BAULKHAM HILLS
HIGH SCHOOL

2018

YEAR 11
YEARLY
ASSESSMENT

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board- approved calculators may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work
- A reference sheet is attached at the back of this question paper

Total marks: 70

Section I – 10 marks (pages 2 – 5)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 10)
- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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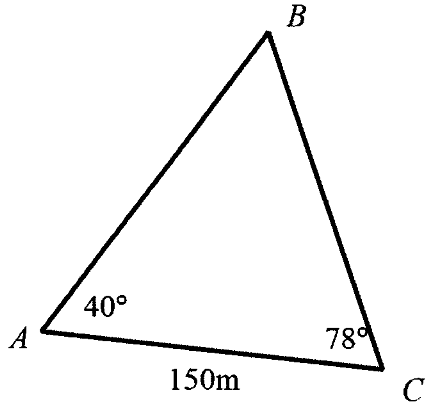
1 The length of BC correct to the nearest metre is:

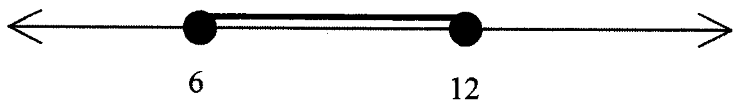


(A) 99 m
(B) 109 m
(C) 166 m
(D) 245 m

2 What is the domain and range of the function $f(x) = \sqrt{4 - x^2}$?

(A) Domain: $-2 \leq x \leq 2$ Range: $-2 \leq y \leq 2$
(B) Domain: $-2 \leq x \leq 2$ Range: $0 \leq y \leq 2$
(C) Domain: $0 \leq x \leq 2$ Range: $-2 \leq y \leq 2$
(D) Domain: $0 \leq x \leq 2$ Range: $0 \leq y \leq 4$

3 The number line represents the solution to the inequality $|x - a| \leq b$.



What are the values of a and b?

(A) $a = 6, b = 3$
(B) $a = 6, b = 6$
(C) $a = 9, b = 3$
(D) $a = 9, b = 6$

4 What is the value of $f'(2)$ if $f(x) = \frac{1}{3x}$?

(A) $-\frac{3}{4}$

(B) $-\frac{1}{6}$

(C) $-\frac{1}{12}$

(D) $\frac{1}{3}$

5 A function $y = f(x)$ has $f'(3) = 0$ and $f''(3) = -1$. At the point where $x = 3$, $y = f(x)$ is:

(A) Stationary and concave up
(B) Decreasing and concave up
(C) Stationary and concave down
(D) Stationary with a horizontal point of inflexion.

6 How many values of x satisfy the equation $(\sin x + 1)(\tan^2 x - 3) = 0$ for $0° \leq x \leq 180°$?

(A) 1

(B) 2

(C) 3

(D) 4

7 Let $f(x) = x^4 - 1$. Which of the following statements is NOT true?

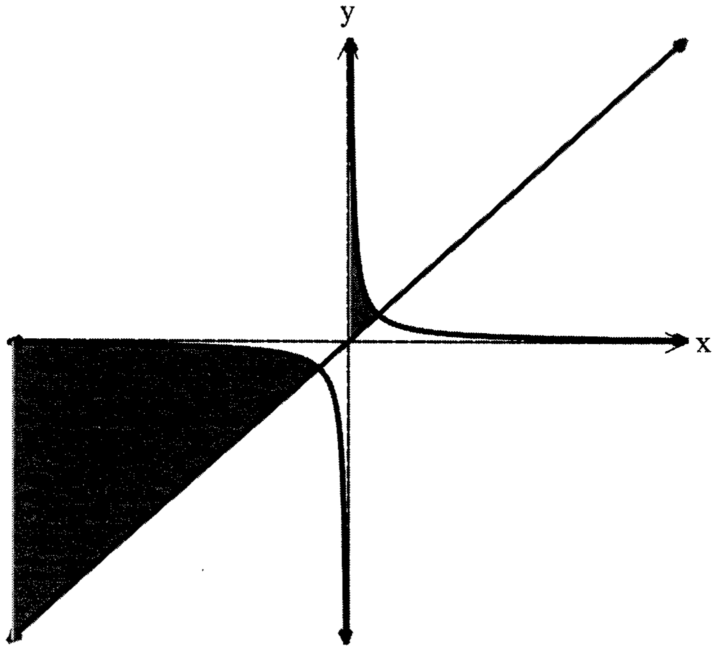
(A) $f(x)$ is an even function.

(B) $f''(x) \geq 0$ for all values of x

(C) $f(x)$ has a horizontal point of inflexion when $x = 0$

(D) $\lim_{x \to \infty} \frac{1}{f(x)} = 0$

8 Which of the following inequalities could describe the shaded region below?



(A) $y \leq \frac{1}{x}$ and $y \leq x$

(B) $y \leq \frac{1}{x}$ and $y \geq x$

(C) $y \geq \frac{1}{x}$ and $y \leq x$

(D) $y \geq \frac{1}{x}$ and $y \geq x$

9 The quadratic equation $3x^2 - 5x + 2 = 0$ has roots α and β.
Which of the following statements is true?

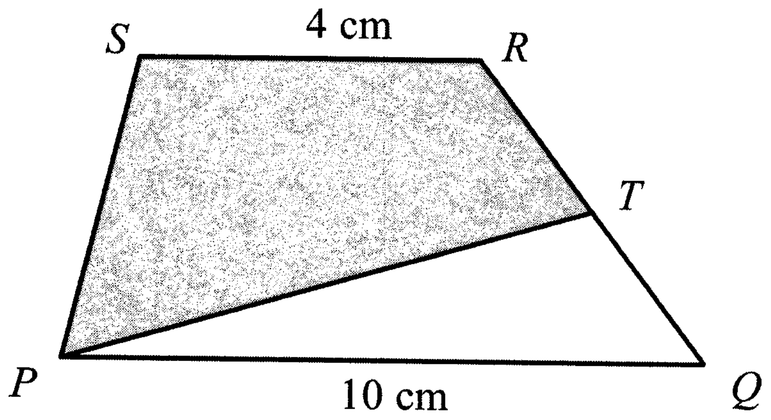
(A) $2\alpha\beta = -\frac{4}{3}$

(B) $\alpha^2 + \beta^2 = \frac{13}{9}$

(C) $2\alpha + 3\beta = \frac{25}{3}$

(D) $\alpha^2\beta^2 = \frac{2}{9}$

10 The lines PQ and SR are parallel and 6 cm apart. T is the midpoint of QR.



What is the area, in square centimetres, of the shaded region?

(A) 21

(B) 26

(C) 27

(D) 34

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate pages of the answer booklet. Extra writing paper is available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start on the appropriate page of your answer booklet.

a) Evaluate $\left(\dfrac{\sqrt{17}}{6 \times 10^{-2}}\right)^3$, correct to three significant figures. **2**

b) Factorise $a^2 - b^2 + 2a + 2b$ **2**

c) Solve the inequality $|2x - 1| > 3$. **2**

d) Simplify $\dfrac{5}{m-2} - \dfrac{2}{m-3}$. **2**

e) Solve the following simultaneous equations: **2**

$3x + 6y = 7$
$2x + 9y = 3$

f) Evaluate $\lim\limits_{x \to 3} \dfrac{3 + 2x - x^2}{x - 3}$ **2**

g) Solve $|3 - x| = x + 1$. **3**

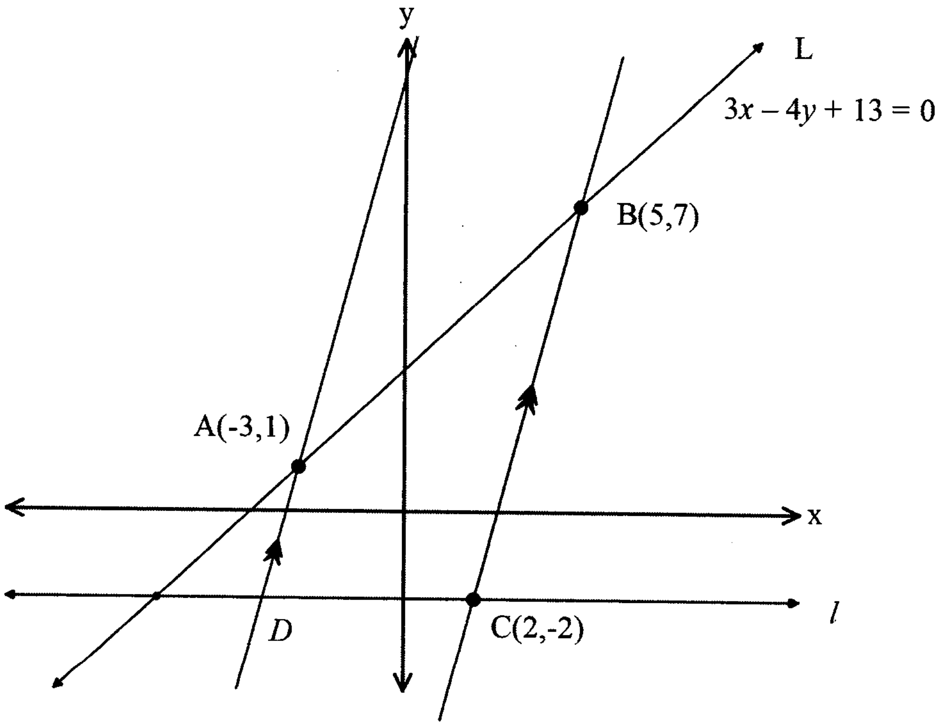
End of Question 11

Question 12 (15 marks) Start on the appropriate page of your answer booklet.

a) Express the following with a rational denominator $\dfrac{4\sqrt{2}-3}{5+\sqrt{2}}$. **2**

b) If $\sec\theta = \dfrac{5}{4}$, and θ is acute, find the value of $\cot(90° - \theta)$. **2**

c) The points A(-3,1) and B(5,7) lie on the line L with equation $3x - 4y + 13 = 0$.
The line l is parallel to the x-axis. The points C(2,-2) and D are two points on l such that $DA \parallel CB$.



i) Calculate the distance AB. **1**
ii) Find the perpendicular distance of C to the line L. **2**
iii) Show that the equation of the line AD is $y = 3x + 10$. **2**
iv) Find the coordinates of D. **1**
v) By joining AC, find the area of quadrilateral $ABCD$. **2**

d) Solve $2\sin^2\theta = \sin\theta$ for $0° \leq \theta \leq 360°$. **3**

End of Question 12

Question 13 (15 marks) Start on the appropriate page of your answer booklet.

a) Consider the function $f(x) = 1 + 3x - x^3$ for $-2 \leq x \leq 2$.

i) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

ii) Find the coordinates of any points of inflection. **2**

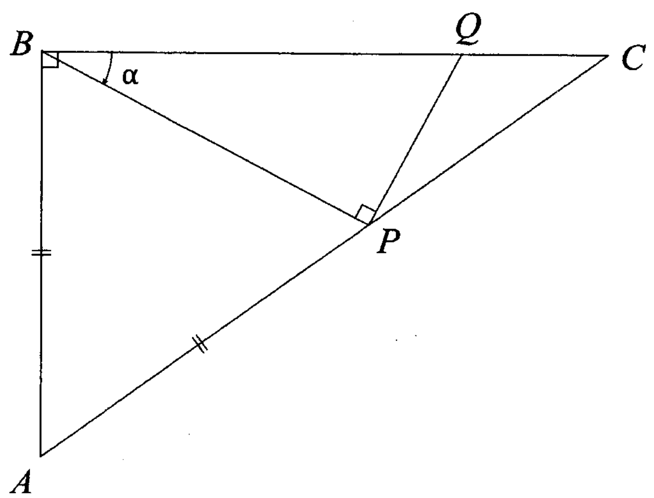
iii) Sketch the curve for $-2 \leq x \leq 2$, clearly labelling any stationary points and the y-intercept. **2**

b) For the parabola $(x+2)^2 = 12(y-6)$, find:

i) The coordinates of the focus. **2**

ii) The equation of the directrix. **1**

c) Triangles ABC and BPQ are right-angled triangles and $AB = AP$. $\angle PBQ = \alpha°$



i) Copy the diagram into your answer booklet.

ii) Prove that $\angle PBQ = \angle CPQ$. **2**

iii) Hence prove that $PC^2 = BC \times QC$. **3**

End of Question 13

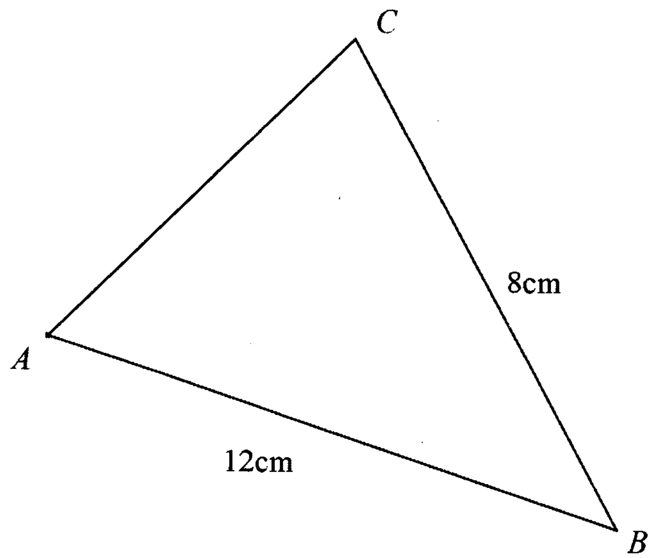
Question 14 (15 marks) Start on the appropriate page of your answer booklet.

a) Find the value(s) of a such that one of the roots of $x^2 + 4ax + 18a = 0$ is equal to three times the other. **2**

b) Show that the curve $y = \dfrac{3x + 5}{x + 1}$ is always decreasing for all values of x in its domain. **3**

c) The quadratic expression $(3 - k)x^2 + kx + 1$ is positive definite. Find the possible values of k. **3**

d) In a triangle ABC the length of side AB is 12 cm and the length of side BC is 8cm.



NOT TO SCALE

Within what range of values does the length of side AC lie? **2**

Question 14 continues on following page

Question 14 (continued)

e) A truck is to travel 1000 kilometres at a constant speed of v km/h. When travelling at v km/h, the truck consumes fuel at the rate of $\left(6+\dfrac{v^2}{50}\right)$ litres per hour.
The truck company pays \$1.50 per litre for fuel and pays each of 2 drivers \$36 per hour whilst the truck is travelling.

i) Let the total cost of fuel and the drivers' wages for the trip be C dollars.
Show that $C = 30v + \dfrac{81000}{v}$. **2**

ii) The truck must take no longer than 12 hours to complete the trip, and speed limits require that $v \leq 100$.

At what speed v should the truck travel to minimise the cost C? **3**
(Answer to 2 decimal places).

End of paper

YEAR 11 YEARLY 2UNIT 2018

1. (B): $\angle ABC = 180^\circ - (40^\circ + 78^\circ)$
$= 62^\circ$

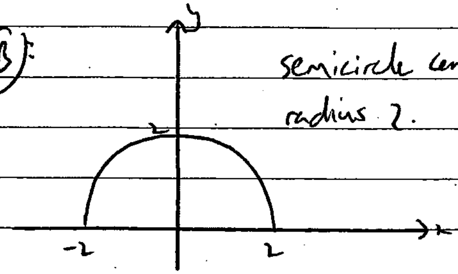
$\frac{BC}{\sin 40^\circ} = \frac{AC}{\sin 62^\circ}$

$BC = \frac{150 \sin 40^\circ}{\sin 62^\circ}$

$= 109.200\ldots$

$BC = 109\text{m}$ (nearest metre)

2. (B): semicircle centre at origin radius 2.



3. (C): $|x - 9| \leq 3$ since $\frac{12-6}{2} = 3$, centre at 9

4. (C) $f(x) = \frac{1}{3} x^{-1}$

$f'(x) = -\frac{1}{3} x^{-2}$

$f'(x) = -\frac{1}{3x^2}$

$f'(2) = \frac{-1}{3 \times 2^2}$

$= -\frac{1}{12}$

5. (C) $f'(3) = 0 \Rightarrow$ stationary point
$f''(3) = -1 < 0$ ∩ ∴ concave down
∴ maximum turning point

6. (B): $(\sin x + 1)(\tan^2 x - 3) = 0$
$\sin x = -1$
No solutions since $\sin x > 0$ for $0^\circ < x < 180^\circ$

$\tan^2 x = 3$
$\tan x = \pm\sqrt{3}$
$\therefore x = 60^\circ, 120^\circ$

∴ 2 solutions

7. (C): Consider (A) $f(-a) = (-a)^4 - 1$
$= a^4 - 1$
$= f(a)$
∴ Even ⇒ (A) is true

Consider (B) $f'(x) = 4x^3$
$f''(x) = 12x^2$
$\therefore f''(x) \geq 0$ for all x

Consider (C) $f''(x) = 0$
$12x^2 = 0$
$x = 0$

Possible point of inflexion when $x = 0$

x	-1	0	1
$f''(x)$	12	0	12

concavity doesn't change
∴ no point of inflexion
∴ (C) is false

(Note: Consider (D) $\lim_{x \to \infty} \frac{1}{f(x)}$
$= \lim_{x \to \infty} \left(\frac{1}{x^4 - 1}\right)$
$= 0$
∴ D is true)

8. (B): Region lies above $y=x$ ie $y \geq x$

Region lies below $y=\frac{1}{x}$ ie $y \leq \frac{1}{x}$

9. (B): for (A) $\alpha\beta = \frac{2}{3}$

$2\alpha\beta = \frac{4}{3}$ ∴ not true

For (B) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{5}{3}\right)^2 - 2\times\frac{2}{3}$

$= \frac{13}{9}$ ∴ true

For (C) $2\alpha+3\beta = 2(\alpha+\beta)+\beta$

∴ $2\times\frac{5}{3}+\beta = \frac{25}{3}$ if true

$\beta = 5$

but $3\times5^2 - 5\times5+2 = 52$

$\neq 0$

∴ $\beta=5$ is not a root.

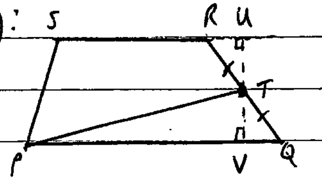
∴ $2\alpha+3\beta \neq \frac{25}{3}$ ∴ not true

For (D) $\alpha^2\beta^2 = (\alpha\beta)^2$

$= \left(\frac{2}{3}\right)^2$

$= \frac{4}{9}$ ∴ not true.

10. (C):



since T is midpoint of RQ, RT = TQ

∴ UT = TV = $\frac{6}{2}$ (ratio of intercepts on parallel lines, 1:1)

$= 3$

$A_{trapezium} - A_{triangle} = \frac{6}{2}(4+10) - \frac{1}{2}\times10\times3$

$= 42 - 15$

$= 27 cm^2$

11 (a) 324503.6835

≈ 325000 3 significant figures

or 3.25×10^5

(2) correct answer

(1) obtains correct unrounded answer.

(b) $a^2-b^2+2a+2b$

$=(a+b)(a-b)+2(a+b)$

$=(a+b)(a-b+2)$

(2) correct answer

(1) factorises a^2-b^2

(c) $2x-1<-3$ or $2x-1>3$

$2x<-2$ $\quad$ $2x>4$

$x<-1$ or $x>2$

(2) correct answer

(1) solves 1 inequality correctly

(d) $\frac{5(m-3)-2(m-2)}{(m-2)(m-3)}$

$= \frac{5m-15-2m+4}{(m-2)(m-3)}$

$= \frac{3m-11}{(m-2)(m-3)}$

(2) correct answer

(1) identifies common denominator

(e) $3x+6y=7$ (1)

$2x+9y=3$ (2)

(1)×2 $\quad 6x+12y=14$ (3)

(2)×3 $\quad 6x+27y=9$ (4)

(4)−(3) $\quad 15y=-5$

$y=-\frac{1}{3}$

sub in (1) $\quad 3x-2=7$

$3x=9$

$x=3$

∴ $x=3$ and $y=-\frac{1}{3}$

(2) correct solution

(1) finds one correct value

11 (f) $\lim_{x\to 3} \frac{(3-x)(1+x)}{x-3}$

$= \lim_{x\to 3} \frac{-(x-3)(1+x)}{x-3}$

$= \lim_{x\to 3} -(1+x)$

$= -(1+3)$

$= -4$

(2) correct solution
(1) factorises numerator

(g) $|3-x| = x+1$

Note $x+1 \geq 0$ ie $x \geq -1$

$3-x = x+1$ $\quad$ $-(3-x) = x+1$

$2x = 2$ $\quad$ $x-3 = x+1$

$x = 1$ $\quad$ $-3 \neq 1$

$\therefore$ no solution

test/check since $x=1$ satisfies $x \geq -1$ it is a solution

$\therefore x=1$ is the only solution

(3) correct solution
(2) obtains $x=1$ and no solution without testing
(1) solves for $x=1$

12 (a) $\frac{4\sqrt{2}-3}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}$

$= \frac{20\sqrt{2}-8-15+3\sqrt{2}}{25-2}$

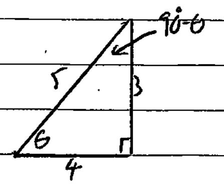
$= \frac{23\sqrt{2}-23}{23}$

$= \frac{23(\sqrt{2}-1)}{23}$

$= \sqrt{2}-1$

(2) correct answer ($\frac{23\sqrt{2}-23}{23}$ is acceptable)
(1) multiplies by conjugate

12 (b) $\cos\theta = \frac{4}{5}$



$\cot(90-\theta)^\circ = \tan\theta$

$= \frac{3}{4}$

(2) correct answer
(1) finds $\tan\theta$
or (1) attempts to use complementary relationship between cot and tan

12 (c) (i) $AD = \sqrt{(5+3)^2 + (7-1)^2}$

$= \sqrt{64+36}$

$= 10$

(1) correct answer

(ii) Perp Dist $= \left|\frac{2\times 3 + 4\times 2 + 13}{\sqrt{3^2+4^2}}\right|$

$= \frac{27}{5}$

(2) correct answer
(1) correctly substitutes into formula.

(iii) $m_{BC} = \frac{7+2}{5-2}$

$= \frac{9}{3}$

$= 3$

$m_{AD} = m_{BC}$ (parallel lines)

$\therefore m_{AD} = 3$

Using $y - y_1 = m(x - x_1)$

$y - 1 = 3(x+3)$

$y - 1 = 3x + 9$

$y = 3x + 10$

(2) correct solution
(1) calculates gradient of AD (must mention parallel lines)

(iv) At D, $y = -2$

$-2 = 3x + 10$

$3x = -12$

$x = -4$

$\therefore D$ is $(-4, -2)$

(1) correct answer

(v) Area ΔDAC + Area ΔBAC = Area of ABCD.

Area $= \left(\frac{1}{2} \times 6 \times 3\right) + \left(\frac{1}{2} \times 10 \times \frac{27}{5}\right)$

$= 9 + 27$

$= 36$ square units

(2) correct answer
(1) calculates area of ΔDAC or ΔABC.
(1) Bald answer or another correct method and answer.

12 (b) $2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0 \quad \sin\theta = \frac{1}{2}$

$\theta = 0, 180°, 360° \quad \theta = 30°, 180° - 30°$

$\therefore \theta = 0°, 30°, 150°, 180°, 360°$

(3) correct answer
(2) solves $\sin\theta = \frac{1}{2}$ to obtain 30°, 150°.
(1) obtains $\sin\theta = 0$ and $\sin\theta = \frac{1}{2}$ and finds at least 1 solution

Question 13

a)(i) $f(x) = 1 + 3x - x^3$

$f'(x) = 3 - 3x^2$

$f''(x) = -6x$

Stat points occur when $\frac{dy}{dx} = 0$

$3 - 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

Testing nature;

$f''(-1) = 6 > 0$ ∪

$\therefore$ Local minimum turning point at $(-1, -1)$

$f''(1) = -6 < 0$ ∩

$\therefore$ Local maximum turning point at $(1, 3)$

(3) correct solution
(2) finds coordinates of stationary points
(2) finds x values of stat points and determines nature
(1) finds x values of stat points

(ii) Points of inflexion may occur when $\frac{d^2y}{dx^2} = 0$

$-6x = 0$

$x = 0$

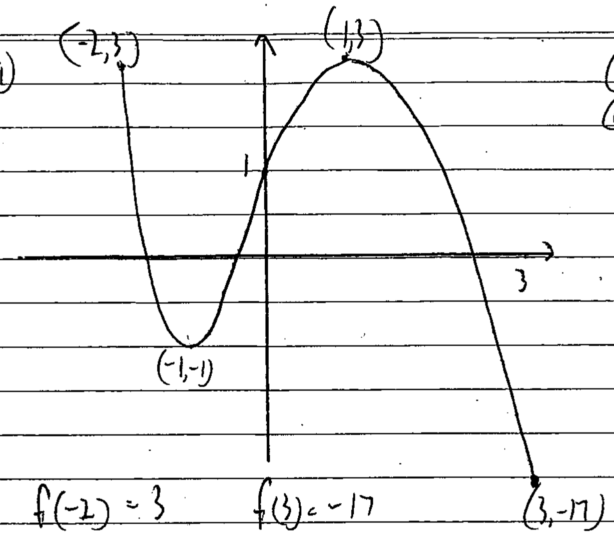
Testing

x	-1	0	1
$f''(x)$	6	0	-6

Concavity changes $\therefore$ Point of inflexion at $(0, 1)$

(2) correct solution
(1) finds coordinates without showing concavity changes

13 a (iii)



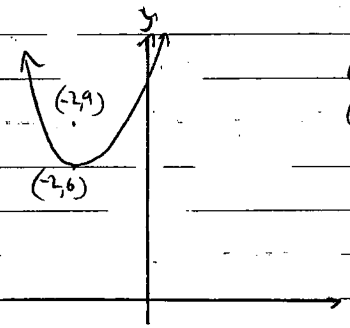
$f(-2) = 3 \quad f(3) = -17$

(2) correct graph
(1) majority of features:
- turning points labelled
- y intercept labelled
- point of inflexion (0,1) where concavity appears to change
- endpoints shown

b (i) $4a = 12$

$a = 3$

vertex at $(-2, 6)$



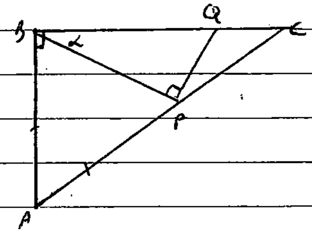
$\therefore$ Focus at $(-2, 9)$

(2) correct answer
(1) calculates focal length (must be positive a)

(ii) $y = 3$

(1) correct answer

(c)(i)



(ii) $\angle CBP + \angle PBA = 90°$ (complementary adjacent angles) $\angle ABC = 90°$

$\therefore \angle PBA = 90 - \alpha$

$\angle BPA = 90 - \alpha$ (= angles opposite = sides in $\triangle ABP$)

$\angle APB + \angle BPQ + \angle QPC = 180°$ ($\angle$ sum on straight line)

$90 - \alpha + 90° + \angle QPC = 180°$

$\therefore \angle QPC = \alpha = \angle QBP$ as required

(2) correct solution
(1) significant prog

13 c(iii) In △'s BPC & PQC

$\angle CBP = \angle CPQ$ (proven in (ii))

$\angle C$ is common

$\therefore \triangle BPC \,|||\, \triangle PQC$ (AA)

$\therefore \frac{BC}{PC} = \frac{PL}{QC}$ (matching sides in same ratio, $\triangle BPC \,|||\, \triangle PQC$)

$\therefore PC^2 = BC \times QC$

(3) correct proof
(2) correct logic without reasons
(2) proves $\triangle BPC \,|||\, \triangle PQC$ with reasons
(1) limited progress (eg incomplete similarity proof)

Q14 (a) let roots be α and 3α

$\alpha + 3\alpha = -4a$

$4\alpha = -4a$

$\alpha = -a$

Also $\alpha(3\alpha) = 18a$

$3\alpha^2 = 18a$

$3a^2 = 18a$

$a^2 = 6a$

$a(a-6) = 0$

$\therefore a = 0$ or $a = 6$.

(2) correct answer (no penalty if finding a=0 and then ignoring as trivial
(1) attempts to use sum or product of roots with α and 3α

(b) $\frac{dy}{dx} = \frac{(x+1).3 - (3x+5)}{(x+1)^2}$

$= \frac{3x+3-3x-5}{(x+1)^2}$

$\frac{dy}{dx} = \frac{-2}{(x+1)^2}$

Always decreasing as negative ÷ by positive ($(x+1)^2 > 0$) is always negative except $x = -1$ where the denominator is 0

(3) correct solution
(2) finds simplified $\frac{dy}{dx}$
(1) Correctly applies the quotient rule.

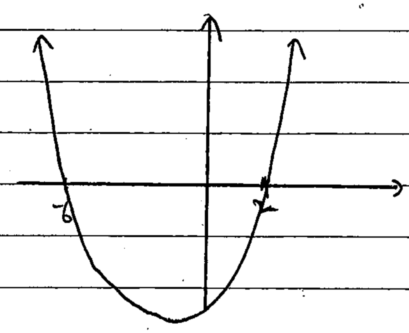
14 (c) For positive definite

$a > 0$ and $\Delta < 0$

$3-k > 0$ and $k^2 - 4(3-k).1 < 0$

$k < 3$ $\quad k^2 + 4k - 12 < 0$.

$(k+6)(k-2) < 0$



$k < 3$ and $-6 < k < 2$.

$\therefore -6 < k < 2$

(3) correct solution (NB must have considered $k<3$)
(2) solves $\Delta < 0$
(1) states $k<3$ or calculates discriminant.

14 (d) Since the sum of any 2 sides of a triangle must exceed the third side

$AC + 8 > 12$ and $12 + 8 > AC$

$AC > 4$ $\quad AC < 20$

$\therefore 4 < AC < 20$ ans.

(2) correct answer
(1) either $AC > 4$ or $AC < 20$

14 e) i) speed = $\frac{\text{distance}}{\text{time}}$

(2) correct solution
(1) calculates cost per hour

time = $\frac{1000}{v}$

cost per hour = driver cost/hr + fuel cost/hr

$= 2 \times 36 + \left(6 + \frac{v^2}{50}\right) \times 1.5$

$= 81 + \frac{3v^2}{100}$

Total cost, C = $\left(81 + \frac{3v^2}{100}\right) \times \frac{1000}{v}$

$C = 30v + \frac{81000}{v}$

ii) $\frac{dC}{dv} = 30 - 81000v^{-2}$

$= 30 - \frac{81000}{v^2}$

(3) correct solution
(2) uses calculus to find minimum cost at 51.96 km/hr, proving it is a minimum
(1) obtains 51.96 by solving $\frac{dC}{dv} = 0$

For max or min, $\frac{dC}{dv} = 0$

$30 - \frac{81000}{v^2} = 0$

$30 = \frac{81000}{v^2}$

$v^2 = 2700$

$v \doteq 51.96$ km/hr.

(Note: an answer of $83\frac{1}{3}$ km/h without any calculus is 0 marks)

$\frac{d^2C}{dv^2} = 162000v^{-3}$

$= \frac{162000}{v^3}$

when $v = 51.96$, $\frac{d^2C}{dv^2} = 1.1547\ldots$

> 0 U

$\therefore$ min cost

But trip must be less than 12 hours $\frac{1000}{v} \leq 12$

$v \geq 83\frac{1}{3}$ km/h.

$\therefore$ Time constraints not met by minimum cost. As speed increases cost increases so $83\frac{1}{3}$ km/hr is the speed the truck should travel.