term correctly.
11b(ii)	$y=(3x-1)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}(3x-1)^{-\frac{1}{2}}\times 3$	**2**	**2 marks** • correct solution. **1 mark** • obtains $\frac{1}{2}(3x-1)^{-\frac{1}{2}}$.
11b(iii)	$\frac{dy}{dx}=\frac{3x^2(x+1)-x^3}{(x+1)^2}$ $=\frac{2x^3+3x^2}{(x+1)^2}$ $=\frac{x^2(2x+3)}{(x+1)^2}$	**2**	**2 marks** • correct solution. **1 mark** • correct use of the quotient or product rule.
11c	<u>$Let\ x=-1$</u> $5(-1)^2+7(-1)+6=A(-1+1)^2+B(-1+1)+C$ $C=4$ <u>equating x^2</u> $5=A$ <u>$Let\ x=0$</u> $6=5+B+4$ $B=-3$ $\therefore A=5, B=-3, C=4$	**2**	**2 marks** • correct solution. **1 mark** • Finds one value.
11d	$\frac{dy}{dx}=14x^6-11$ $at\ x=1:$ $m_T=3$ $m_N=-\frac{1}{3}$ $y=2(1)^7-11(1)+1$ $y=-8$ Equation of the tangent: $y+8=-\frac{1}{3}(x-1)$ $x+3y+23=0$	**3**	**3 marks** • correct solution. **2 marks** • Finds the gradient of the normal. **1 mark** • Finds the gradient of the tangent •Uses the gradient function as the gradient of the normal.
11e(i)	$g(-3)=(-3)^2+1$ $=10$	**1**	**1 mark** • correct answer.
11e(ii)	$2=x^2+1$ $x^2=1$ $x=\pm 1$	**1**	**1 mark** • correct solution.

12a	$-3+5k-7=0$ $k=2$	**1**	**1 mark** • correct answer.
12b	$\lim_{x\to3}\frac{x^3-27}{x-3}=\lim_{x\to3}\frac{(x-3)(x^2+3x+9)}{x-3}$ $=\lim_{x\to3}x^2+3x+9$ $=3^2+3\times3+9$ $=27$	**2**	**2 marks** • correct solution. **1 mark** • Finds a limit with incorrect factorisation but able to cancel out the denominator
12c(i)	$g(-x)=\sqrt{36-(-x)^2}$ $=\sqrt{36-x^2}$ $g(-x)=g(x)$ $\therefore g(x)$ is even	**1**	**1 mark** • correct proof.
12c (ii)	Max value when $x^2=0\rightarrow y=6$ Min value when $x=6\rightarrow y=0$ $\therefore\ 0\le y\le 6$	**1**	**1 mark** • correct answer.
12d (i)	$m=\frac{4-0}{5--3}=\frac{1}{2}$ $y-0=\frac{1}{2}(x+3)$ $2y=x+3$	**2**	**2 marks** • correct solution. **1 mark** • Finds the gradient.
12d (ii)	*If B intersects 4y = 7x + 21 and 4y = 31 −3x, then it must satisfy both equations* Sub (1,7) into $4y=7x+21$ $LHS=4\times7=28$ $RHS=7(1)+21=28$ $\therefore$ lies on $4y=7x+21$ Sub (1,7) into $4y=31-3x$ $LHS=4\times7=28$ $RHS=31-3\times1=28$ $\therefore$ lies on $4y=31-3x$	**2**	**2 marks** • correct solution. **1 mark** • Shows the point lies on one line. • Finds either x or y.
12d(iii)	$d=\frac{\|ax_1+by_1+c\|}{\sqrt{a^2+b^2}}$ $d=\frac{\|1-2(7)+3\|}{\sqrt{1^2+2^2}}$ $d=\frac{10}{\sqrt5}$ $d=2\sqrt5$	**2**	**2 marks** • correct solution. **1 mark** • manipulates equation into general form and applies it to the perpendicular distance formula.
12d(iv)	$d=\sqrt{(5+3)^2+(4-0)^2}$ $d=4\sqrt5$	**1**	**1 mark** • correct answer.
12d(v)	$A=\frac{1}{2}\times4\sqrt5\times2\sqrt5$ $A=20$ units^2	**1**	**1 mark** • correct answer.
12e	The equation $kx^2-4x+k+1=0$ will have equal roots when $\Delta=0$ $\Delta=b^2-4ac$ $0=16-4k(k+1)$ $0=16-4k-4k^2$ $k^2+k-4=0$ $k=\frac{-1\pm17}{2}$	**2**	**2 marks** • correct solution. **1 mark** • correct algebraic expression for $\Delta=0$.

<table>
<tr><td>13a</td><td>

Since $0 \le \theta \le 180°$ and $\tan\theta < 0$
then θ is in the 2nd quadrant.
$\therefore \cos\theta = -\frac{3}{\sqrt{34}}$</td><td>2</td><td>2 marks
• correct solution.
1 mark
• recognising $\cos\theta < 0$
• finds $\sqrt{34}$</td></tr>
<tr><td>13b(i)</td><td>In ΔABC and ΔAED
$\angle BAC$ is common
$\angle AED = \angle ABC = 90°$ $(AC \perp DE)$
$\Delta ABC ||| \Delta AED$ (AA)</td><td>2</td><td>2 marks
• correct solution.
1 mark
• Proves twp pairs of angles equal and not stating the rule used.</td></tr>
<tr><td>13b(ii)</td><td>$Let\ DB = x$
$\frac{AD}{AC} = \frac{AE}{AB}$ (matching sides of similar triangles in ratio)
$\frac{8}{5+11} = \frac{5}{8+x}$
$8 + x = \frac{5 \times 16}{8}$
$x = 2$</td><td>3</td><td>3 marks
• correct solution.
2 marks
• Finds an appropriate ratio stating the correct reason.
• Finds x without any reasons.
1 mark
• Finds the ratio.</td></tr>
<tr><td>13c(i)</td><td>$f'(x) = -3 + 3x^2$
stationary points occur when $f'(x) = 0$
$0 = -3(1 - x^2)$
$x = \pm 1$
$f(-1) = 3$ and $f(1) = -1$
$f''(-1) = 6x$
$f''(-1) = -6 < 0$ $\therefore$ concave down
$\therefore$ max at $(-1, 3)$
$f''(-1) = 6 > 0$ $\therefore$ concave up
$\therefore$ min at $(1, -1)$</td><td>3</td><td>3 marks
• correct solution.
2 marks
• correct solution.
1 mark
• Finds $x = \pm 1$</td></tr>
<tr><td>13c(ii)</td><td>Inflection occurs when $f''(x) = 0$
$6x = 0$
$x = 0$
$f(0) = 1$
<u>Test for inflexion:</u>
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td></tr>
<tr><td>$f''(x)$</td><td>-6</td><td>0</td><td>6</td></tr>
<tr><td></td><td>$\cap$</td><td>$-$</td><td>$\cup$</td></tr>
</table>
$\therefore$ $(0,1)$ is an inflection point</td><td>2</td><td>2 marks
• correct solution (must test inflexion point)
1 mark
• Finds $x = 0$ by second derivative.</td></tr>
<tr><td>13c(iii)</td><td>
</td><td>2</td><td>2 marks
• correct solution.
1 mark
• draws a graph without showing keys points such as max point, min point or endpoints.</td></tr>
<tr><td></td><td>Max value is 19.</td><td>1</td><td>1 mark
• correct answer.</td></tr>
</table>

14a(i)	 $\frac{\sin\alpha}{9}=\frac{\sin 35}{6}$ $\alpha = 59°21'$ or $120°39'$ $\therefore \theta = 85°39'$ or $24°21'$	**2**	**2 marks** • correct solution. **1 mark** • Finds an angle for the triangle.
14b	$LHS=\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}$ $=\sin\theta\left(\frac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta}\right)$ $=\frac{2}{\sin^2\theta}$ $=2\text{cosec}\,\theta$	**2**	**2 marks** • correct solution. **1 mark** • recognising $1-\cos^2\theta=1$
14c	$\frac{4^x-2^x}{6^x-3^x}=\frac{2^x(2^x-1)}{3^x(2^x-1)}$ $=\frac{2^x}{3^x}$	**2**	**2 marks** • correct solution. **1 mark** • attempts to factorise 2^x or 3^x
14d	$PA=2PB$ $PA^2=4PB^2$ $(x-1)^2+(y-5)^2=4(x-4)^2+4(y+1)^2$ $x^2-2x+1+y^2-10y+25=4x^2-32x+64+4y^2+8y+4$ $0=3x^2-30x+3y^2+18y+42$ $0=x^2+y^2-10x+6y+14$ $x^2-10x+5^2+y^2+6y+3^2=5^2+3^2-14$ $(x-5)^2+(y+3)^2=20$ Centre is $(5,-3)$	**3**	**3 marks** • correct solution. **2 marks** • Completes the square to find a centre. **1 mark** • Finds a simplified expression for $PA^2=4PB^2$.
14e (i)	$PM=12cm$ $PK=P_1K$ (same side folded) $PK=12-x$ $P_1K=12-x$	**1**	**1 mark** • correct solution.
14e (ii)	$MP_1^2=(12-x)^2-x^2$ $=144-24x$ $MP_1=\sqrt{144-24x}$ $MP_1=2\sqrt{36-6x}$ $A=\frac{1}{2}\times MK\times MP_1$ $A=\frac{1}{2}\times x\times 2\sqrt{36-6x}$ $A=x\sqrt{36-6x}$	**2**	**2 marks** • correct solution. **1 mark** • Finds MP_1.
14e (iii)	Stationary points when $\frac{dA}{dx}=0$ $u=x \quad v=(36-6x)^{\frac{1}{2}}$ $u'=1 \quad v'=-3(36-6x)^{-\frac{1}{2}}$	**3**	**3 marks** • correct solution. **2 marks** • shows max occurs at = 4. . **1 mark** • Finds $\frac{dA}{dx}$

$$\frac{dA}{dx} = \sqrt{36-6x} - \frac{3x}{\sqrt{36-6x}}$$

$$\text{if } A' = 0$$

$$\frac{3x}{\sqrt{36-6x}} = \sqrt{36-6x}$$

$$3x = 36 - 6x$$

$$9x = 36$$

$$x = 4$$

x	3	4	5
A'	2.12	0	-4.22
	/	–	\

$\therefore$ max occurs at $x = 4$.

$$A_{\max} = 4\sqrt{36 - 6\times 4}$$

$$A_{\max} = 4\sqrt{12}$$

$$A_{\max} = 8\sqrt{3}\,\text{cm}^2$$