Caringbah High School

Year 11 2016

Mathematics

Preliminary HSC Course

Semester 2 Exam

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet with mathematical formulae is provided.
- In Questions 6 - 12, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 85

Section I **5 marks**

Attempt Questions 1-5

Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **80 marks**

Attempt Questions 6 - 12

Write your answers on the answer sheets provided. Ensure your name or student number is clearly visible.

Name: _____ **Class:**

Marker's Use Only									
Section I	**Section II**							**Total**	
Q 1-5	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	**Q11**	**Q12**		
/5	/12	/12	/12	/11	/11	/11	/11	/85	%

Question 1 - 5 (1 mark each) Answer on the page provided.

1) If $f(x) = x^2 - x + 1$ and $f(a) = 3$, the values of a are?

A) 1 and -2 B) -1 and -2

C) -1 and 2 D) 1 and 2

2) The curve of $y = \frac{1}{x}$ is ?

A) an odd function. B) an even function.

C) neither an odd nor even function. D) a relation.

3) The linear function with equation $2x - 4y - 3 = 0$ has ?

A) gradient $-\frac{3}{4}$, y-intercept $\frac{1}{2}$ B) gradient $\frac{1}{2}$, y-intercept $-\frac{3}{4}$

C) gradient $\frac{3}{4}$, y-intercept $-\frac{1}{2}$ D) gradient $-\frac{1}{2}$, y-intercept $\frac{3}{4}$

4) Which of the expressions below is equivalent to $\sin(270 + \theta)$?

A) $\sin\theta$ B) $-\sin\theta$ C) $\cos\theta$ D) $-\cos\theta$

5) What is the size of each interior angle of a regular 18-sided polygon?

A) $157.5°$ B) $160°$

C) $162°$ D) $165°$

End of multiple choice questions

Question 6 (12 marks) Start a NEW page. **Marks**

a) Determine the midpoint of the interval AB with $A\ (-1, 3)$ and $B\ (11, -6)$. **1**

b) Rationalise the denominator and simplify $\dfrac{2\sqrt{3}}{2\sqrt{2}+3}$ **2**

c) If $3x - ky + 7 = 0$ is parallel to $y = 4x - 5$, find the value of k. **2**

d) Simplify $\dfrac{\cos(90° - \theta)}{\cos\theta}$ **2**

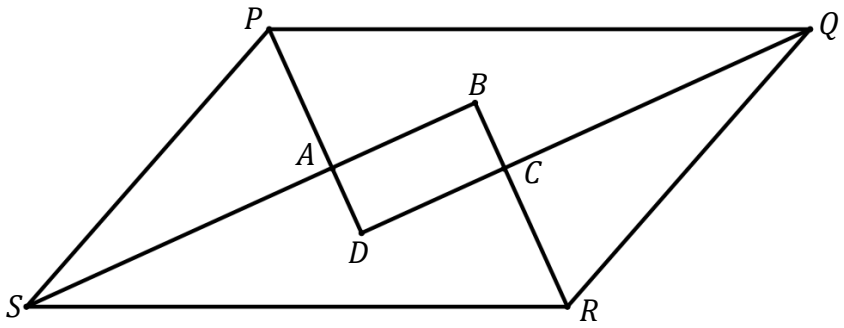
e) Simplify $2\sqrt{54} + \sqrt{150}$ **2**

f) Find the equation of the line that is perpendicular to $5x + 2y - 4 = 0$ that also passes through the point $(-1, 2)$. Write your answer in general form. **3**

Question 7 (12 marks) Start a NEW page. **Marks**

a) If $f(x) = 1 - x$, write a simplified expression for $f(1 - x)$ **1**

b) The quadrilateral $PQRS$ is a parallelogram with $SR \neq RQ$. **3**
The bisectors of its angles intersect at A, B, C and D as shown.



Copy or trace the diagram onto your answer page and prove that quadrilateral $ABCD$ is a rectangle.

c) i) Sketch the following piecemeal function in the domain $-2 \leqslant x \leqslant 6$ **3**

$$f(x) = \begin{cases} -3 & \text{for } x < -1 \\ -x^2 - 2 & \text{for } -1 \leqslant x \leqslant 2 \\ x - 8 & \text{for } x > 2 \end{cases}$$

ii) What is the range of $f(x)$ for the given domain? **1**

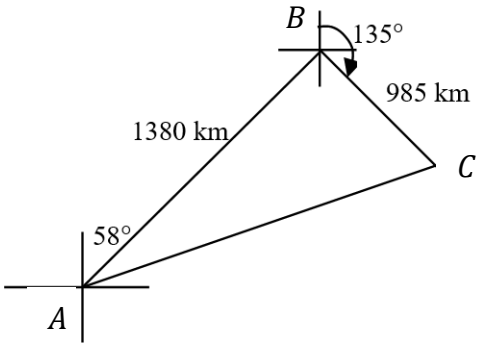
d) Solve $x^2 - x - 2 > 0$ **2**

e) Find the domain and range of **2**

$$y = \frac{1}{\sqrt{4 - x^2}}$$

Question 8 (12 marks) Start a NEW page. **Marks**

a) A plane travels 1380 km from A to B on a bearing of 58°T.
The plane then travels 985 km on a bearing of 135°T to a point C.



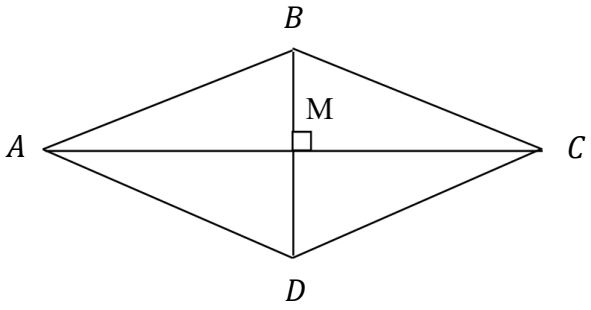
i) Show that $\angle ABC$ is 103°. **1**

ii) Find the distance of CA, correct to one decimal place. **2**

iii) Find the size of $\angle BAC$, to the nearest degree. **2**

iv) Find the bearing of A from C, to the nearest degree. **1**

b) The area of rhombus $ABCD$ is 80 cm^2. One diagonal is 2½ times as long as the other.



i) Find the length of the shorter diagonal. **2**

ii) Find the length of the sides of the rhombus **2**

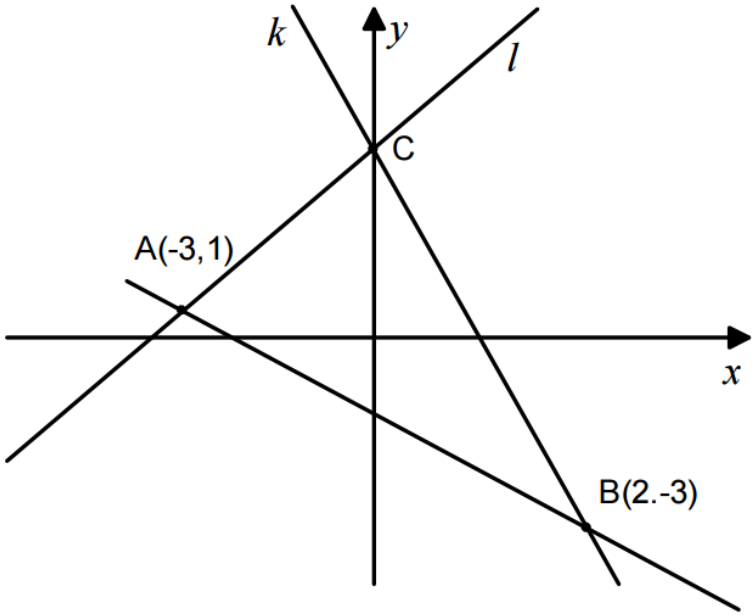
c) Evaluate $\lim_{x\to -4} \dfrac{x^2+3x-4}{x+4}$ **2**

Question 9 (11 marks) Start a NEW page. **Marks**

a) Prove that $\sec^2\theta - 2\tan\theta = (\tan\theta - 1)^2$. **2**

b) Given points $A(-3, 1)$ and $B(2, -3)$.

Point A lies on the line l given by the equation $4x - 3y + 15 = 0$ and
Point B lies on the line k given by the equation $4x + y - 5 = 0$.



The questions below refer to the diagram drawn above.

Copy the diagram onto your answer page.

(i) Show that the point C, the point of intersection of the lines l and k, must lie on the y-axis. **2**

(ii) Find the gradient of the line AB. **1**

(iii) Show the equation of the line AB is $4x + 5y + 7 = 0$. **2**

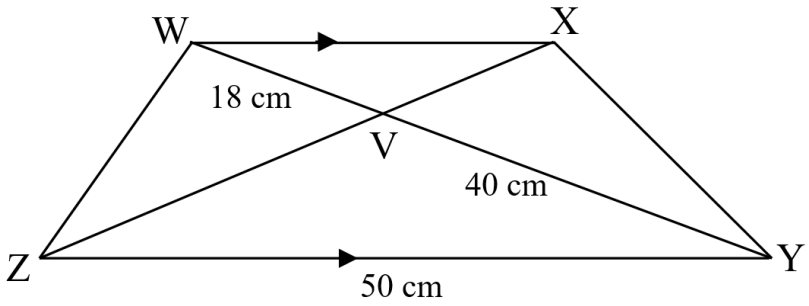
(iv) Find the perpendicular distance from point C to the line AB. **2**

(v) Find the exact area of ΔABC. **2**

Question 10 (11 marks) Start a NEW page. **Marks**

a) Solve $|x - 4| = 2x + 1$. **3**

b) $WXYZ$ is a trapezium with $WX \parallel ZY$. WV = 18 cm, VY = 40 cm and ZY = 50 cm.



i) Show that $\Delta WXV \Vert\Vert\Vert \Delta YZV$, giving reasons. **3**

ii) Find the length of WX. **2**

c) Shade the region given by $y > x^2 - 1$ and $y \leqslant 3^x$ **3**

Question 11 (11 marks) Start a NEW page. **Marks**

a) i) Given that a circle has an equation $x^2 + 4x + y^2 - 6y - 36 = 0$, find the centre and radius of the circle. **2**

ii) Shade the region defined by $x^2 + 4x + y^2 - 6y - 36 \leqslant 0$ **2**

b) Find the value of x if $2 \times (2^{2x}) - 2^x = 0$. **3**

c) i) On the same number plane sketch the graphs $y = x^2 - 4$ and $y = 4x + 1$. **2**

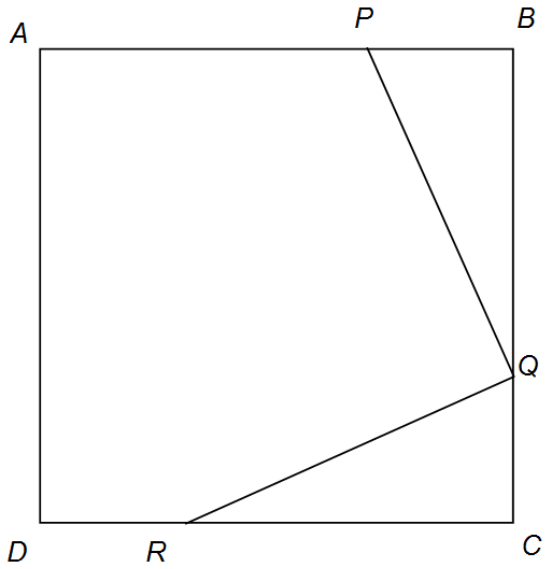
ii) Hence or otherwise, find the points of intersection of these graphs. **2**

Question 12 (11 marks) Start a NEW page. **Marks**

a) Factorise fully $-192x^3 - 3$ **2**

b) Show that the line $3x + 4y - 12 = 0$ does not intersect the circle $x^2 + y^2 = 4$ **3**

c) $ABCD$ is a square P, Q and R are points on AB, BC and CD respectively such that $AP = BQ = CR$.



i) Prove that $\Delta PBQ \equiv \Delta QCR$ **3**

ii) Hence or otherwise, prove that $\angle PQR = 90^{\circ}$. **3**

END OF EXAM

Name: _____

Multiple choice answer page. Fill in either A, B, C or D for questions 1-5.

This page must be handed in with your answer booklets

1.	
2.	
3.	
4.	
5.	

Year 11 2016 Mathematics Preliminary HSC Course Semester 2 Exam Answers

Q1) $\boldsymbol{f(a) = a^2 - a + 1 = 3}$
$\boldsymbol{\therefore a^2 - a - 2 = 0}$
$\boldsymbol{\therefore (a-2)(a+1) = 0}$
$\boldsymbol{\therefore a = -1 \text{ or } 2}$
(C)

Q2) $\boldsymbol{y = \frac{1}{x}}$ is an odd function (A)

Q3) $\boldsymbol{2x - 4y - 3 = 0}$
$\boldsymbol{\therefore y = \frac{1}{2}x - \frac{3}{2}}$
$\boldsymbol{\therefore m = \frac{1}{2}, b = -\frac{3}{4}}$ (B)

Q4) $\boldsymbol{\sin(270 + \theta)}$
$\boldsymbol{= \sin(360 - (90 - \theta))}$
$\boldsymbol{= -\sin(90 - \theta)}$
$\boldsymbol{-\cos\theta}$ (D)

Q5) $\boldsymbol{\text{Exterior Angle} = 360°/18 = 20°}$
$\boldsymbol{\therefore \text{Interior Angle} = 180° - 20°}$
$\boldsymbol{\therefore \text{Interior Angle} = 160°}$ (B)

Q6)

a) $\boldsymbol{\text{Midpoint} = \left(\frac{(-1) + 11}{2}, \frac{3 + (-6)}{2}\right)}$
$\boldsymbol{\therefore \text{Midpoint} = \left(5, -\frac{3}{2}\right)}$

b) $\boldsymbol{\frac{2\sqrt{3}}{2\sqrt{2}+3} \times \frac{2\sqrt{2}-3}{2\sqrt{2}-3} = \frac{4\sqrt{6} - 6\sqrt{3}}{-1}}$
$\boldsymbol{= -4\sqrt{6} + 6\sqrt{3}}$

c) $y = 4x - 5 \ \therefore \text{m} = 4$
$3x - ky + 7 = 0 \ \therefore y = \frac{3}{k}x + \frac{7}{k}$
$\therefore \frac{3}{k} = 4 \quad \therefore k = \frac{3}{4}$

d) $\frac{\cos(90° - \theta)}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

e) $2\sqrt{54} + \sqrt{150} = 6\sqrt{6} + 5\sqrt{6}$
$= 11\sqrt{6}$

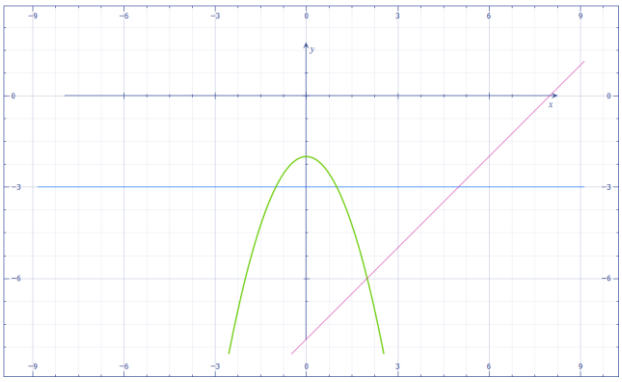
f) $5x + 2y - 4 = 0 \quad \therefore y = -\frac{5}{2}x - 2$
$\therefore \text{m} = -\frac{5}{2}, \quad \therefore \text{m}_\perp = \frac{2}{5}$
$\therefore$ line throught $(-1,2)$
$y - 2 = \frac{2}{5}x - (-1)$
$\therefore y = \frac{2}{5}x + 3$

Q7)

a) $f(1-x) = 1 - (1-x) = x$

b) Let $\theta = \angle PSA = \angle ASR$ (bisector of $\angle PSR$)
$= \angle PQC = \angle CQR$ (bisector of $\angle PQR$)
$\therefore \angle SPA + \angle APQ = 180° - 2\theta$ (co-int PQ//SR)
$\therefore \angle SPA = \angle APQ = 90° - \theta$ (bisectors of $\angle SPQ$)
$\therefore \angle PDQ = 180° - (\angle APQ + \angle PQC)$ (sum ΔPDQ)
$\therefore \angle PDQ = 90°$
$\therefore$ similar for $\angle SBR = 90° \quad \therefore$ rectangle

c) i)



ii) Range $y \geqslant -6$

d) $x^2 - x - 2 > 0$
$\therefore (x-2)(x+1) > 0$
$\therefore -1 < x < 2$

e) Domain $-2 < x < 2$, Range $y \geqslant \frac{1}{2}$

Q8)

a) i) $\angle ABC = 56 + 45 = 103°$
ii) $CA^2 = 1380^2 + 985^2 - 2 \times 1380 \times 985 \times \cos 103$
$\therefore CA = 1867.1km$
iii) $\frac{\sin A}{985} = \frac{\sin 103}{1867.1}, \therefore A = 30°56' \approx 31°$
iv) $269°T$

b) i) $A = \frac{1}{2}xy, \therefore 80 = \frac{1}{2} \times x \times \frac{5}{2}x$
$\therefore 80 = \frac{5}{4}x^2, \quad \therefore x = 8$
ii) $AB = \sqrt{4^2 + 10^2} = \sqrt{116}$

c) $\lim_{x \to -4} \frac{x^2+3x-4}{x+4} = \lim_{x \to -4} \frac{(x+4)(x-1)}{x+4}$
$= \lim_{x \to -4} (x - 1) = -5$

Q9)

a) $RHS = (\tan\theta - 1)^2 = \tan^2\theta - 2\tan\theta + 1$
$= (\sec^2\theta - 1) - 2\tan\theta + 1$
$= \sec^2\theta - 2\tan\theta \ = LHS$

b) i) $l: 4x - 3y + 15 = 0 \ \ k: 4x + y - 5 = 0$
$\therefore 4x - 3y + 15 = 0 \quad -①$
$\therefore 4x + y - 5 = 0 \quad -②$
$\therefore ① - ② \ -4y + 20 = 0$
$\therefore y = 5, x = 0 \ \therefore C$ is on $y-$axis

ii) $\text{m} = -\frac{4}{5}$

iii) $y - 1 = -\frac{4}{5}(x + 3)$
$\therefore 5y - 5 = -4x - 12$
$\therefore 4x + 5y + 7 = 0$

iv) $d_\perp = \frac{|4\times0+5\times5+7|}{\sqrt{4^2+5^2}} = \frac{32}{\sqrt{41}}$

v) $AB = \sqrt{5^2 + 4^2} = \sqrt{41}$
$Area = \frac{1}{2} \times \sqrt{41} \times \frac{32}{\sqrt{41}} = 16\ (u^2)$

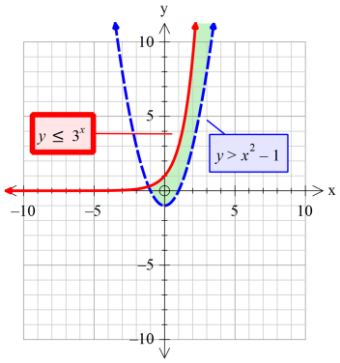
Q10)

a) $x - 4 = 2x + 1$ or $-x + 4 = 2x + 1$
$\therefore x = -5$ or $x = 1$

b) i) In the Δ's WXY & YZV
$\angle XWV = \angle VYZ\ (alt\ \angle's, WX//YZ)$
$\angle WVX = \angle YVZ\ (vert\ opp.)$
$\angle WXV = \angle YZV\ (\angle's\ in\ \Delta)$
$\therefore \Delta WXV \ \|\|\| \ \Delta YZV$

ii) $\frac{WX}{50} = \frac{18}{40} \quad \therefore WX = 22.5$

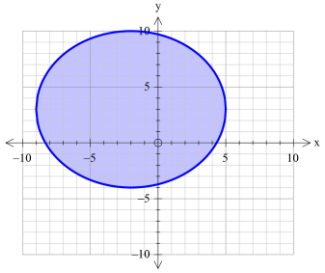
c)



Q11)

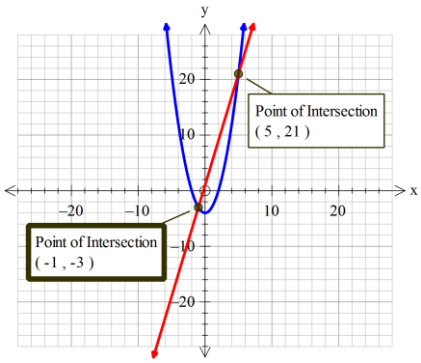
a) i) $x^2 + 4x + 4 + y^2 - 6y + 9 = 36 + 4 + 9$
$\therefore (x + 2)^2 + (y - 3)^2 = 49$
$\therefore$ Circle: Centre $(-2,3)$ Radius $= 7$

ii)



b) $Let\ u = 2^x, \ \therefore 2u^2 - u = 0$
$\therefore u(2u - 1) = 0$
$\therefore u = 0$ or $\frac{1}{2}, \ \therefore 2^x = 0$ or $\frac{1}{2}$,
But $2^x = 0$ has no solutions.
$\therefore x = -1$

c) i)



ii) $y = x^2 - 4$ and $y = 4x + 1$
$\therefore x^2 - 4 \ = 4x + 1$
$\therefore x^2 - 4x - 5 = 0$
$\therefore (x + 1)(x - 5) = 0$
$\therefore x = -1$ or 5
$\therefore (-1, -3)$ or $(5,21)$

Q12)

a) $-192x^3 - 3 = \ -3(64x^3 + 1)$
$= \ -3(4x + 1)(16x^2 - 4x + 1)$

b) Circle $x^2 + y^2 = 4 \ \therefore$
centre $(0,0)$ radius 2
But perpendicular distance from (0,0) to the line $3x + 4y - 12 = 0$
$d_\perp = \frac{|3\times0+4\times0-12|}{\sqrt{3^2+4^2}} = \frac{12}{5} > 2$
$\therefore$ no point(s)of interscetion

c) i) Since $AP = BQ = CR$ then $BP = QC = DR$

In the Δ's PBQ & QCR
$PB = QC\ (above)$
$\angle PBQ = \angle QCR\ (prop.\ of\ square)$
$BP = CR\ (given)$
$\therefore \Delta PQB \equiv \Delta QCR$ (SAS)

ii) Let $\theta = \angle BQP$
$\therefore \angle BPQ = 90 - \theta$
$(\angle's\ in\ \Delta\ since\ \angle PBQ = 90)$
$\therefore \angle RQC = 90 - \theta$
$(\text{corr.}\angle's\ in\ cong.\Delta's)$
$\therefore \angle CQR + \angle RQP + \angle BQP = 180$
$\therefore \angle CQR = 180 - (90 - \theta) - \theta$
$\therefore \angle CQR = 90°$