CHELTENHAM GIRLS HIGH SCHOOL

YEAR 11
YEARLY EXAMINATION
2009

Mathematics

General Instructions

o Working time: $1\frac{1}{2}$ hours. o Write using black or blue pen. o Board-approved calculators may be used. o All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work. o Each **question** is to be started on a **new page**. o Write your **name** on every page.	**Total Marks - 75** o Attempt all Questions 1-5 o All questions are of equal value.

Name :________________________ Class Teacher __________________

Student Number : ____________________________

Q1	Q2	Q3	Q4	Q5	Total	%
/15	/15	/15	/15	/15	/75	

Answer each question on a new page

Question 1 (15 marks) Start on a new page **Marks**

(a) Find, correct to two decimal places, the value of $\dfrac{(4.32)^2}{7.53-2.48}$ **1**

(b) Factorise $2x^2 + 7x - 15$ **2**

(c) Find integers a and b such that $(2-\sqrt{3})^2 = a - b\sqrt{3}$ **3**

(d) Express $\dfrac{3x-2}{5} - \dfrac{x+1}{2}$ as a single fraction in its simplest form **2**

(e) Given that $\cos\theta = \dfrac{-3}{4}$ and $\sin\theta > 0$, find the <u>exact</u> value of $\tan\theta$: **2**

(f) Differentiate

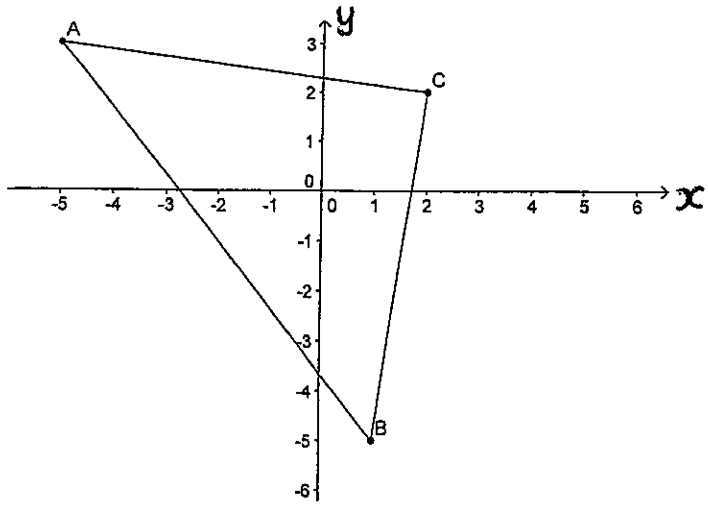
(i) $y = 6 - 3x^4$ **1**

(ii) $y = \dfrac{x+1}{x-2}$ **2**

(iii) $y = \sqrt{(3x-2)^3}$ **2**

Question 2 (15 marks) Start on a new page **Marks**

(a) A, B and C are three points on a number plane whose coordinates are (-5,3), (1,-5) and (2,2) respectively.



(i) Calculate the length of AB **2**

(ii) Find the equation of the line AB **2**

(iii) The line through C, perpendicular to AB, meets AB at N. Find the equation of CN **2**

(b) Solve $2x^2 < x$ **3**

(c) The equation $2x^2 - x + 4 = 0$ has roots α and β. Find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **2**

(iv) $\alpha^2 + \beta^2$ **2**

Question 3 (15 marks) Start on a new page **Marks**

(a) Differentiate $y=(2x^2+3)(5-x)$ **2**

(b) Solve $|2x-4|=3x-1$ **3**

(c) The roots of the quadratic equation $mx^2-20x+n=0$ are 3 and -5. **3**
Find the values of m and n, where m and n are both integers.

(d) A function is defined by the following rule

$$f(x)=\begin{cases} 0 & \text{if} \quad x \le -2 \\ -1 & \text{if} \quad -2 < x < 0 \\ x & \text{if} \quad x \ge 0 \end{cases}$$

Find:

(i) $f(-3)+f(-1)-f(0)$ **2**

(ii) $f(a^2)$ **1**

(e) Sketch the region bounded by $x^2+y^2<16$ and $2x-3y+6\ge 0$ **4**

Question 4 (15 marks) Start on a new page **Marks**

(a) Find the centre and the radius of the circle with equation **3**

$x^2 + y^2 - 18y - 40 = 0$

(b) Solve $4^x - 5 \times 2^x - 24 = 0$ **3**

(c) Find $\lim_{x \to 4} \dfrac{3x^2 - 48}{x - 4}$ **2**

(d) Given $f(x) = 5x^2 - 15x - 12$

(i) Find the equation of its axis of symmetry. **1**

(ii) Find the minimum value of the function **1**

(e) Find the <u>exact</u> value of :

(i) $\tan 60^0$ **1**

(ii) $\sin 240^0$ **1**

(f) Find the values of k for which $f(x) = 3x^2 - (k+2)x + (k+2)$ is positive definite . **3**

Question 5 (15 marks) Start on a new page **Marks**

(a) The focus of a parabola is (2,-5) and its directrix is the line $y = -1$

(i) Sketch the parabola and indicate the coordinates of the vertex V. **3**

(ii) Write down the focal length of the parabola. **1**

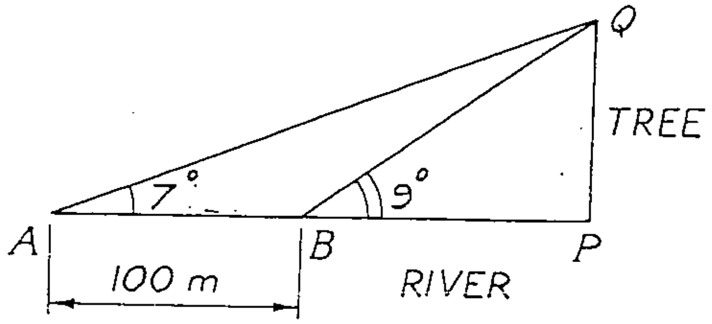
(iii) Find the equation of the parabola. **2**

(b) The function $f(x)$ is defined by $f(x) = \dfrac{1}{x^2+1}$

(i) Find $f'(x)$ **2**

(ii) Find the coordinates of the stationary point and determine its nature. **2**

(c)



The diagram above was sketched by a surveyor, who measured the angle of elevation of a tree top on the other side of the river to be 7^0 at the point A. At point B, 100metres directly towards the tree from A, the angle of elevation was 9^0 .
Let the height of the tree PQ be h and the width of the river BP be r

(i) Write an expression for r in terms of h. **1**

(ii) Derive an expression for h independent of r. **3**

(iii) Calculate the height of the tree correct to three significant figures. **1**

END OF EXAM

Year 11, Yearly Examination solutions - 2009 Mathematics

Question 1

(a) $\dfrac{(4\cdot32)^2}{7\cdot53-2\cdot48}$

$= \dfrac{18\cdot6624}{5\cdot05}$

$= 3\cdot70$

(b) $2x^2+7x-15$

$(2x-3)(x+5)$

(c) $(2-\sqrt{3})^2 = a-b\sqrt{3}$

$4-4\sqrt{3}+3 = a-b\sqrt{3}$

$7-4\sqrt{3} = a-b\sqrt{3}$

$a=7$, $b=4$

(d) $\dfrac{3x-2}{5} - \dfrac{x+1}{2}$

$\dfrac{(3x-2)}{5} - \dfrac{(x+1)}{2}$

$= \dfrac{2(3x-2)-5(x+1)}{10}$

$= \dfrac{6x-4-5x-5}{10}$

$= \dfrac{x-9}{10}$

(e) $\cos\theta = \dfrac{-3}{4}$, $\sin\theta > 0$

$\tan\theta = -\dfrac{\sqrt{7}}{3}$

(f) (i) $y = 6-3x^4$

$\dfrac{dy}{dx} = -12x^3$

(ii) $y = \dfrac{x+1}{x-2}$

$\dfrac{dy}{dx} = \dfrac{(x-2)-(x+1)}{(x-2)^2}$

$= \dfrac{x-2-x-1}{(x-2)^2}$

$= \dfrac{-3}{(x-2)^2}$

(iii) $y = \sqrt{(3x-2)^3}$

$y = (3x-2)^{\frac{3}{2}}$

$\dfrac{dy}{dx} = \dfrac{3}{2}(3x-2)^{\frac{1}{2}} \times 3$

$= \dfrac{9}{2}(3x-2)^{\frac{1}{2}}$

$= \dfrac{9\sqrt{3x-2}}{2}$

Question 2

(a)

(i) $A = (-5,3)$, $B = (1,-5)$

$$AB = \sqrt{(1 - {}^-5)^2 + (-5-3)^2}$$
$$= \sqrt{(6)^2 + (-8)^2}$$
$$= \sqrt{100}$$
$$= 10 \text{ units.}$$

(ii) $m_{AB} = \dfrac{-5-3}{1-{}^-5} = -\dfrac{8}{6} = \dfrac{-4}{3}$

using point $A = (-5,3)$

$$(y-3) = -\frac{4}{3}(x - {}^-5)$$
$$3(y-3) = -4(x+5)$$
$$3y - 9 = -4x - 20$$
$$4x + 3y + 11 = 0$$

$\therefore$ Equation of Line AB is $4x+3y+11=0$.

(iii) $m_{CN} = \dfrac{3}{4}$ $(CN \perp AB)$

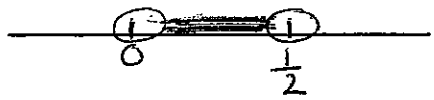
using point $C = (2,2)$

$$(y-2) = \frac{3}{4}(x-2)$$
$$4(y-2) = 3(x-2)$$
$$4y - 8 = 3x - 6$$
$$3x - 4y + 2 = 0$$

$\therefore$ Equation of Line CN is $3x-4y+2=0$

(b) $2x^2 < x$

$$2x^2 - x < 0$$
$$x(2x-1) < 0$$



$$0 < x < \frac{1}{2}$$

(c) $2x^2 - x + 4 = 0$

(i) $\alpha + \beta = -\dfrac{(-1)}{2} = \dfrac{1}{2}$

(ii) $\alpha\beta = \dfrac{4}{2} = 2$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

$$= \frac{\beta + \alpha}{\alpha\beta}$$
$$= \frac{1}{2} \div 2$$
$$= \frac{1}{4}$$

(iv) $\alpha^2 + \beta^2$

$$= (\alpha+\beta)^2 - 2\alpha\beta$$
$$= \left(\frac{1}{2}\right)^2 - 2(2)$$
$$= \frac{1}{4} - 4$$
$$= -3\frac{3}{4}$$

Question 3

(a) $y = (2x^2+3)(5-x)$

$$\frac{dy}{dx} = 4x(5-x)+(-1)(2x^2+3)$$

$$= 20x - 4x^2 - 2x^2 - 3$$

$$= -6x^2 + 20x - 3$$

(b) $|2x-4| = 3x-1$

$(2x-4) = (3x-1)$ or $(2x-4) = -(3x-1)$

$2x-4 = 3x-1$
$-x = 3$
$x = -3$

$2x-4 = -3x+1$
$5x = 5$
$x = 1$

Check
$x = -3$
$|2x-4| = 3x-1$
$|2(-3)-4| = 3(-3)-1$
$|-10| = -10$
$10 = -10$
$\therefore x = -3$ is not a solution.

Check
$x = 1$
$|2x-4| = 3x-1$
$|2(1)-4| = 3(1)-1$
$|2-4| = 3-1$
$|-2| = 2$
$2 = 3$ $x = 1$ is a solution.

$\therefore$ the solution is $x = 1$

(c) $mx^2 - 20x + n = 0$

roots $= \alpha, \beta$.

$$\alpha+\beta = -\frac{(-20)}{m} = 3 + -5$$

$$\frac{20}{m} = -2$$

$-2m = 20$
$m = -10$

$$\alpha\beta = \frac{n}{m} = 3\times(-5)$$

$$\frac{n}{m} = -15$$

$$\frac{n}{-10} = -15$$

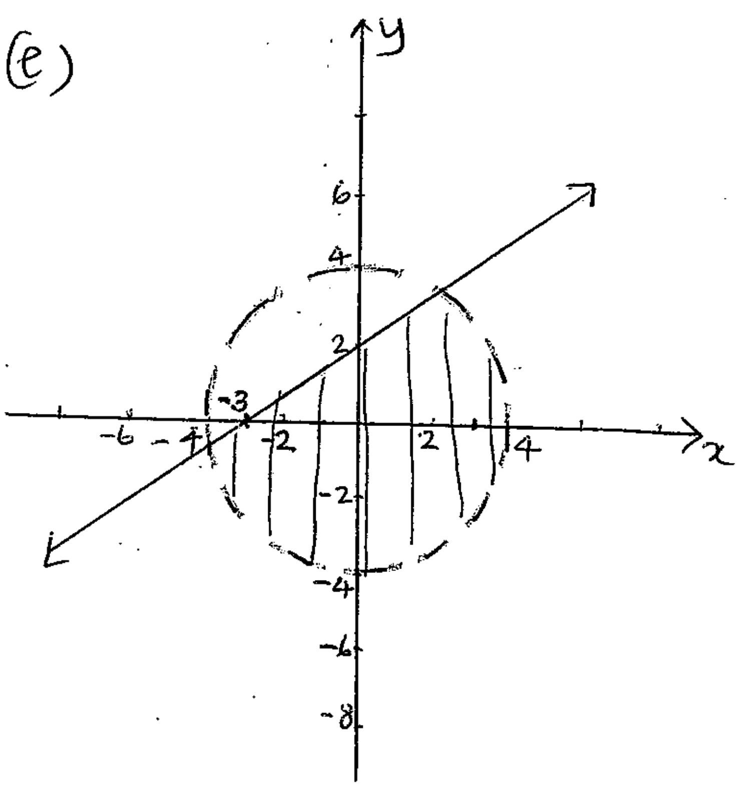
$n = +150$

$\therefore m = -10$ and $n = +150$

(d) (i) $f(-3)+f(-1)-f(0)$
$= 0 + -1 + 0$
$= -1$

(ii) $f(a^2) = a^2$
since $a^2 > 0$

(e)



Question 4

(a) $x^2 + y^2 - 18y - 40 = 0$

$x^2 + y^2 - 18y + 81 = 40 + 81$

$(x-0)^2 + (y-9)^2 = 121$

radius = 11 units.

centre = $(0, 9)$

(b) $4^x - 5 \times 2^x - 24 = 0$

$(2^x)^2 - 5(2^x) - 24 = 0$

take $y = 2^x$

$y^2 - 5y - 24 = 0$

$(y+3)(y-8) = 0$

$y = -3$ or $y = 8$

$2^x = -3$ or $2^x = 8$

$2^x \neq -3$ $\quad$ $2^x = 2^3$

$\therefore x = 3$ is the solution.

(c) $\lim\limits_{x \to 4} \dfrac{3x^2 - 48}{x - 4}$

$= \lim\limits_{x \to 4} \dfrac{3(x^2 - 16)}{x - 4}$

$= \lim\limits_{x \to 4} 3(x+4)$

$= 3(4+4)$

$= 3 \times 8$

$= 24$

(d) $f(x) = 5x^2 - 15x - 12$

(i) axis of symmetry:

$x = \dfrac{-b}{2a}$

$x = \dfrac{-(-15)}{2(5)}$

$= \dfrac{15}{10}$

$x = \dfrac{3}{2}$

(ii) minimum value

$= 5\left(\dfrac{3}{2}\right)^2 - 15\left(\dfrac{3}{2}\right) - 12$

$= -23\dfrac{1}{4}$

(e) (i) $\tan 60^\circ = \sqrt{3}$

(ii) $\sin 240$

$= \sin(180 + 60)$

$= -\sin 60$

$= -\dfrac{\sqrt{3}}{2}$

(f) $f(x) = 3x^2 - (k+2)x + (k+2)$

$a = 3 > 0$, $\Delta < 0$

$\Delta = [-(k+2)]^2 - 4(3)(k+2) < 0$

$= k^2 + 4k + 4 - 12k - 24 < 0$

$= k^2 - 8k - 20 < 0$

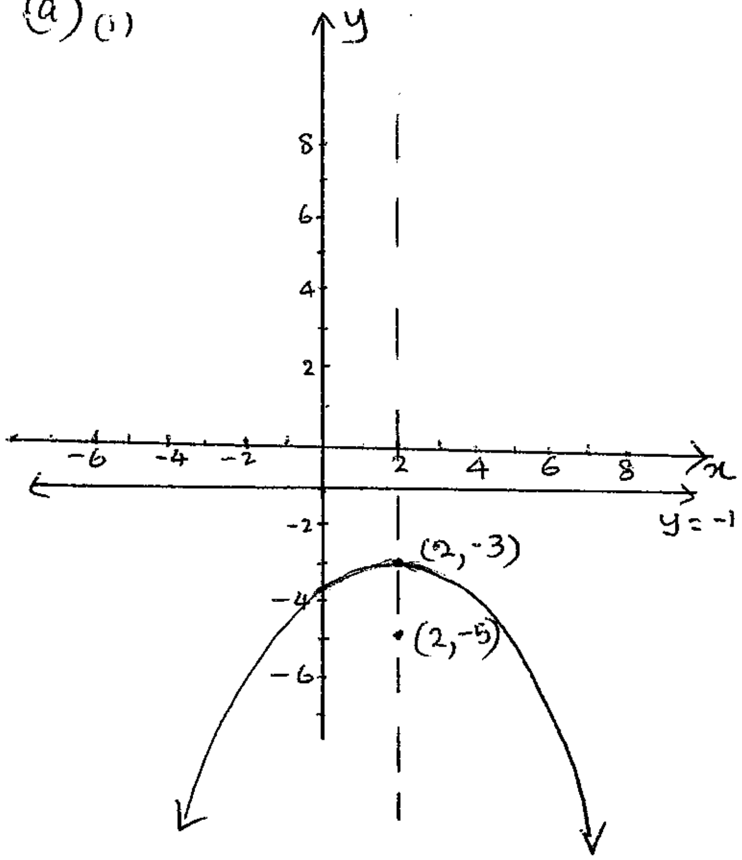
$k^2 - 8k - 20 < 0$

$(k-10)(k+2) < 0$

$-2 < k < 10$

Question 5

(a) (i)



vertex = $(2, -3)$

(ii) Focal length = 2

$a = 2.$

(iii) $(x-h)^2 = -4a(y-k)$

$(x-2)^2 = -4(2)(y - {-3})$

$(x-2)^2 = -8(y+3)$

(b) (i) $f(x) = \frac{1}{(x^2+1)}$

$f(x) = (x^2+1)^{-1}$

$f'(x) = -(x^2+1)^{-2} \times 2x$

$= \frac{-2x}{(x^2+1)^2}$

(b)(ii) At stationary points

$f'(x) = 0$

$\frac{-2x}{(x^2+1)^2} = 0$

$-2x = 0$

$x = 0.$

x	-1	0	1
$f'(x)$	$\frac{2}{4}$	0	$-\frac{2}{4}$
	/	—	\

The turning point is a maximum at $(0, 1)$

(c)(i) $\tan 9° = \frac{h}{r}$

$\therefore r = \frac{h}{\tan 9°}$

(ii) using Δ BPQ

$r = \frac{h}{\tan 9°}$

using Δ APQ

$\tan 7° = \frac{h}{r+100}$

$r + 100 = \frac{h}{\tan 7°}$

$r = \frac{h}{\tan 7°} - 100$

(c) (ii) continued

$$r = \frac{h}{\tan 9^\circ} \longrightarrow (1)$$

$$r = \frac{h}{\tan 7^\circ} - 100 \rightarrow (2)$$

$(2) = (1)$

$$\frac{h}{\tan 7^\circ} - 100 = \frac{h}{\tan 9^\circ}$$

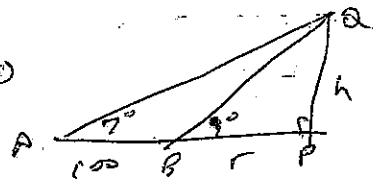
$$\frac{h}{\tan 7^\circ} - \frac{h}{\tan 9^\circ} = 100$$

$$[\tan 9^\circ - \tan 7^\circ]h$$
$$= 100 \times \tan 9^\circ \times \tan 7^\circ$$

$$h = \frac{100 \tan 9^\circ \tan 7^\circ}{[\tan 9^\circ - \tan 7^\circ]}.$$

(iii) 54.6 m.

c) (i)



(i) $\tan 9^\circ = \frac{h}{r}$

$\therefore r = \frac{h}{\tan 9^\circ}$

(ii) In ΔAPQ $\tan 7^\circ = \frac{h}{100+r}$

$= \frac{h}{\cancel{100}\,9^\circ + \frac{h}{\tan 9^\circ}}$

$\tan 7^\circ\left(100 + \frac{h}{\tan 9^\circ}\right) = h$

$\therefore h = 100 \tan 7^\circ + h\frac{\tan 7^\circ}{\tan 9^\circ}$

$\therefore h\left(1 - \frac{\tan 7}{\tan 9}\right) = 100 \tan 7^\circ$

$$h = \frac{100 \tan 7^\circ}{1 - \frac{\tan 7^\circ}{\tan 9^\circ}}$$

$= 54.627\ldots \doteq 54.6\text{ m}$