

2017 Year 11
Semester 2 Exam

Mathematics (2 Unit)

General Instructions

- Approved calculators may be used.
- Start each question in a new booklet.
- Answers not supported with working may not attract full marks.
- Marks may not be awarded for carelessly arranged work.
- Write using a black or blue pen
- A reference sheet with mathematical formulae is provided

Time Allowed

2 hours + 3 minutes reading time

Answer questions 1-5 on the multiple choice answer sheet provided

Multiple Choice (1 Mark each)

Question 1

If $x = a\left(b - \frac{1}{y}\right)$ then

A) $y = \frac{a}{b-x}$

B) $y = \frac{a}{ab-x}$

C) $y = \frac{1}{ab-x}$

D) $y = \frac{x}{a} - b$

Question 2

The solution to the inequality

$6 - x - x^2 \leq 0$ is

A) $-3 \leq x \leq 2$

B) $x \leq -2$ or $x \geq 3$

C) $x \leq -3$ or $x \geq 2$

D) $-2 \leq x \leq 3$

Question 3

What is the perpendicular distance between the lines $y = x + 5$ and $x - y = 3$

A) $4\sqrt{2}$

B) 4

C) $\sqrt{2}$

D) $2\sqrt{2}$

Question 4

Which of the following trigonometric expressions is equivalent to $\tan(90-x)^\circ$?

- A) $\tan x^\circ$
- B) $\cot x^\circ$
- C) $-\tan x^\circ$
- D) $-\cot x^\circ$

Question 5

The midpoint of $A(x, 2), B(-5, y)$ is $M(8, -10)$

The value of x, y is,

- A) $x = -21, y = 22$
- B) $x = 21, y = 22$
- C) $x = -21, y = -22$
- D) $x = 21, y = -22$

Question 6 (12 Marks)

Marks

- a) Find the value of y such that

$$2\sqrt{50} - \sqrt{72} = \sqrt{y} \quad (2)$$

- b) Express with a rational denominator

$$\frac{\sqrt{2}}{\sqrt{3}+1} \quad (2)$$

- c) Solve the equation $2\cos\theta = -1$ for $0^\circ \leq \theta \leq 360^\circ$ (2)

- d) Find $\lim_{x \rightarrow -4} \frac{x^2 + x - 12}{x + 4}$ (2)

- e) Graph the region ($\frac{1}{3}$ page size) where the inequations hold simultaneously
 $y > x^2$ and $y \geq x + 6$ (3)

- f) Show that $\cos\theta \tan\theta = \sin\theta$ (1)

Question 7 (12 Marks)

Marks

a) State the domain and range of $y = \sqrt{x-2}$ (2)

b) If $f(x) = \begin{bmatrix} x^2 - 1 & x \geq 1 \\ 2 - x & -1 < x < 1 \\ \frac{1}{x} & x \leq -1 \end{bmatrix}$

Find the value of $2f(3) - f(-3)$ (2)

c) Simplify $\frac{16}{2^{3x} \times 8^{1-x}}$ (2)

d) Draw separate sketches ($\frac{1}{3}$ page size) of the following. Show any intercepts with the axes, and any asymptotes.

i) $y = 2x - 1$ (1)

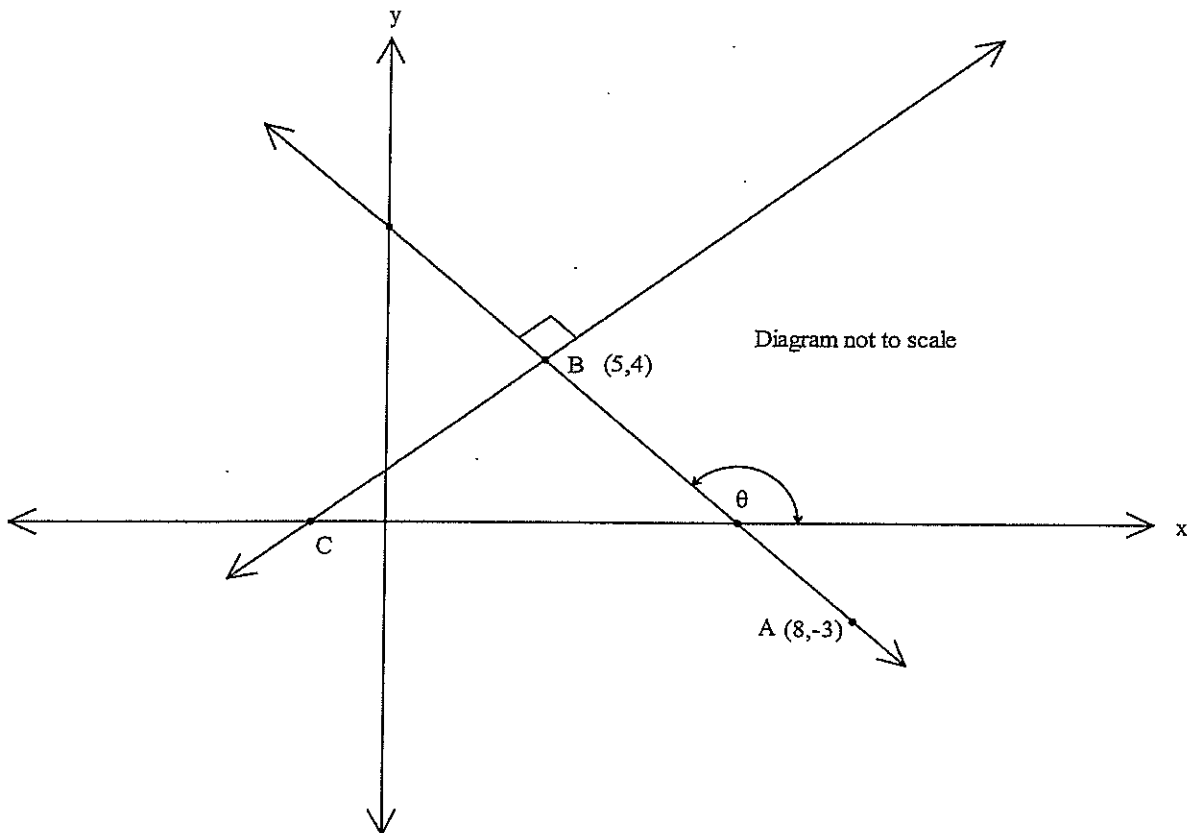
ii) $y = -\sqrt{4 - x^2}$ (2)

iii) $y = 2^x + 1$ (3)

Question 8 (12 Marks)

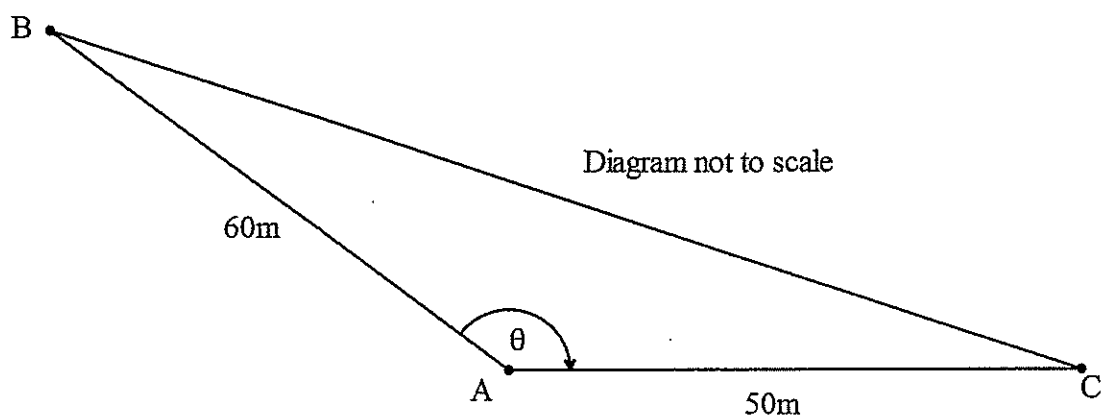
Marks

- a) The points $A(8, -3)$ and $B(5, 4)$ are shown in the diagram below. The line through AB makes an angle of θ with the positive x axis and the point C lies on the x axis.



- i) Find the gradient of the line AB (1)
- ii) Find the value of θ to the nearest degree. (1)
- iii) Find the coordinates of C given $AB \perp BC$ (2)
- iv) Find the coordinates of M , the midpoint of AB (1)
- v) Find the equation of the line through M parallel to the line BC in general form (2)

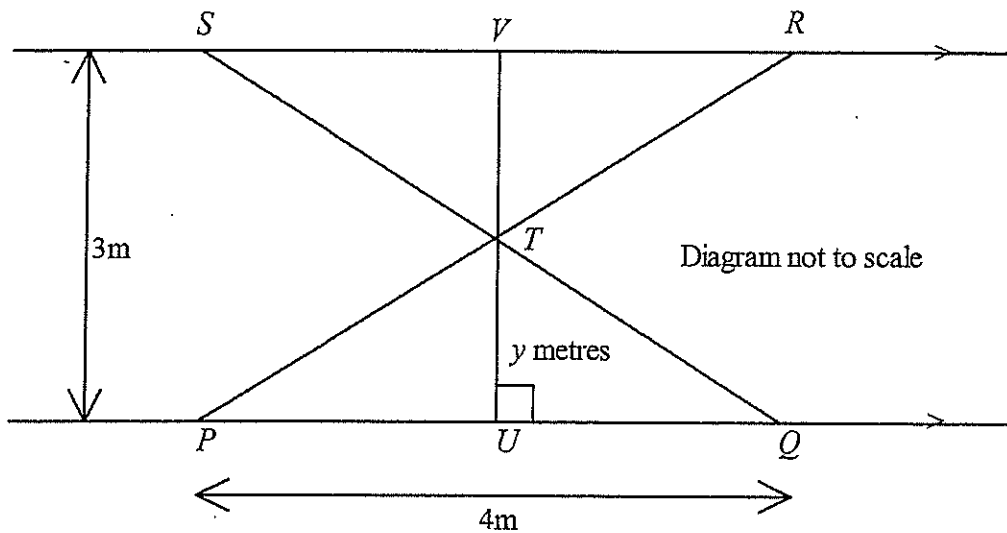
- b) Two sides of a triangular field are $AB = 60$ m, $AC = 50$ m and the included angle $\angle BAC = \theta$ is obtuse.



- i) If the area of the triangle is 750m^2 , find the size of $\angle BAC$ (2)
- ii) Find the length of the side BC (1 decimal place) (2)
- c) Make b the subject if $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ (1)

Question 9 (11 Marks)

Marks



a) In the diagram PQ and SR are parallel railings 3 metres apart. The points P and Q are fixed 4 metres apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T perpendicular to PQ intersects PQ at U and SR at V . The length UT is y metres.

i. By using similar triangles, or otherwise, show that $\frac{SR}{PQ} = \frac{VT}{UT}$ (3)

ii. Show that $SR = \frac{12}{y} - 4$ (2)

iii. Find the total area, A of $\triangle PTQ$ and $\triangle RTS$ in terms of y in simplest form (3)

b) Solve for x

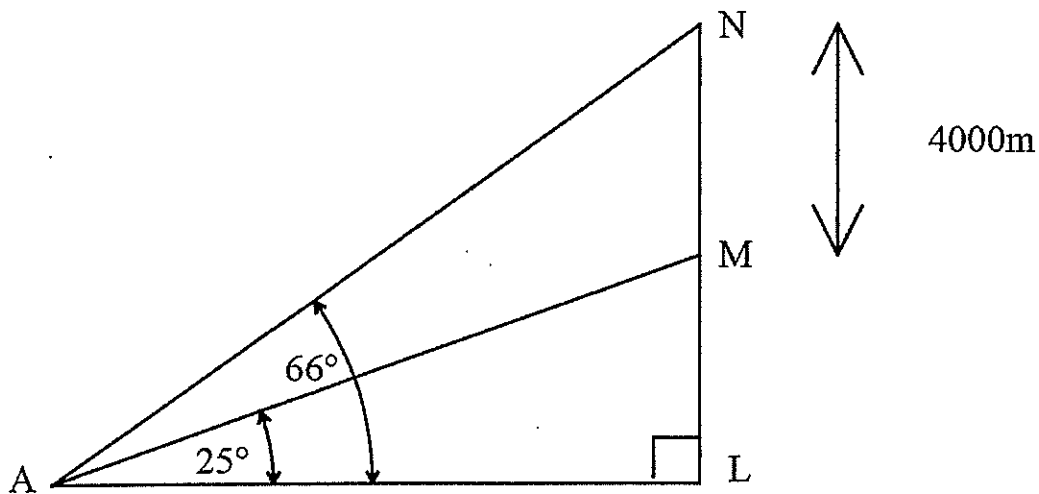
$$\frac{1}{2\sqrt{5} + \sqrt{x}} + \frac{1}{2\sqrt{5} - \sqrt{x}} = 2\sqrt{5}$$

(3)

Question 10 (11 Marks)

Marks

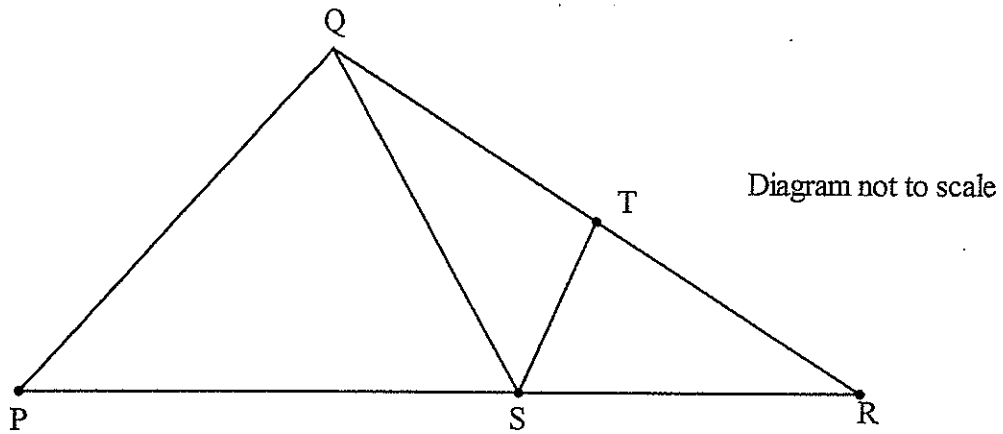
a) A rocket launched vertically from L is observed on from A . Soon after launch when at position M its angle of elevation is 25° . After it climbs 4000 metres from this position to N its angle of elevation is 66°



- i) Find $\angle ANL$ (1)
- ii) Find length of AM (to nearest metre) (2)
- iii) Find how far the observer is from the launch pad (to nearest metre) (2)

Question 10 (continued)

b)



In $\triangle PQR$, T lies on side QR and S lies on side PR such that
 $QT = TR$, $QS = QP$, $ST \perp QT$

- i) Copy the diagram into your answer booklet and show all the given information. (1)
- ii) Prove that $\triangle QTS \cong \triangle RTS$ (2)
- iii) Prove that $\angle QPS = 2\angle TQS$ (3)

Question 11 (11 Marks)

Marks

a. Consider the circle $(x-1)^2 + (y-2)^2 = 18$

i) Find the centre of the circle and its radius (2)

ii) Show that the line $y = x - 5$ is tangent to the circle (3)

b. Consider the function $g(x) = \frac{2x}{x^2 - 1}$

i) State the domain of this function (1)

ii) Determine with justification whether the function is odd, even, or neither (2)

c. Given $f(x) = \sqrt{5x-25} - \sqrt{x-1}$ find the value of x for which $f(x) = 2$. (3)

Question 12 (11 Marks)

Marks

a. A regular polygon has an exterior angle of 20° .
How many sides does the polygon have? (1)

b. Prove that $\cos^2(90^\circ - \theta) \cot \theta = \sin \theta \cos \theta$ (2)

c. Show that $\lim_{x \rightarrow \infty} \frac{4x^2 - x^3 + 2}{3x^3 - x^2 + 1} = \frac{-1}{3}$ (2)

d. Find the equation of the line passing through the point of intersection of the lines
 $4x - 2y + 3 = 0$ and $x + 4y + 6 = 0$ and which has a gradient of $\frac{2}{3}$. (3)

e. A large tank can be filled by 2 similar small pumps and 1 larger pump working together in
1 hour and 12 minutes. The larger pump B alone takes 1 hour less than the smaller pump A
alone to fill the tank. Find out how long each pump takes, given that $\frac{V}{P} = T$ where
 $V =$ Volume of the tank, $P =$ Pump, $T =$ Time (3)

END OF EXAM