



# Year 11 Mathematics

## Semester 2 Examination

### September 2010

**Time Allowed:** 2 Hours

**Instructions:** Start each question on a new page  
Show all necessary working  
Silent, non-programmable calculators may be used  
Pencil or liquid paper should not be used  
Answer all questions  
Marks may not be awarded for partial or incomplete answers

+ Solutions  
Caringbah 2010 Y11 24 Yearly  
@ & S  
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✓ Basic  
✓ Calculus  
✓ Trig  
✓ Geometry  
✓ Coordinate Ge  
✓ Functions

#### QUESTION 1: (13 marks)

**Marks**

- i) State the domain and range of  $y = 9 - x^2$  2
- ii) Simplify:  $\frac{x^2 + 3x + 2}{x^3 + 8}$  2
- iii) Solve:  $5x^2 + 7x - 12 = 0$  2
- iv) Differentiate 1
- (a)  $y = 2x^3 - 12x + 7$  2
- (b)  $y = \frac{3}{x^4}$  2
- v) Simplify:  $\frac{a+2}{5} - \frac{a-1}{3}$  2
- vi) Factorise fully:  $x^3 - x + x^2y - y$  2

**QUESTION 2: (13 marks)****Marks**

- i) If  $\frac{5}{2\sqrt{2}+3} = a + b\sqrt{2}$ , find the values of  $a$  and  $b$  2
- ii) Express  $y^{\frac{-2}{3}}$  without a negative or fractional index 1
- iii) Draw a neat sketch of the following functions:
- (a)  $y = \frac{-2}{x}$  2
- (b)  $y = \sqrt{4-x^2}$  2
- iv) Find  $\frac{dy}{dx}$  given  $y = (5-2x)^3$  2
- v) Given  $\sin x = \frac{3}{7}$  and  $\tan x < 0$ , find the exact value of  $\cos x$  2
- vi) Evaluate:  $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2}$  2

**QUESTION 3: (13 marks)**

- i) If  $f(x) = \sqrt{4x-3}$ , find the value of  $f'(3)$  2
- ii) Solve simultaneously:  $\begin{cases} x-y=1 \\ xy=2 \end{cases}$  2
- iii) Simplify:  $\frac{2^n \times 4^{n+1}}{8^{n-2}}$  2
- iv) Differentiate:  $y = \frac{x^2}{3x+1}$  2

Question 3 continued over page

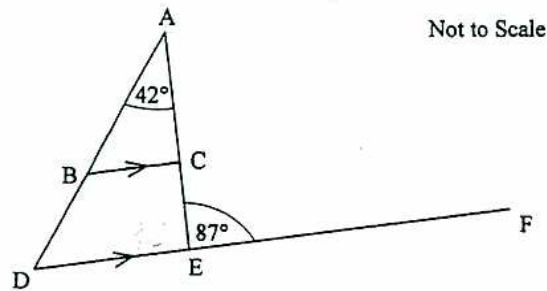
**QUESTION 3: (cont)****Marks**

- v) In the diagram below, ADE is a triangle. B and C lie on AD and AE respectively such that BC is parallel to DE. Line DE is produced to F.

**3**

$$\angle AEF = 87^\circ \text{ and } \angle DAE = 42^\circ$$

Find the size of  $\angle ABC$ , giving reasons for your answer.



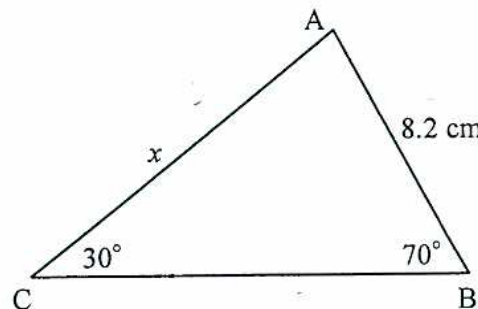
- vi) Given  $f(x) = x^2 - 1$ , is  $f(a+b) = f(a) + f(b)$ ? Justify your answer.

**2****QUESTION 4: (13 marks)**

- i) Find:

(a) the length of AC, correct to 1 decimal place

(b) the area of the triangle ABC, to the nearest  $\text{cm}^2$ .



NB. Diagram not to scale

- ii) Solve:  $|2x - 5| = 3x + 1$

**3**

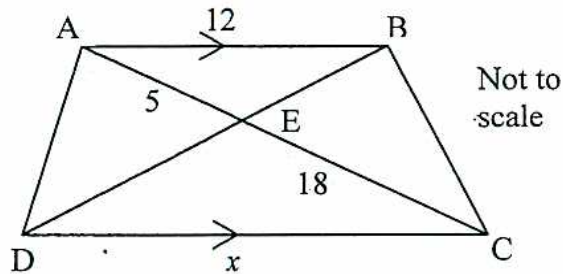
- iii) Differentiate by first principles:  $f(x) = x^2 - 2x$

**3**

Question 4 continued over page

**QUESTION 4: (cont)****Marks**

iv)



(a) Copy the diagram into your book.

(b) Prove  $\triangle ABE$  is similar to  $\triangle CDE$ .**3**(c) Find the length of  $x$ , giving reasons.**1****QUESTION 5: (13 marks)**Given the points  $A(1,4)$ ,  $B(-2,-1)$  and  $C(4,3)$ ,

(a) Find the length of the interval joining B to C

**1**

(b) Find M, the midpoint of the interval BC

**1**

(c) Find the equation of the circle, centre M, passing through both B and C

**2**

(d) Find the gradient of the line joining B to C

**1**(e) Show that the equation of the line BC is  $2x-3y+1=0$ **1**

(f) Find the perpendicular distance from A to the line BC

**2**

(g) Hence find the area of triangle ABC

**1**

(h) Find the equation of the line through B perpendicular to the line BC

**2**(i) Find the co-ordinates of the point  $D(x,y)$  such that A is the midpoint of BD**2**

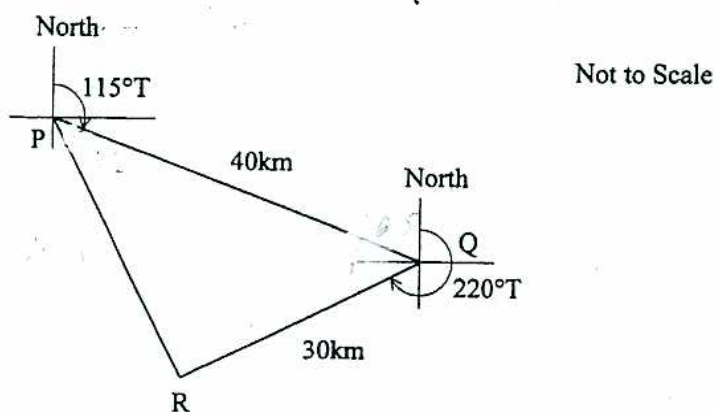
**QUESTION 6: (13 marks)**

**Marks**

i) Prove:  $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = 2\sec^2 x$

**3**

- ii) From P the bearing of a Lighthouse Q, 40 kilometres distant from P, is  $115^\circ\text{T}$ . From Q the bearing of a headland R, 30 kilometres from Q, is  $220^\circ\text{T}$ . This is illustrated in the diagram below.



- (a) Find the size of  $\angle\text{PQR}$  **1**
- (b) Use the Cosine Rule to find the length of PR. Give your answer correct to 2 decimal places. **1**
- (c) Find the bearing of R from P. Give your answer to the nearest whole degree. **2**

iii) For  $y = \frac{6}{\sqrt{x}}$  find:

- (a)  $\frac{dy}{dx}$  **2**
- (b) The gradient of the curve  $y = \frac{6}{\sqrt{x}}$  at the point where  $x = 9$  **1**

iv) A function is defined as:  $f(x) = \begin{cases} 1 & \text{for } x < 0 \\ 2^x & \text{for } x \geq 0 \end{cases}$

- (a) Sketch the graph of this function for  $-3 \leq x \leq 3$  **2**
- (b) Find the value of  $f(-2) + f(2)$  **1**

**QUESTION 7: (13 marks)****Marks**

- i) Solve  $\sin\theta = \frac{-\sqrt{3}}{2}$  for  $0^\circ \leq \theta \leq 360^\circ$  **2**
- ii) (a) Sketch the graphs of  $y = x^2 + x - 6$  and  $y = 9 - x$  on the same number plane. Your graph should be at least  $\frac{1}{3}$  page. **2**
- (b) On your graph mark the intercepts of both curves on the x-axis and y-axis, as well as the co-ordinates of their points of intersection. **2**
- (c) Hence shade the region defined by  $y \geq x^2 + x - 6$  and  $y \leq 9 - x$  **2**
- iii) Differentiate  $y = x^3(x + 1)^4$ , expressing your answer in fully factored form **3**
- iv) Sketch the graph of  $y = |x| + x$  **2**

**THE END!**

# Year 11 Mathematics : Solutions - Semester 2 Exam 2010

i) Domain: all real  $x$   
 Range:  $y \leq 9$

ii)  $\frac{x^2+3x+2}{x^3+8} = \frac{(x+2)(x+1)}{(x+2)(x^2-2x+4)}$   
 $= \frac{x+1}{x^2-2x+4}$

iii)  $5x^2+7x-12=0$   
 $(5x+12)(x-1)=0$   
 $x = -\frac{12}{5}, 1$

iv) a)  $y = 2x^3 - 12x + 7$   
 $y' = 6x^2 - 12$

b)  $y = \frac{3}{x^4} = 3x^{-4}$   
 $y' = -12x^{-5} = -\frac{12}{x^5}$

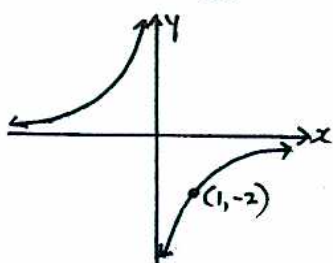
v)  $\frac{a+2}{5} - \frac{a-1}{3} = \frac{3(a+2) - 5(a-1)}{15}$   
 $= \frac{3a+6-5a+5}{15} = \frac{11-2a}{15}$

vi)  $x^3-x+x^2y-y = x(x^2-1)+y(x^2-1)$   
 $= (x+y)(x^2-1)$   
 $= (x+y)(x-1)(x+1)$

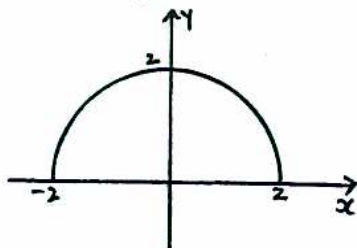
② i)  $\frac{5}{2\sqrt{2}+3} \times \frac{2\sqrt{2}-3}{2\sqrt{2}-3} = \frac{5(2\sqrt{2}-3)}{8-9}$   
 $= -5(2\sqrt{2}-3)$

ii)  $y^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{y^2}}$  or  $\frac{1}{(\sqrt[3]{y})^2}$

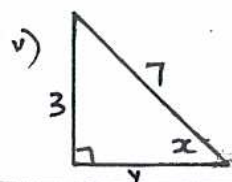
iii) a)  $y = -\frac{2}{x}$



b)  $y = \sqrt{4-x^2}$



iv)  $y = (5-2x)^3$   
 $y' = 3(5-2x)^2 \times -2 = -6(5-2x)^2$



$\sin x = \frac{3}{7}$   
 $y = \sqrt{49-9} = \sqrt{40}$

$\tan x < 0$   
 $\therefore \text{quad 2}$   
 $\cos x = -\frac{\sqrt{40}}{7}$

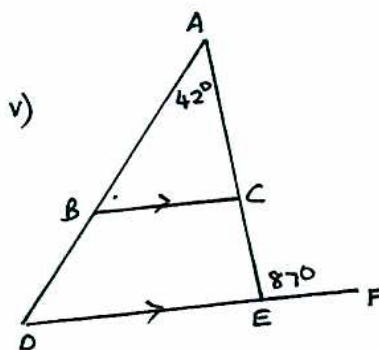
vi)  $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2} = \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(x+2)}$   
 $= \lim_{x \rightarrow 1} \frac{1}{x+2}$   
 $= \frac{1}{3}$

③ i)  $f(x) = \sqrt{4x-3} = (4x-3)^{\frac{1}{2}}$   
 $f'(x) = \frac{1}{2}(4x-3)^{-\frac{1}{2}} \times 4 = \frac{2}{\sqrt{4x-3}}$   
 $f'(3) = \frac{2}{\sqrt{12-3}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$

ii)  $x-y=1 \rightarrow y=x-1$   
 $xy=2$   
 $x(x-1)=2$   
 $x^2-x-2=0$   
 $(x-2)(x+1)=0$   
 $x=2, -1$   
 $\therefore (x,y) = (2,1) \text{ or } (-1,-2)$

iii)  $\frac{2^n \times 4^{n+1}}{8^{n-2}} = \frac{2^n \times (2^2)^{n+1}}{(2^3)^{n-2}}$   
 $= \frac{2^{3n+2}}{2^{3n-6}} = 2^8$

iv)  $y = \frac{x^2}{3x+1}$   
 $y' = \frac{(3x+1) \times 2x - x^2 \times 3}{(3x+1)^2}$   
 $= \frac{3x^2 + 2x}{(3x+1)^2}$   
 $= \frac{x(3x+2)}{(3x+1)^2}$



$\angle BCE = 87^\circ$  (alt L's,  $BC \parallel DF$ )  
 $\angle ACB = 93^\circ$  (L sum straight L)  
 $\angle ABC = 45^\circ$  (L sum  $\triangle ABC$ )

vi)  $f(x) = x^2 - 1$   
 $f(a+b) = (a+b)^2 - 1 = a^2 + 2ab + b^2 - 1$   
 $f(a) = a^2 - 1$   
 $f(b) = b^2 - 1$   
 $f(a) + f(b) = a^2 - 1 + b^2 - 1 = a^2 + b^2 - 2 \neq f(a+b)$

④ i) a)  $\frac{x}{\sin 70} = \frac{8.2}{\sin 30}$   
 $x = \frac{8.2 \sin 70}{\sin 30}$   
 $\therefore AC = 15.4 \text{ cm}$

b) Area =  $\frac{1}{2} \times 15.4 \times 8.2 \sin 80$   
 $= 62.2 \text{ cm}^2$

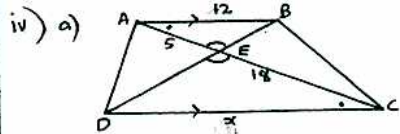
ii)  $|2x-5| = 3x+1$

$2x-5 = 3x+1$        $2x-5 = -(3x+1)$   
 $-6 = x$                $2x-5 = -3x-1$   
 $5x = 4$

LHS =  $|2(-6)-5| = 17$        $x = \frac{4}{5}$   
RHS =  $3(-6)+1 = -17$       LHS =  $|2(\frac{4}{5})-5| = 3\frac{2}{5}$   
 $\therefore x \neq -6$               RHS =  $3(\frac{4}{5})+1 = 3\frac{2}{5}$   
 $\therefore x = \frac{4}{5}$

iii)  $f(x) = x^2 - 2x$   
 $f(x+h) = (x+h)^2 - 2(x+h)$   
 $= x^2 + 2xh + h^2 - 2x - 2h$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$   
 $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - (x^2 - 2x)}{h}$   
 $= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 2h}{h}$   
 $= \lim_{h \rightarrow 0} 2x + h - 2$   
 $= 2x - 2$

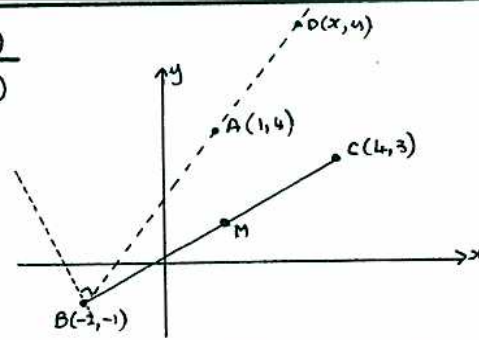


b) In  $\triangle ABE$ ,  $\triangle CDE$   
 $\angle AEB = \angle CED$  (vert opp  $\angle$ s)  
 $\angle BAE = \angle ECD$  (alt  $\angle$ s,  $AB \parallel DC$ )  
 $\therefore \triangle ABE \sim \triangle CDE$  (equiangular)

c)  $\frac{x}{12} = \frac{18}{5}$  (ratios of corresp side)  
 $\therefore x = 43.2$

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⑤



a)  $BC = \sqrt{(4-(-2))^2 + (3-(-1))^2} = \sqrt{36 + 16}$   
 $= \sqrt{52} = 2\sqrt{13}$

b)  $M = (\frac{4-2}{2}, \frac{3-1}{2}) = (1, 1)$

c)  $C(1, 1)$      $R = \sqrt{13}$   
 $(x-1)^2 + (y-1)^2 = 13$

d)  $m = \frac{3-1}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

e) eqn BC:  $y - 3 = \frac{2}{3}(x - 4)$   
 $3y - 9 = 2x - 8$   
 $0 = 2x - 3y + 1$

f)  $D = \frac{2(1) - 3(4) + 1}{\sqrt{2^2 + (-3)^2}} = \frac{9}{\sqrt{13}}$

g)  $A = \frac{1}{2} \times 2\sqrt{13} \times \frac{9}{\sqrt{13}} = 9 \text{ units}^2$

h)  $m_1 = -\frac{3}{2}$   
 $y+1 = -\frac{3}{2}(x+2)$   
 $2y+2 = -3x-6$   
 $3x+2y+8=0$

i)  $B(-2, -1)$      $A(1, 4)$      $D(x, 4)$   
 $1 = \frac{-2+x}{2}$        $4 = \frac{-1+y}{2}$   
 $2 = -2+x$        $8 = -1+y$   
 $x = 4$                $y = 9$   
 $\therefore D(4, 9)$

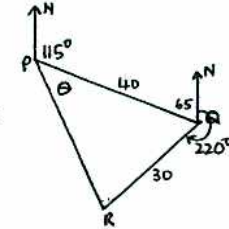
Page ③

⑥

i) LHS =  $\frac{1}{1-\sin x} + \frac{1}{1+\sin x}$   
 $= \frac{1+\sin x + 1-\sin x}{(1-\sin x)(1+\sin x)}$   
 $= \frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x}$   
 $= 2 \sec^2 x = \text{RHS}$

ii) a)  $\angle POR = 75^\circ$

b)  $PR^2 = 40^2 + 30^2 - 2 \times 40 \times 30 \cos 75$   
 $= 1878.8$   
 $PR = 43.35 \text{ km}$



c)  $\frac{\sin \theta}{30} = \frac{\sin 75}{43.35}$   
 $\sin \theta = 0.66846$   
 $\theta = 42^\circ$

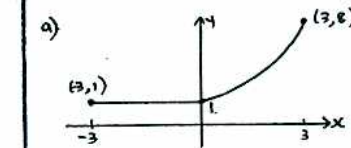
$\therefore$  Bearing is  $115 + 42 = 157^\circ \text{T}$

iii)  $y = \frac{6}{\sqrt{x}} = 6x^{-1/2}$

a)  $\frac{dy}{dx} = -3x^{-3/2} = \frac{-3}{\sqrt{x^3}}$

b) at  $x=9$ ,  $m = \frac{-3}{\sqrt{9^3}} = \frac{-1}{9}$

iv)  $f(x) = \begin{cases} 1 & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$



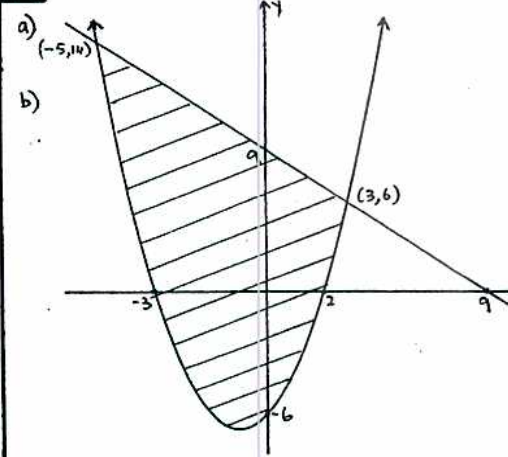
b)  $f(-2) + f(2) = 1 + 4 = 5$

⑦ i)  $\sin \theta = \frac{-\sqrt{3}}{2}$      $0^\circ \leq \theta < 360^\circ$   
 $\theta = 240^\circ, 300^\circ$     Quad 3, 4     $60^\circ$

ii)  $y = x^2 + x - 6$   
 $= (x+3)(x-2)$

$y=0$ ,  $x=-3, 2$   
 $x=0$ ,  $y=-6$

⑧



$x^2 + x - 6 = 9 - x$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = 5, 3$

$x=5$ ,  $y=14$      $x=3$      $y=6$

iii)  $y = x^3(x+1)^4$

$\frac{dy}{dx} = x^3 \cdot 4(x+1)^3 + (x+1)^4 \cdot 3x^2$   
 $= 4x^3(x+1)^3 + 3x^2(x+1)^4$   
 $= x^2(x+1)^3 [4x + 3(x+1)]$   
 $= x^2(x+1)^3 (7x+3)$

iv)  $y = |x| + x$

